Temperature Distribution Analysis of Circular Cross Section Fin by Analytical and Finite Difference Method using C++ Coding

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Abstract

This work presents a method, based on finite difference formulation for the determination of temperature distribution along the length of the fin with uniform circular cross-section. The temperature values that are obtained are considered for two different boundary conditions, adiabatic and convective tips. Numerical and analytical investigation is carried out and the temperature distributions are plotted and observed. And this process is repeated for different materials having different thermal properties; by varying the thermal parameters of these materials their temperature distribution is analyzed. Here pin fin having a diameter of 20 mm and length of 200 mm is considered for evaluation (l>>d).

Keywords- *Circular fin, Numerical, Analytical, Adiabatic tip, Convective tip, Uniform, FDM, Microsoft visual C++.*

I. INTRODUCTION

The uniform extended surface is investigated in this paper. In the study of heat transfer, a Fin is a surface that extends from an object to increase the rate of heat transfer to or from the environment by increasing convection. The amount of Heat conduction, convection, or radiation of an object determines the amount of heat it transfers.

Increasing the temperature difference between the object and the Natural environment, increasing the convection heat transfer coefficient, or increasing the surface area of the object increases the heat transfer. Sometimes it is not economical or it is not feasible to change the first two options. Adding a fin to an object, however, increases the surface area and can sometimes be an economical solution to heat transfer problems. Fins are probably the most common method of enhancing heat dissipation from a hot surface. The principle of operation is to provide a larger area over which convective heat transfer may occur than the original surface area. Pin fins are rather common for example, as cooling devices for micro chips.

Of course, optimization is a key concern, for reasons of weight, dimensions and cost. The fin shape and dimension that provides the most heat removal for a given addition in mass and bulk must be found. Generally, optimization involves some type of compromise. The pin fin of circular cross-section is the simplest type of fin for theoretical analysis.

In this analysis a numerical and analytical study using MICROSOFT VISUAL C++ will be conducted to study temperature distribution analysis through circular shaped fin.

In a recent times, majority of research has been done in the field of heat transfer .N.Nagarani[1] has analyzed (2010) the heat transfer rate and efficiency for circular and elliptical annular fins for different environmental conditions. She found that the elliptical fin efficiency is more than circular fin efficiency and an elliptical fin could be a good choice if there is a space restriction in one particular direction. Amol B. Dhumne[2] Has experimented (2013) on heat transfer enhancement and the corresponding pressure drop over a flat surfaces equipped with cylindrical cross sectional perforated pin fins in a rectangular channel. The experimental implementation shows that the use of cylindrical perforated pin fins leads to heat transfer enhancement than the solid cylindrical fins. Pooja P. Shirjose[15] has conducted numerical simulations for fins with various shapes of perforations. The size of the perforation for this work was considered as 6 mm. These fins were studied for different flow conditions characterized by Reynolds Number (Re = 21,000 to Re = 87,000). The numerical results were validated against the existing experimental data. Based on the results comparison, in terms of Nusselt Number, the elliptical perforations on the fins provide better heat transfer rate. Ibrahim GIRGIN, Cuneyt EZGI[14] has performed the numerical solutions for circular fin of rectangular profile using finite difference method, and the results are compared to the analytical solutions.

Y. Pratapa Reddy, B.Jithendra Kumar, D. Srinivasulu,, Dr.Ch. Srinivasa Rao[13] has experimented to find out the temperature distribution within the pin fin made of composite metals and steady state heat transfer analysis has been carried using a finite element software ANSYS to test and validate results. The temperature distribution at different regions of pin fin are evaluated by FEM and compared with the results obtained by experimental work. Ziad M. Al-Makhyoul [12] has performed on an analytical study of a two dimensional (radial and transverse) heat flow through annular fin variable thickness analytically depending on the group of resistances by derivating the major equations of conduction and convection .Yuh-Wei Chiu, Yi-Xiong Lin, Jiin-Yuh Jang [11] has performed the numerical and analytical analysis to study the thermal-hydraulic characteristics in elliptical finned-tube heat exchanger under the dry and air/water spray cooled system. . The numerical results indicated that the pressure drop of circular finned-tube heat exchanger is 3 times that of the elliptic finned-tube heat exchanger, while the heat transfer coefficient of the circular finned-tube is 1.51 times that of the elliptic finned-tube.

