SSRG International Journal of Mechanical Engineering Volume 5 Issue 7, 9-15, July 2018
ISSN: 2348 – 8360 /doi:10.14445/23488360/IJME-V5I7P102

Measurement of Gear Stiffness of Healthy and Cracked Spur Gear by Strain Gauge Technique

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Abstract

Due to excessive service load, inappropriate operating conditions, or simply the end of fatigue life, damage can occur in gears. A fault either distributed or localized is incurred by gear stiffness, and ultimately vibration characteristics of the damaged tooth will change. An experimental technique based on a strain gauge was used to measure the gear mesh stiffness of healthy and cracked spur gear. The crack length was also identified by calculating the stress intensity factor of the cracked spur gear. Gear mesh stiffness varies during the meshing of gear, and it is the main excitation source of machine noise and vibration. It is also affected due to any fault in gear tooth-like crack. Tooth becomes less stiff due to a crack in gear tooth, and vibration response varies from the original. Also, it affects the dynamic behavior of transmission and may lead to an abrupt loss in efficiency and damages. Cracks are one of the most dangerous faults that can also affect gear teeth due to excessive loads transmitted and material fatigue, especially at the teeth root where a stress concentration is observed. Tooth damage causes a reduction in gear tooth stiffness. It is a key parameter in gear dynamics determining factors such as the gear system's load-carrying capacity, dynamic tooth loads, and vibration characteristics. The tooth's stiffness is calculated from deflections due to bending, fillet foundation deflections, and contact deflection.

Keywords — *Cracked Gear, Spur Gear, Strain Gauge, Strain energy, Stress intensity.*

I. INTRODUCTION

Gearboxes are used in automobiles and industrial types of machinery. Due to excessive service load, inappropriate operating conditions, damage can occur in gears, and gear faults are responsible for approximately 60% of gearbox failures. Gears are used to transmit power from the driving shaft to the drive shaft. Gear pair mesh stiffness is an important parameter that affects the vibration response of the gearbox. Stiffness of gear pair is varying during engagement to disengagement. Mesh stiffness of gear is one of the focused areas of research, and it comes under the effect of the growth of gear tooth fault like cracking, pitting and spalling.

Mesh is a barrier made to connect or to bring into contact, and stiffness is the rigidity of an object, the extent to which it resists deformation in response to an applied force. Gear mesh stiffness is nothing but the ability of a material to resist deformation when gears in mesh. It depends on several parameters, particularly on the load and rotational position of gears, considering that the gear mesh stiffness is a major excitation source of the system gear pairs. Cracks are one of the most dangerous faults that can also affect gear teeth due to excessive loads transmitted and material fatigue, especially at the teeth root where stress concentration is observed.

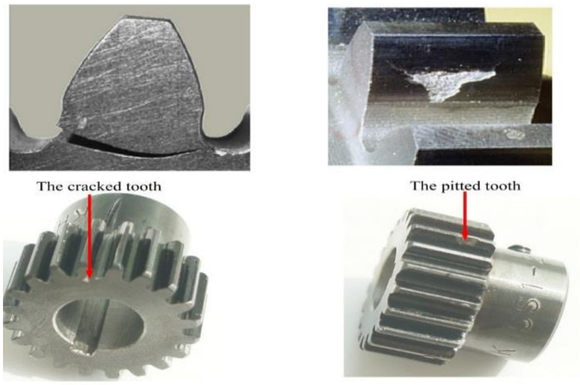

Fig 1: Damaged Spur Gears

The gear mesh stiffness is a time-varying parameter that reflects gear mesh conditions as the number of teeth in contact varies and as the line of contact of the engaged gear, teeth vary. The tooth geometry function, the position of the contact point, gear tooth deflections, gear tooth profile errors, gear hub torsional deformation and finally, the local faults on the tooth. The tooth's stiffness is calculated from deflections due to bending δ_b, fillet foundation deflections δ_f, contact deflections δ_h.

A strain gauge is an experimental technique to measure the strain at a point. A strain gauge measures the surface strain at a point. The strain gauge is widely used to measure strain in static and dynamic systems and, thus, for static and dynamic stress intensity factors (SIFs) in the isotropic and composite structures. Irwin first suggested the application of strain gauge for SIFs calculation. Daily and Sanford developed a strain gauge method to investigate the model I SIF in two dimension isotropic bodies.

