Double Diffusive Flow in a Porous Medium Embedded with Vertical Annulus with Power Law Heating

Abdullah A.A.A. Al-Rashed[a], Salman Ahmed N.J[b], Sarfaraz Kamangar[b] , H.M.T. Khaleed[c] , T.M.Yunus Khan[b] , Abdulwahab A. Alnaqi[a*]

[a]*Dept. of Automotive and Marine Engineering Technology, College of Technological Studies, The Public Authority for Applied Education and Training, Kuwait*

[b]*Dept. of Mechanical Engineering, University of Malaya, Kuala Lumpur, 50603 Malaysia*

[c]*Dept. of Mechanical Engineering, Faculty of Engineering, Islamic University, Madinah Munawwarra, Kingdom of Saudi Arabia*

Abstract

The present paper deals with the evaluation of double diffusive flow in a porous medium fixed with vertical wall. The boundary conditions are such that the vertical plate is heated with power law coefficient and far away medium is maintained at lower temperature Tc. The vertical plate is maintained at higher concentration Cw and far away medium is subjected to low concentration Cc. The study is carried out to know the heat and mass transfer behavior inside the porous region due to power law heating. The heat and mass transfer behavior is explored with respect to various parameters such as Lewis number, buoyancy ratio, Rayleigh number etc.

Keywords — Porous media, vertical plate, power law heating, Finite element method

I. INTRODUCTION

The study of heat transfer and fluid flow behaviour in porous medium has found numerous applications in industry as well as engineering disciplines that have led to an extensive research by many eminent researchers during the last few decades. The various challenges related to the applications such as; the contamination of chemicals in the soil, grain storage installations, cryogenic containers, migration of moisture through the air contained in fibrous insulations and so on, have motivated towards extensive research in this particular area. The fundamental concept pertaining to the flow through porous medium has been dealt meticulously in the books [1-5]. The details of the free convective heat transfer is reported by many researchers recently [6-30]. However, the research related to the combined heat and mass transfer has found more intricate challenges due to the complexity of the phenomenon, which has also been addressed, yet comparatively to the lesser extent [31-47]. The effect of Rayleigh number, Lewis number and buoyancy ratio on Nusselt and Sherwood number in a porous enclosure was reported by Bourich et al. [31] and the correlation between the various parameters were discussed. However, in another study, they investigated the partially heated porous enclosure, to study the Double-diffusive natural convection [32]. The buoyancy induced heat and mass transfer from a vertical plate was reported by Lai and Kulacki [33]. They concluded that the effect of the Lewis number has most pronounced effect on the concentration field than on the temperature and flow fields. Similar study pertaining to the convective heat and mass transfer was investigated to explore various aspects in the literature [34-45]. An attempt has been made to demonstrate the evaluation of double diffusive flow due to power law heating at inner radius in a porous medium fixed with vertical annulus. This is an extension of previous work [45] where isothermal heating was considered. However, the current work focusses the variable heating at hot surface. The mathematical model is described in detail in [45] thus we are not going too detail into it. The governing equations for natural convection in porous annulus are given as:

$$u = \frac{-K}{\mu}\frac{\partial p}{\partial r} \quad , \quad w = \frac{-K}{\mu}\left(\frac{\partial p}{\partial z} + \rho g\right) \qquad (1)$$

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \alpha\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right) \qquad (2)$$

$$u\frac{\partial C}{\partial r} + w\frac{\partial C}{\partial z} = D\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right) + \frac{\partial^2 C}{\partial z^2}\right) \qquad (3)$$

Corresponding boundary conditions are:

at $r = r_i$, $T = T_\infty + (y)^\lambda$, $C = C_w$, $u = 0$, (4)

at $r = r_o$, $T = T_\infty$, $C = C_\infty$, $w = 0$

where λ is power law coefficient.