J.Y Jang and J.T Lai[9] has analysed numerically and experimentally of fluid flow and heat transfer over a four row circularfinned tube heat exchanger. Two types of finned tube configurations have been investigated under dry and wet conditions for different values of inlet frontal velocity ranging from 2 to 6m/s.The experimental results indicated that the sensible colburn factor for the wet coils is 20% higher than that for the dry coils; the friction factor f for the wet coils is 15% higher than that for the dry coil.

II. MATHEMATICAL MODEL

The temperature distribution of circular fin is calculated using the fact that the base of the fin is attached to the wall which is at temperature of 100^0C.

A. *Physical Model and Formulation.*

1) *Heat Flow Through a Straight Fin of Uniform Circular Cross Section:*

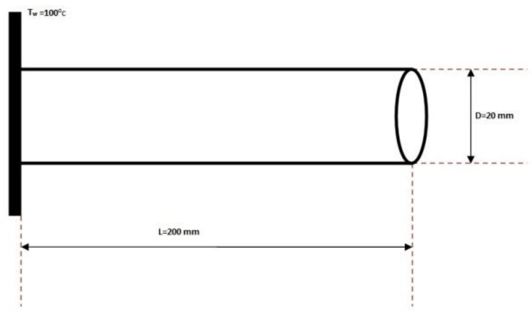


Fig.1. Fin taken for analysis

2)Abbreviations:

Here 'L' stands for the length by which the fin protrudes or extends from the primary surface is called protruding length.

K = thermal conductivity of fin material

h = convective heat transfer coefficient

T_∞ = ambient or surrounding temperature

$A = (\pi/4)D^2$ (cross sectional area of fin)

$P = \pi D$ (perimeter of fin)

Temperature is a function of 'x' direction

i.e., T = f(x) x = z (axial direction)

B. *Assumptions in Analysis of Fins*

(i) In the analysis of fins, we consider steady operation with no heat generation in the fin.

(ii) Thermal conductivity *k* of the material to remain constant.

(iii) We also assume the convection heat transfer coefficient *h* to be constant and uniform over the entire surface of the fin for convenience in the analysis.

(iv) We recognize that the convection heat transfer coefficient *h*, in general, varies along the fin as well as its circumference, and its value at a point is a strong function of the fluid motion at that point.

C. *Analytical Formulation*

1) *Fin with Adiabatic Tip*

Fins are not likely to be so long that their temperature approaches the surrounding temperature at the tip. A more realistic situation is for heat transfer from the fin tip to be negligible since the heat transfer from the fin is proportional to its surface area, and the surface area of the fin tip is usually a negligible fraction of the total fin area.

The temperature distribution along the length of the fin is given by:

$\theta/\theta_0 = [\ T_x - T_a\ ]/[\ T_w - T_a\ ] = \cosh[m(l-x)]/\cosh(ml)$.

The rate of heat flow from the fin is given by

$Q_{fin} = (\ T_w - T_a\ )\tanh(ml)\ \sqrt{(hPkA_{cs})}$.

2) *Heat dissipation from a fin losing heat at the tip (convective tip):*

The boundary conditions are

(i) At x = 0 , $\theta = \theta_0$

(ii) Heat conducted to the fin at x = l = heat convected from the end to the surroundings.

i.e. $-kA_{cs}[dt/dx]_{x=l} = h\,A_{su}(t - t_a)$

Where A_{cs} (cross sectional area for heat conduction) equals A_{su} (surface area from which the convective heat transport takes place), at the tip of the fin; i.e., $A_{cs} = A_{su}$.