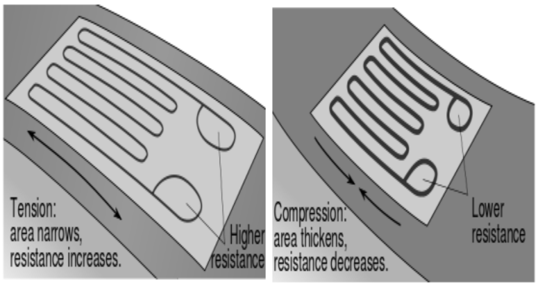

Fig 2: Working Principle of Strain Gauge

A strain gauge is an experimental technique to measure the strain at a point. A strain gauge measures the surface strain at a point. A strain gauge is widely used to measure strain in static and dynamic systems. Thus, it is for both static and dynamic stress intensity factors (SIFs) in the isotropic and composite structures. Irwin first suggested the application of strain gauge for SIFs calculation. Daily and Sanford developed a strain gauge method to investigate the model I SIF in two dimension isotropic bodies.

The stress field near crack tips can be categorized as Mode I: opening mode, Mode II: sliding and Mode III: tearing. Each of them is characterized by a "local mode of deformation," as illustrated in Figure 2.

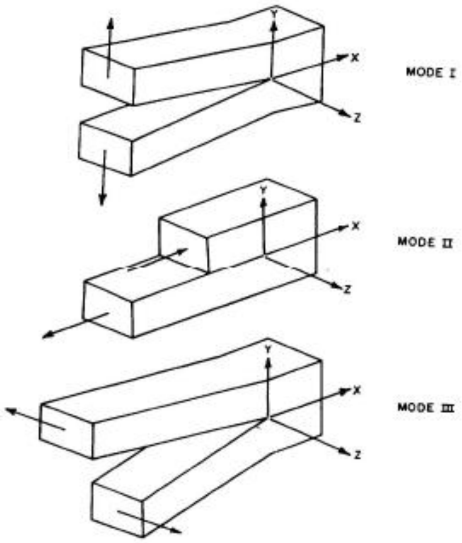

Fig 3: Three Basic Modes of Crack Surface Displacement

The opening mode, I, is related to local displacement in which the crack surfaces move directly apart (symmetric concerning the x-y and x-z planes). The sliding mode, II, is related to local displacement. The crack surfaces slide over one another perpendicular to the crack's leading edge (symmetric concerning the x-y plane and skew-symmetric concerning the x-z plane). The tearing mode, III, is related to local displacement. The crack surfaces slide concerning one another parallel to the leading edge (skew-symmetric concerning the x-y and x-z planes). Although these three modes can be superposed to "describe the most general case of crack tip deformation and stress fields," Mode I is the primary focus of this paper.

II. LITERATURE REVIEW

Isa Yesilyurt et al. [1] present vibration analysis in the detection, quantification, and advanced monitoring of damage incurred by spur gear teeth. The stiffness of a single spur gear tooth is analyzed theoretically. Due to the difficulties in measuring the gear tooth stiffness, an experimental procedure based on the modal analysis is developed to assess the gear tooth damage severity.

Fakher Chaari et al. [2] have analyzed the time-varying gear mesh stiffness was derived. A comparison with the finite element method showed good agreement associated with a reduced computation time. The obtained time-varying gear mesh stiffness can be used to model a complete gear set to check the dynamic behavior in the presence of such a fault.

Omar D. Mohammed et al. [3] have work on the purpose of vibration-based condition monitoring. To prevent catastrophic gear failures, it is important to improve the studied gear model's simulated dynamic response.

Yogesh Pandya et al. [4]in this paper, the conventional photoelasticity technique has been revisited to explore the possibility of using it as a supplementary technique to measure gear mesh variation stiffness experimentally.

Naresh K. Raghuwanshi et al. [5] presented the work; photoelasticity technique has been used to measure the stress intensity factor (SIF) for cracked gear tooth. Subsequently, SIF has been used to calculate the gear mesh stiffness.

Anand Parey et al. [6] have studied an experimental technique based on strain gauge has been proposed to measure the gear mesh stiffness of healthy spur gear and cracked spur gear pair system.

H. Sarangi et al. [7] present investigation experimental verification of optimal strain gage locations and their importance inaccurate determination of mode I stress intensity factors (SIFs) using Dally and Sanford's single strain gage technique (DS technique) has been carried out.