II. RESULTS AND DISCUSSION

The above mentioned equations subjected to boundary condition (4) are solved with the help of finite element method by using triangular elements. Results are obtained for different values of power law coefficient. Figure 1 illustrates the variation in isotherms, iso-concentration and streamlines for two values of λ. This figure is obtained by keeping other parameters as *Ra=50, N=0.2 and Le=2, Rr=1 and Ar=1*. The temperature distribution is different from the case of constant heating of inner radius [45]. It is seen that the high temperature lines appear in the upper section of annulus whereas low temperature lines start from lower section of annulus. This could be attributed to high thermal energy in upper section due to nonzero value of λ. It is further seen that the higher amount of porous medium towards the outer radius is occupied by low thermal energy at $\lambda = 1$ as compared to the case of $\lambda = 0.33$. It is seen that the iso-concentration lines move towards the hot surface due to increase in the value of λ. This indicates that the mass transfer increases with increase in power law coefficient. The flow cell moves towards the cold surface due to increase in power law coefficient.

III. CONCLUSION

An attempt is made to understand the heat and mass transfer behaviour in an annular cylinder containing porous medium. Finite element method is used to solve the governing equations. The results indicated that the porous medium has low thermal energy at outer region due to high value of λ. The mass transfer increases with increase in λ.

REFERENCES

[1] D. Nield and A. Bejan, Convection in Porous Media, ed. 3rd. New York: Springer Verlag.,2006.

[2] D.B. Ingham, I.Pop (Eds.),Transport phenomena in porous media, Pergamon, Oxford, 1998.

[3] K .Vafai, Hand book of porous media, Marcel Dekker, New York, 2000.

[4] I. Pop, D.B. Ingham, Convective heat transfer: Mathematical and computational modeling of viscous fluids and porous media, Pergamon, Oxford, 2001.

[5] A.D. Bejan, Kraus (Eds), Heat transfer handbook Wiley, New York, 2003.

[6] N.J.S. Ahmed, I.A. Badruddin, Z.A. Zainal, H.M.T. Khaleed, J. Kanesan, Heat transfer in a conical cylinder with porous medium, Int. J. Heat Mass Transfer. 52(13-14), 3070-3078 (2009).

[7] I.A. Badruddin, A.A.A.A. Abdullah, N. J. S. Ahmed, S. Kamangar, Investigation of heat transfer in square porous-annulus', Int. J Heat Mass Transfer. 55 (7-8), 2184-2192 (2012).

[8] I.A. Badruddin, Z.A. Zainal, P.A. Narayana, K.N. Seetharamu, L.W. Siew, Free convection and radiation for a vertical wall with varying temperature embedded in a porous medium, Int. J. Therm. Sci. 45(5), 487-493(2006).

[9] I.A. Badruddin, Ahmed N. J. S, A. A. A. Al-Rashed, N. Nik-Ghazali, M Jameel, S. Kamangar, H. M. T. Khaleed, T. M. Yunus Khan, Conjugate Heat Transfer in an Annulus with Porous Medium Fixed Between Solids, Transport in Porous media, 109(3), 589-608 (2015).

[10] I.A. Badruddin, Z.A. Zainal, P.A. Narayana, K.N. Seetharamu, Heat transfer by radiation and natural convection through a vertical annulus embedded in porous medium, Int. Commun. Heat Mass Transfer. 33(4), 500-507(2006).

[11] Irfan Anjum Badruddin and G. A. Quadir, Radiation and viscous dissipation effect on square porous annulus, AIP Conf. Proc. 1738, 480127 (2016); http://dx.doi.org/10.1063/1.4952363

[12] Irfan Anjum Badruddin1, Z. A. Zainal, P. A. Aswatha Narayana, K. N. Seetharamu and Lam Weng Siew,Free convection and radiation characteristics for a vertical plate embedded in a porous medium, Int. J for Num. Methd. in Engg., 65 (13), 2265–2278, 26(2006).

[13] V. Prasad, F.A Kulacki, Natural convection in a vertical porous annulus, Int. J. Heat Mass Transfer. 27, 207-219(1984).

[14] R.C Rajamani, C. Srinivas, P. Nithiarasu, K.N. Seetharamu, Convective Heat-Transfer in Axisymmetrical Porous Bodies, Int. J of Numer Methods Heat Fluid Flow. 5(9), 829-837(1995).