Thus $dt/dx = -h\theta/k$ at x = l

Temperature distribution along the length of the fin is given by:

$\theta/\theta_0 = (t - t_a)/(t_0 - t_a) = \{ \cosh[m(l - x)] + (h/km)\sinh[m(l - x)] \}/\{ \cosh(ml) + (h/km)\sinh(ml) \}$

The rate of heat flow from the fin is given by

$Q_{fin} = \sqrt{(PhkA_{cs})}\,(t_0 - t_a)\{ \tanh(ml) +$

$(h/km) \}/\{ 1 + (h/km)\tanh(ml) \}$.

D. Finite Difference Formulation

Here the fin under analysis is discretized into N equal parts as shown in the figure given below

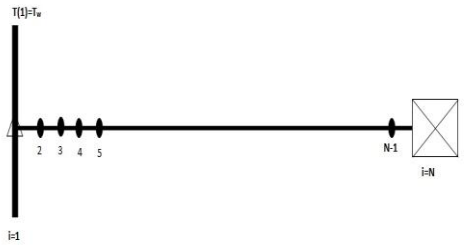


Fig.2. Discretized fin.

1) Adiabatic Tip

a) At i = 1

$T_i = T_w$

b) At i = 2 to N−1

c) $d^2T_i/dx^2 = 0$

⇨ $(T_{i+1} - 2T_i + T_{i-1})/(\Delta x)^2 = 0$

⇨ $T_i = (T_{i+1} + T_{i-1})/2$

⇨ $Q_{cond,in} = Q_{cond,out} + Q_{conv}$

⇨ $Q_{cond,in} = Q_{cond,in} + (d/dx)Q_{cond,in} + hA(T_i - T_a)$

⇨ $kA(d^2T_i/dx^2)\Delta x = hA(T_i - T_a)$

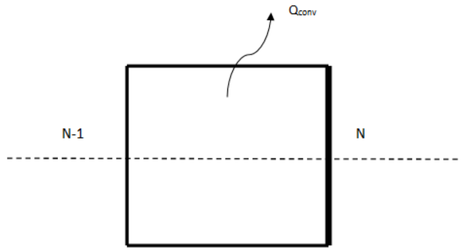
⇨ $k(\pi D^2/4)(d^2T_i/dx^2)\Delta x = h(\pi D)\Delta x(T_i - Ta)$

⇨ $d^2T_i/dx^2 = (4h/kD)(T_i - T_a)$

⇨ $(T_{i-1} - 2T_i + T_{i+1})/(\Delta x)^2 = (4h/kD)(T_i - T_a)$

⇨ $T_i = [T_{i-1} + T_{i+1} + \{4h(\Delta x)^2/kD\}T_a]/[2 + 4h(\Delta x)^2/kD]$.

d. For i = N



$Q_{cond} = Q_{conv}$

$-kA(dT_i/dx) = hA(T_i - T_a)$

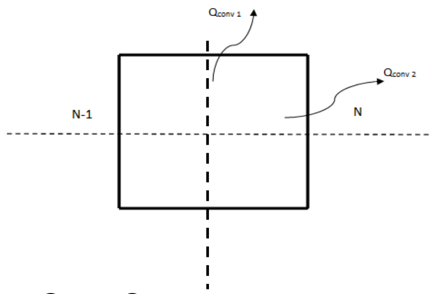
$-k(\pi D^2/4)[(3T_i - 4T_{i-1} + T_{i-2})/2(\Delta x)] = h(\pi D)(\Delta x/2)(T_i - T_a)$

$-3T_i + 4T_{i-1} - T_{i-2} = [8h(\Delta x)^2/kD](T_i - T_a)$

The temperature distribution for adiabatic tip is given by:

⇨ $T_i = [(4h/kD)(\Delta x)^2T_a + 4T_{i-1} - T_{i-2}]/[4h(\Delta x^2)/kD + 3]$.