III.PROBLEM STATEMENT AND OBJECTIVE

A. Problem Definition

Mesh stiffness is the key parameter of gear-pair, and crack is the key element of gear fault, which affects the tooth flank and angular position of gears. The stress intensity factor (SIF) is used to calculate gear mesh stiffness of healthy and cracked spur gear by applying strain gauge and comparing their results to predetermine the problems and improve its stiffness to avoid damage and wear of teeth.

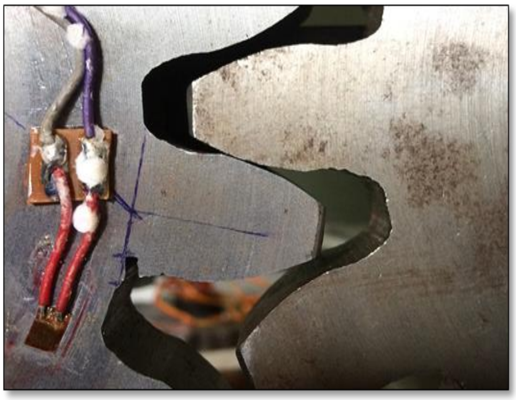

Fig 4: Pinion Tooth with 5mm Crack

B. Objective

(a) To calculate mesh stiffness of healthy and cracked spur gear tooth by potential energy method.

(b) To calculate the effect of crack length on mesh stiffness by strain gauge technique which are based on strain energy and strain energy release rate

(c) To establish an improved analytical model for time-varying mesh stiffness calculation of cracked spur gear.

IV. THEORETICAL ANALYSIS

Table I: Parameters selected for Spur Pinion and Gear

Sr. No.	Parameters	Symbol	Gear and Pinion
1	Number of Teeth	Z	17
2	Module	M	12
3	Pressure Angle	Ø	20°
4	Face Width	B	0.025
5	Contact ratio	CR	1.44
6	Young's Modulus	E	2.1×10^{11} Pa
7	Modulus of Rigidity	G	8×10^{10} Pa
8	Poisson's ratio	U	0.3

A. Theoretical Calculations

Bending stiffness (k_b)

$$k_b = \frac{F^2}{2U_b}$$

Shear stiffness (k_s)

$$k_s = \frac{F^2}{2U_s}$$

Axial compressive stiffness (k_a)

$$k_a = \frac{F^2}{2U_a}$$

B. The potential energy stored in the gear/pinion tooth is given by

Bending energy (U_b)

$$U_b = \frac{M^2}{2EI} = \frac{[F_a(d-x) - F_b h]^2}{2EI}$$

Shear energy (U_s)

$$U_s = \frac{1.2F_a^2}{2GA} = \frac{[1.2Fcos\emptyset]^2}{2GA}$$

Axial compressive energy (U_a)

$$U_a = \frac{[Fsin\emptyset]^2}{2EA}$$

Bending and shear effect (F_a)

$$F_a = Fcos\alpha$$

Compressive effect (F_b)

$$F_b = Fsin\alpha$$

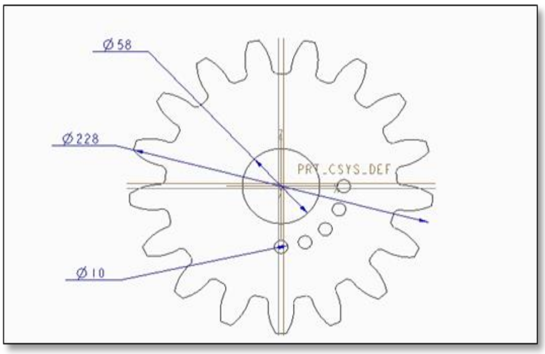

Fig5: Creo Drawing of Healthy Spur gear and Pinion

C. Tooth stiffness of uncracked gear/pinion (k_{uc})

$$k_{uc} = \frac{1}{\frac{1}{k_b} + \frac{1}{k_s} + \frac{1}{k_a}}$$

Moment of inertia (I)

$$I = \frac{1}{12} \times (2h_x)^3 \times B$$

Area of cross-section (A)