[15] N.J.S. Ahmed, S. Kamangar, I.A. Badruddin, A. A. A. A. Al-Rashed, G.A. Quadir, H.M.T. Khaleed, T.M.Y. Khan, Conjugate heat transfer in porous annulus, J. Porous Media. 19(12), 1109-1119(2014).

[16] T.W. Ting, Y.M. Hung, N. Guo, Entropy generation of viscous dissipative nanofluid flow in thermal non-equilibrium porous media embedded in microchannels, Int. J. Heat Mass Transfer. 81, 862-877(2015).

[17] N.J.S. Ahmed, I.A. Badruddin, J. Kanesan, Z.A. Zainal, K.S.N. Ahamed, Study of mixed convection in an annular vertical cylinder filled with saturated porous medium, using thermal non-equilibrium model, Int. J. Heat Mass Transfer. 54(17-18), 3822-3825(2011).

[18] D.M. Manole and J.L. Lage, Numerical benchmark results for natural convection in a porous medium cavity, Heat and Mass Transfer in Porous Media, ASME Conference, HTD, 216 55 (1992).

[19] C. Bekermann, R. Viskanta and S. Ramadhyani, A numerical study of non-Darcian natural convection in a vertical enclosure filled with a porous medium, Numerical Heat Transfer Part A, 10, 557-570(1986).

[20] S.L. Moya, E. Ramos and M. Sen , Numerical study of natural convection in a tilted rectangular porous material, Int. J. of Heat Mass Transfer, 30, 741-756(1987).

[21] A.C. Baytas and I. Pop, Free convection in oblique enclosures filled with a porous medium, Int. J. of Heat Mass Transfer, 42,1047 –1057(1999).

[22] A. Misirlioglu, A.C. Baytas and I. Pop, Free convection in a wavy cavity filled with a porous medium, Int. J. of Heat Mass Transfer, 48, 1840-1850(2005).

[23] I.A. Badruddin, A.A.A.A. Al-Rashed, N.J.S. Ahmed, S. Kamangar, K. Jeevan, Natural convection in a square porous annulus, Int. J. Heat Mass Transfer. 55(23–24), 7175-7187(2012).

[24] G. A. Quadir and Irfan Anjum Badruddin, Heat transfer in porous medium embedded with vertical plate: Non-equilibrium approach - Part B, AIP Conf. Proc. 1738, 480125 (2016); http://dx.doi.org/10.1063/1.4952361

[25] Irfan Anjum Badruddin and G. A. Quadir, Heat transfer in porous medium embedded with vertical plate: Non-equilibrium approach - Part A, AIP Conf. Proc. 1738, 480124 (2016); http://dx.doi.org/10.1063/1.4952360

[26] I.A. Badruddin, Z. A Zainal, P.A. Narayana, K.N. Seetharamu, Thermal non-equilibrium modeling of heat transfer through vertical annulus embedded with porous medium, Int. J. Heat Mass Transfer. 49(25-26), 4955-4965(2006).

[27] Irfan Anjum Badruddin1, Z. A. Zainal, P. A. Aswatha Narayana, K. N. Seetharamu and Lam Weng Siew,Free convection and radiation characteristics for a vertical plate embedded in a porous medium, Int. J for Numerical Methods in Engineering, 65 (13), 2265–2278, 26(2006).

[28] I.A. Badruddin, Z.A. Zainal, Z. A Khan, Z. Mallick, Effect of viscous dissipation and radiation on natural convection in a porous medium embedded within vertical annulus, Int. J. Therm. Sci. 46 (3), 221-227(2007).

[29] I.A. Badruddin, Z.A. Zainal, P.A. Narayana, K.N. Seetharamu, Heat transfer in porous cavity under the influence of radiation and viscous dissipation, Int. commun. Heat Mass Transfer. 33(4), 491-499(2006).

[30] I.A. Badruddin, Z.A. Zainal, P.A. Narayana, K.N. Seetharamu, Numerical analysis of convection conduction and radiation using a non-equilibrium model in a square porous cavity, Int. J. Therm. Sci. 46(1), 20-29(2007).