2) Convective Tip



$Q_{cond,in} = Q_{conv} + Q_{cond,out}$

$-k(\pi D^2/4)dT/dx = h\pi D(\Delta x/2)(T_i - T_a) + h(\pi D^2/4)(T_i - T_a$

$dT/dx = -(4h/kD)(T_i - T_a)[\Delta x + d/4]$

$[3T_i - 4T_{i-1} + T_{i-2}]/2(\Delta x) = -(4h/kD)(T_i - T_a)[\Delta x + d/4]$

The temperature distribution for convective tip is given by:

⇨ $T_i = T_a[(2\Delta x)(4h/kD)\{(\Delta x/2) + (d/4)\} + 4T_{i-1} - T_{i-2}]/ [3 + (2\Delta x)(4h/kD)\{ (\Delta x/2) + d/4 \}]$.

III. RESULTS AND DISCUSSIONS

A. Comparison of Numerical and Analytical Results.

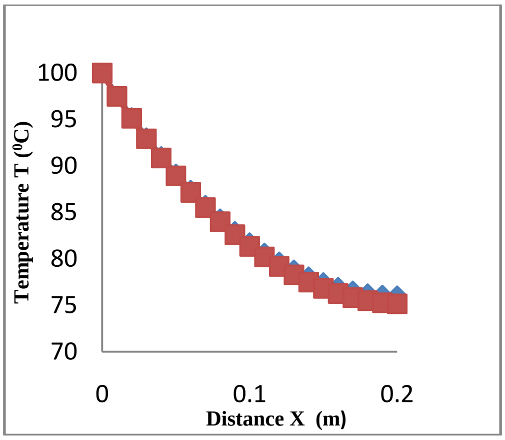


Fig.3.Temperature Profile for Adiabatic tip.

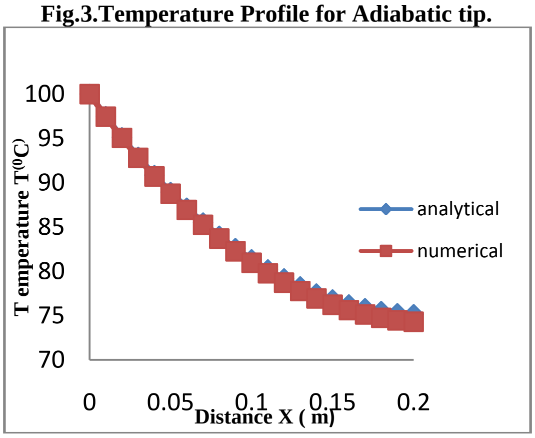


Fig.4. Temperature Profile for Convective Tip.

Above figures shows the relation between the temperature and the length of the fin. Analytical and numerical results are compared for adiabatic and convective tips. In both the figures analytical results are in good agreement with the numerical results.

B. Temperature Distribution Profiles For Different Materials.

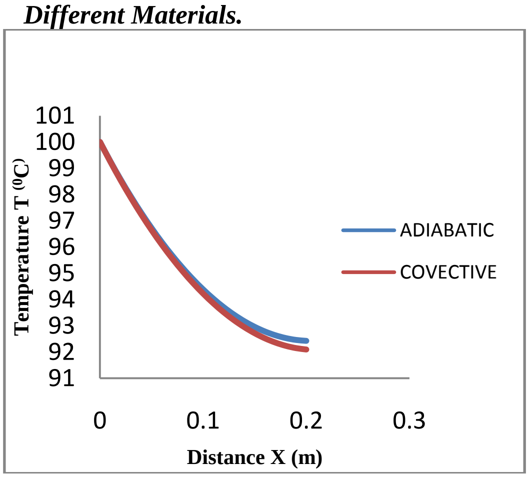


Fig.5.Temperature Profile for Alluminium Fin.