$$A = (2h_x) \times B$$

D. Tooth stiffness of cracked gear (k_c)

$$k_c = \frac{1}{\frac{1}{k_b} + \frac{1}{k_s} + \frac{1}{k_a}}$$

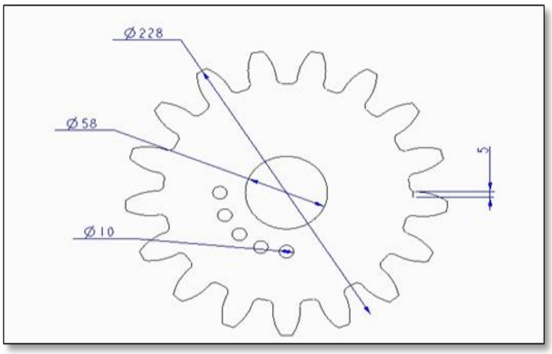

Fig6: Creo Drawing of Cracked Spur Pinion

E. *Theoretical Calculation for Single Spur Gear Pair*

Tooth stiffness of uncracked gear $(k_{uc})_g$

$$(k_{uc})_g = \frac{1}{\frac{1}{k_b} + \frac{1}{k_s} + \frac{1}{k_a}}$$

Tooth stiffness of uncracked pinion $(k_{uc})_p$

$$(k_{uc})_p = \frac{1}{\frac{1}{k_b} + \frac{1}{k_s} + \frac{1}{k_a}}$$

Tooth stiffness of cracked pinion $(k_{uc})_p$

$$(k_c)_p = \frac{1}{\frac{1}{k_b} + \frac{1}{k_s} + \frac{1}{k_a}}$$

Hertzian contact stiffness (k_h)

$$k_h = \frac{EB\pi}{4(1-\vartheta^2)}$$

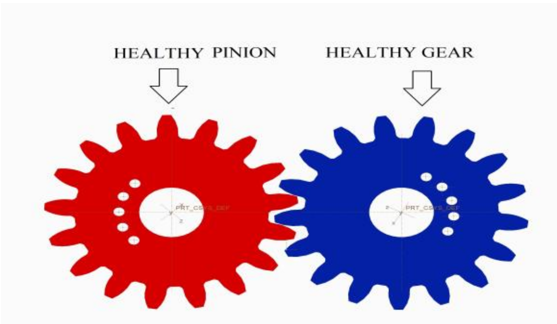

Fig7: Creo 3D Model of Healthy Spur gear Pair

F. *Mesh stiffness for single uncracked spur gear pair (kuc)sp*

$$(k_{uc})_{sp} = \frac{1}{\frac{1}{(k_{uc})_g} + \frac{1}{(k_{uc})_p} + \frac{1}{k_h}}$$

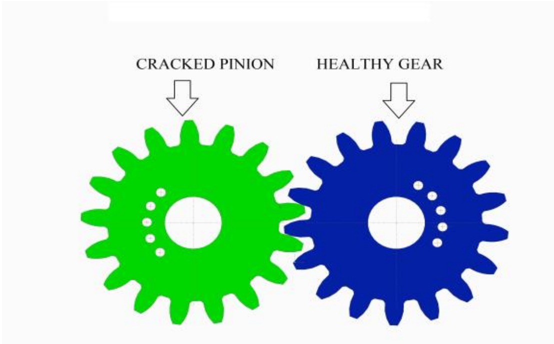

Fig8: Creo 3D Model of Cracked Spur gear Pair

V. FINITE ELEMENT ANALYSIS

A. *Material Selection*

Material used for spur gear is C45 (Medium Carbon Steel) having density 7850 kg/m^-3 whose chemical composition is ferro (Fe), Carbon(C) (0.43-0.50%), Silicon (Si) (0.17-0.4%), Magnese (Mn)(0.5-0.8%). Also having Young's Modulus 2.1*10^11Pa and Modulus of Rigidity 8*10^10 Pa.