[31] M. Bourich, M. Hasnaoui, and A.A Amahmid, Scale analysis of thermosolutal convection in a saturated porous

enclosure submitted to vertical temperature and horizontal concentration gradients. Energy conversion and management. 45, 2795- 2811(2004).

[32] M. Bourich, M. Hasnaoui, and A. Amahmid, Double-diffusive natural convection in a porous enclosure partially heated from below and differentially salted, Int .J. of Heat and Fluid Flow. 25, 1034-1046(2004).

[33] F. C. Lai, and F. A. Kulacki, Coupled heat and mass transfer by natural convection from vertical surfaces in porous media, Int. J. Heat Mass Transfer 34.4-5, 1189-1194(1991).

[34] W. A. Khan, and A. Aziz. Double-diffusive natural convective boundary layer flow in a porous medium saturated with a nanofluid over a vertical plate: Prescribed surface heat, solute and nanoparticle fluxes. Int. J. Therm. Sci. 50.11,2154-2160(2011).

[35] Irfan Anjum Badruddin and G. A. Quadir, Heat and mass transfer in porous cavity: Assisting flow, AIP Conf. Proc. 1738, 480126 (2016); http://dx.doi.org/10.1063/1.4952362

[36] I.A. Badruddin, T. M. Yunus Khan, Salman Ahmed N. J., Sarfaraz Kamangar, Effect of variable heating on double diffusive flow in a square porous cavity, AIP Conference Proceedings 1728, 020689 (2016); doi: 10.1063/1.4946740

[37] Cheng, Ching-Yang, Combined heat and mass transfer in natural convection flow from a vertical wavy surface in a power-law fluid saturated porous medium with thermal and mass stratification, Int. Commun. Heat Mass Transfer, 36.4,351-356(2009).

[38] Griffiths, R. W. Layered double-diffusive convection in porous media, J. Fluid Mech. 102, 221-248 (1981).

[39] Tai, Bo-Chen, and Ming-I. Char, Soret and Dufour effects on free convection flow of non-Newtonian fluids along a vertical plate embedded in a porous medium with thermal radiation, Int. Commun. Heat Mass Transfer,37.5 480-483 (2010).

[40] V. Trevisan, Osvair and Adrian Bejan, Combined heat and mass transfer by natural convection in a porous medium, Advances in Heat Transfer 20 315-352(1990).

[41] A. A. Mohamad, and R. Bennacer, Double diffusion, natural convection in an enclosure filled with saturated porous medium subjected to cross gradients; stably stratified fluid. Int. J. Heat Mass Transfer,45.18: 3725-3740(2002).

[42] N. Nik-Ghazali, Irfan Anjum Badruddin, A. Badarudin, S. Tabatabaeikia, Dufour and Soret Effects on Square Porous Annulus, Adv. Mech. Engg. , January-December, 6, 209753 (2014).

[43] Azeem, T. M. Yunus Khan, I.A. Badruddin, N. Nik-Ghazali, Mohd Yamani Idna Idris, Influence of radiation on double conjugate diffusion in a porous cavity, AIP Conference Proceedings 1728, 020283 (2016); doi: 10.1063/1.4946334.

[44] D. Angirasa, G. P. Peterson, and I. Pop, Combined heat and mass transfer by natural convection with opposing buoyancy effects in a fluid saturated porous medium, Int. J. Heat Mass Transfer, 40.12 2755-2773(1997).

[45] Irfan Anjum Badruddin, N. J. Salman Ahmed, Abdullah A. A. A. Al-Rashed, Jeevan Kanesan, Sarfaraz Kamangar, H. M. T. Khaleed, Analysis of Heat and Mass Transfer in a Vertical Annular Porous Cylinder Using FEM, Transport in Porous Media, 91(2), 697-715(2012).

[46] S. K. Rastogi, and D. Poulikakos. Double-diffusion from a vertical surface in a porous region saturated with a non-Newtonian fluid, Int. J. Heat Mass Transfer 38.5, 935-946 (1995).

[47] Cheng, Ching-Yang, Natural convection heat and mass transfer of non-Newtonian power law fluids with yield stress in porous media from a vertical plate with variable wall heat and mass fluxes, Int. Commun. Heat Mass Transfer, 33.9, 1156-1164(2006).