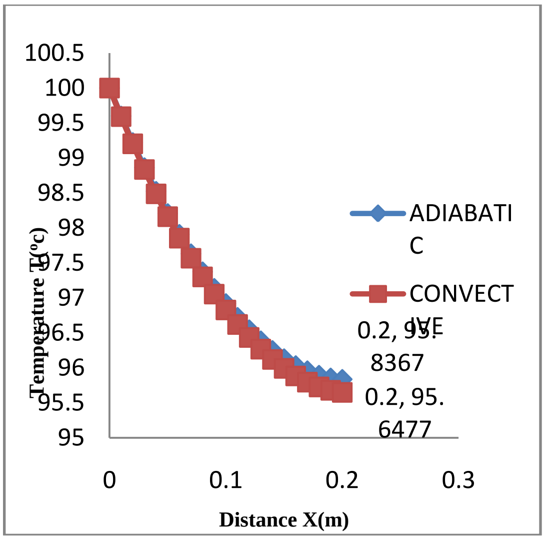


Fig.6.Temperature Profile for Copper Fin.

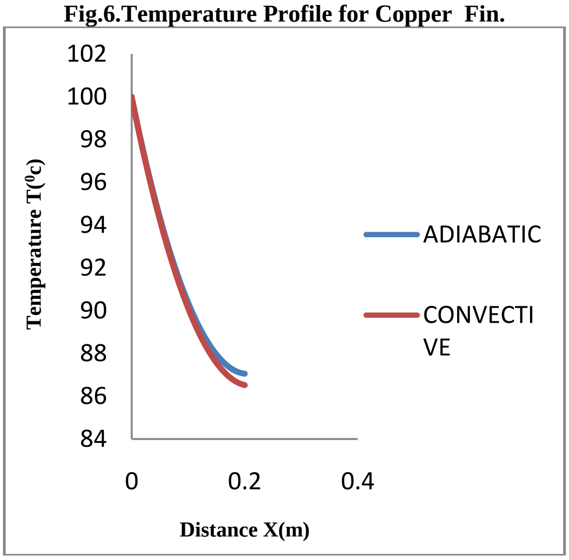


Fig.7.Temperature Profile for Zinc Fin.

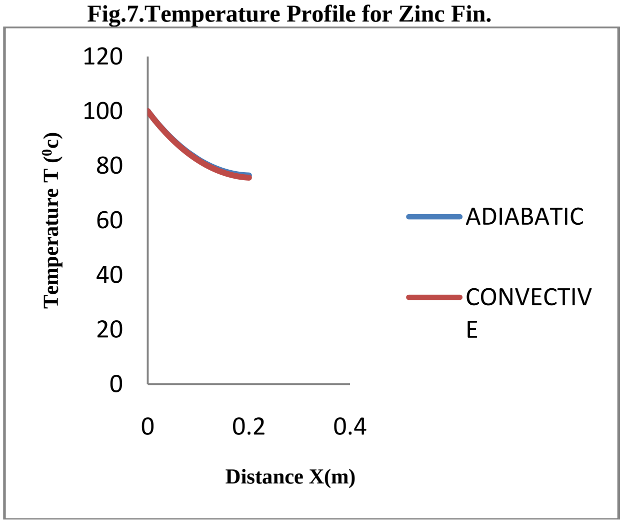


Fig.8.Temperature Profile for Carbon Steel Fin.

Above figures shows the relation between the temperature and the length of the fin for various materials. With increase in the value of the thermal conductivity of the fin material, the temperature at the tip of the fin move closer to the wall temperature, thus the rate of heat transfer increases. For half of the length of the fin for adiabatic tip fin and convective tip fin have approximately same values. For remaining half the length, the adiabatic tip fin had higher temperature than the convective tip fin . Higher thermal conductivity materials had more heat transfer rate than the lower thermal conductivity values.

C. Temperature Distribution Profile at Different Thermal Conductivities.

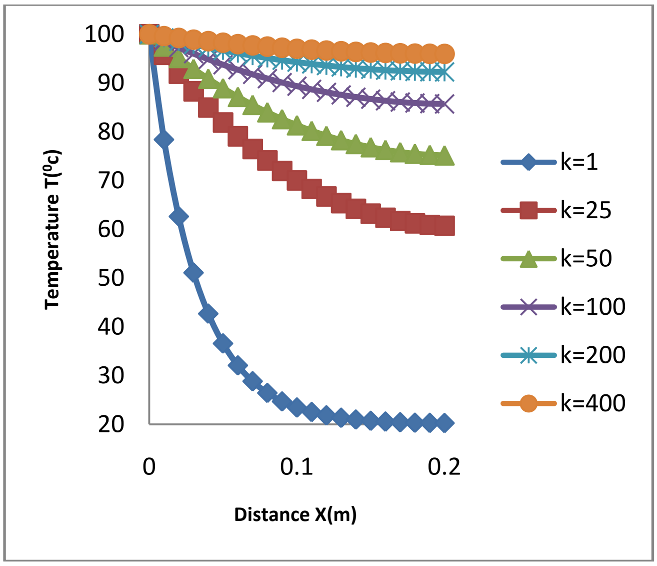


Fig.9.Temperature Profile for Adiabatic Tip.

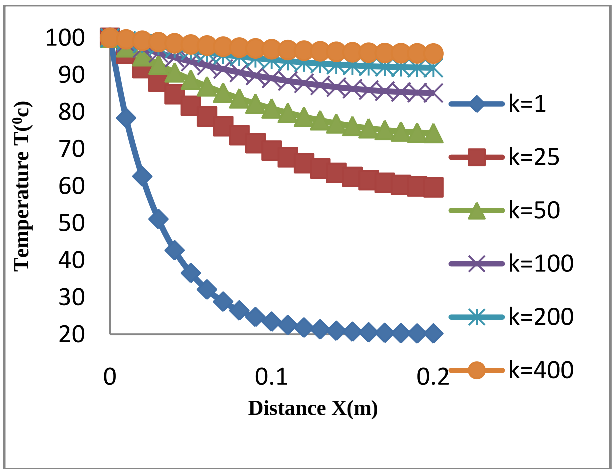


Fig.10.Temperature Profile for Convective Tip.

Above figures shows the temperature distribution of the fin at different thermal conductivities.

For both adiabatic tip and convective tip with increase in the value of the thermal conductivity the slope of the graph decreases and temperature at the tip of the fin increases. At the higher values of thermal conductivity, difference between the slopes of the graphs decreases. So at higher thermal conductivities, heat transfer rates are approximately same when compared at different thermal conductivities. Even though copper has high thermal conductivity than aluminium, Aluminium is used in most of the fin materials because of its less weight, less cost and also approximately same heat transfer rate as copper.

IV. CONCLUSIONS

1) With increase in value of thermal conductivity of fin material, the temperatures on surface on fin move closer to wall temperature of the material to which fin is attached. Thus the rate of heat transfer is more.

2) Root of the fin temperature is at wall surface temperature to which fin is attached. The temperature gradually decreases as we proceed to the tip of the fin.

3) Temperature distribution depends upon the thermal conductivity of the material, higher the value of it lower will be the temperature difference between wall and tip.

4) In all the cases, up to half the length of the fin the temperatures are approximately same for adiabatic tip and convective tip. For the remaining half of the length the temperature for the adiabatic tip will be more than the convective tip.

5) With increase in the thermal conductivity value, the slope of the graph decreases and the heat transfer rate increases. At higher thermal conductivities the difference between the slopes of the graph decreases. So heat transfer rates are approximately same at higher thermal conductivities.

6) Comparing the analytical and numerical results, the numerical results are in good agreement with the analytical results.

REFERENCES.

[1] N.Nagarani, K.Mayilsamy "Experimental heat transfer analysis on annular circular and elliptical fins" , International Journal of Engineering Science and technology Vol.2(7), 2010, 2839-2845.

[2] Amol B.Dhumne, Hemant S.Farkade "Heat transfer analysis of cylindrical perforated fins in staggered arrangement", International Journal of Innovative technology and Exploring Engineering (IJITEE) ISSN: 2278-3075, Vol.2, Issue-5, April 2013.