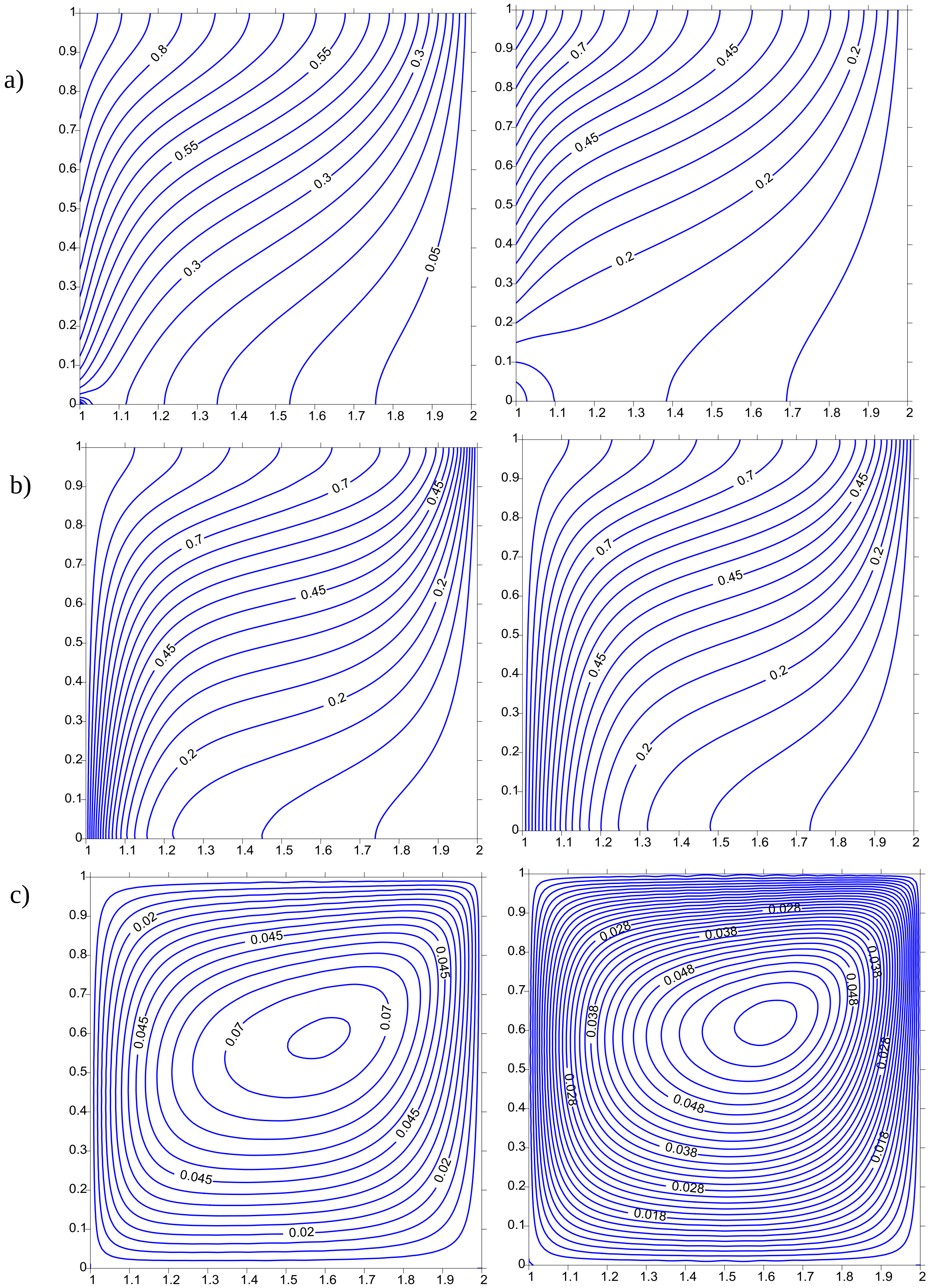

Figure 1: a) Isotherms b) Isoconcentration lines c) Streamlines at Left $\lambda = 0.33$ Right $\lambda = 1$