[3] L.A.0.Rocha, F.E.M.Saboya, J.V.C.Vargas "A comparative study of elliptical and circular sections in one and two row tubes and plate fin heat exchangers", International Journal of Heat and Fluid flow 18:247-252, 1997.

[4] Abdurrahman Moinuddin, Mostafa H.Sharqawy, Syed M.Zubair "Heat and mass transfer from annular fins of different cross sectional area. Part 2. Optimal dimensions of fins", International Journal of Refrigeration 35(2012) 377-385.

[5] Chee n Su An, Do Hyung Choi, "Analysis of heat transfer performance of cross-flow fin- tube heat exchangers under dry and wet conditions", International journal of Heat and Mass transfer 55 (2012) 1496-1504.

[6] Md. Farhad Ismail, M.O. Reza, M.A. Zobaer, Mohammad Ali "Numerical investigation of turbulent heat convection from solid and longitudinally perforated rectangular fins", International Conference on Thermal Engineering 56 (2013) 497-5.

[7] G.Zhang ,B.T.F Chung ,"Design Charts for Circular Fins of Arbitrary Profile Subject to Radiation and Convection with Wall Resistances", The Open Thermodynamics Journal, 2012, 6, 15-24.

[8] Sethuraman Ramasamy and Pakkirisamy Balashanmugam, "Thermal Performance Analysis of the Solar Water Heater with Circular and Rectangular Absorber Fins",IJISET -

International Journal of Innovative Science, Engineering & Technology, Vol. 2, Issue 1, January 2015.

[9] J.Y Jang and J.T Lai, "Numerical and Experimental Analysis of Heat Transfer AFluid Flow in 3-D Circular Finned Tube Heat Exchanger";Intel conf on CFD in and metal processing and power generation CSIRO 1997.

[10] Saeed J. Almalowi, "Analytical and Numerical Solution of One-Dimensional a Rectangular Fin with an Additional Heat source", International Research Journal of Engineering and Technology (IRJET) Volume: 02 Issue: 08 | Nov-2015.

[11] Yuh-Wei Chiu, Yi-Xiong Lin, Jiin-Yuh Jang, "Numerical and Experimental Analysis of Elliptic Finned-Tube Heat Exchangers under Misted Conditions", Proceedings of Clima 2007 WellBeing Indoors.

[12] Ziad M. Al-Makhyoul," Numerical and Analytical Study of Two Dimensional Heat Transfer Thorough an Annular Fin of Variable Thickness".

[13] Y. Pratapa Reddy , B. Jithendra Kumar , D. Srinivasulu , Dr.Ch. Srinivasa Rao , "Temperature Distribution Analysis of Composite Pin Fin By Experimental and Finite Element Method", International Journal of Innovative Research in Science,Engineering and Technology, Vol. 4, Issue 10, October 2015.

[14] Ibrahim GIRGIN, Cüneyt EZGI**,** " Finite difference model of a circular fin with rectangular profile", Journal of Naval Science and Engineering2015, Vol.11, No.1, pp.53-67.

[15] Pooja P. Shirjose,"Numerical investigation of elliptical and triangular perforated fins under forced convection", International Journal of Scientific Research Engineering & Technology (IJSRET), ISSN 2278 – 0882 Volume5, Issue4, April 2016.

[16] Kuang Yuan Kung* and Shih-Ching Lo, "Transient Analysis of Two-Dimensional Cylindrical Fin with Various Surface Heat Effects".

[17] Manasa N, Vijaya Raghu B, Dr. V Seshadri, Yogesh Kumar K J, Puneeth Kumar SB, "Numerical analysis of heat dissipation through rectangular channel with elliptical pin fins", International Journal of Emerging Technology and Advanced Engineering Volume 5, Issue 10, October 2015**.**

[18] D.D Ganji and A.S Dogonchi, "Analytical investigation of convective heat transfer of a longitudinal fin with temperature dependent thermal conductivity, heat transfer coefficient and heat generation", International journal of physical sciences,vol.9(21),pp.466-474,16 November 2014.