Meshed Model

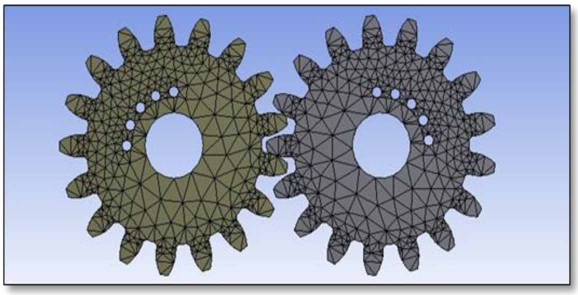

Fig9: Meshed Model of Single Healthy Spur Gear Pair

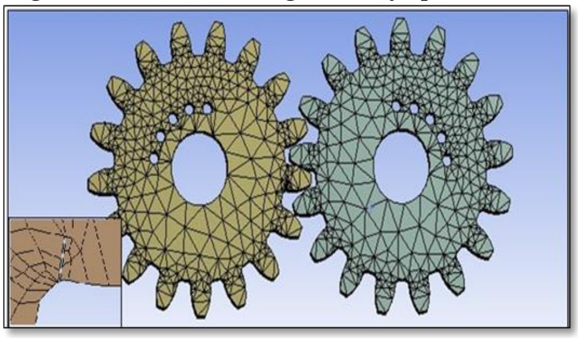

Fig10: Meshed Model of Single Cracked Spur Gear Pair

Moment

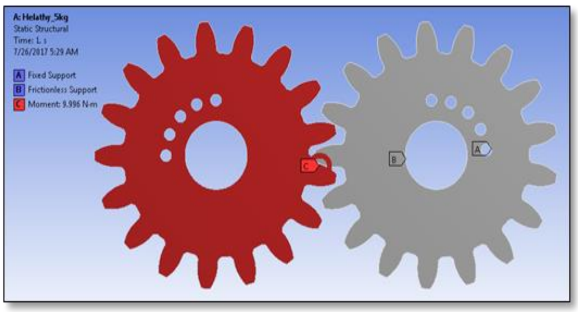

Fig11.1: Moment Applied on Pinion

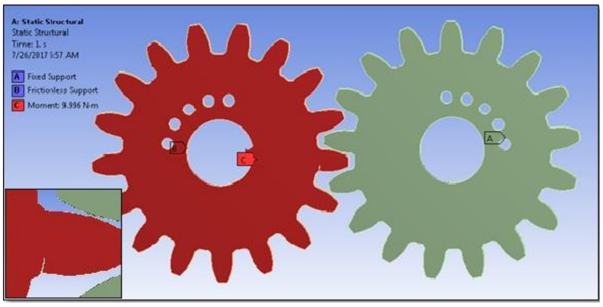

Fig11.2: Moment Applied on Pinion

VI. EXPERIMENTAL TESTING

To Strain gauge experimental set-up has been fabricated with the help of specimen supporting frame, spur gear loading arrangement, angular rotation of gears measuring device (protector), gear fixing arrangement, strain gauges and data acquisition system as shown in Fig. 12.

Fig12: Experimental Set-up

In the set-up, the pinion is free to rotate, and gear is also free to rotate, but it can be fixed at different angular positions for the observation purpose. During the experiment, gear has to be fixed at a particular meshing position. Torque is applied on the pinion with the help of lever arrangement on the pinion in an anticlockwise direction.

VII. RESULT AND DISCUSSION

(a) FEA Result

A. Healthy Spur gear Pair

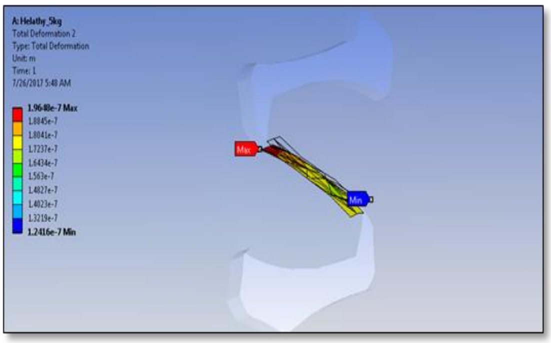

Fig. 13: Total Deformation of Spur Healthy Gear Pair (Tooth Face only)

Maximum Total Deformation = 1.96×10^{-7} m
Minimum Total deformation = 1.24×10^{-7} m

Fig. 14: Strain Energy of Healthy Spur Gear Pair

Maximum Strain Energy = 8×10^{-8} J
Minimum Strain Energy = 2.32×10^{-15} J

Fig. 15: Strain Intensity of Healthy Spur Gear Pair

Maximum Strain Intensity = 9.42×10^{5} Pa
Minimum Strain Intensity = 28.59 Pa

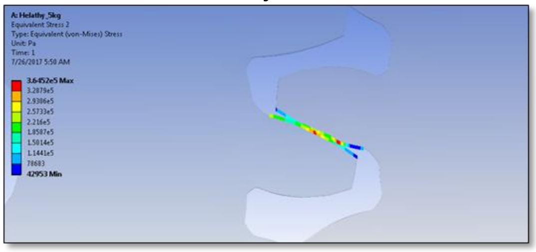

Fig. 16: Equivalent Stress of Healthy Spur Gear Pair (Tooth Edge Only)

Maximum Equivalent Stress = 3.64×10^{5} Pa
Minimum Equivalent Stress =42953 Pa

B. Cracked Spur gear Pair

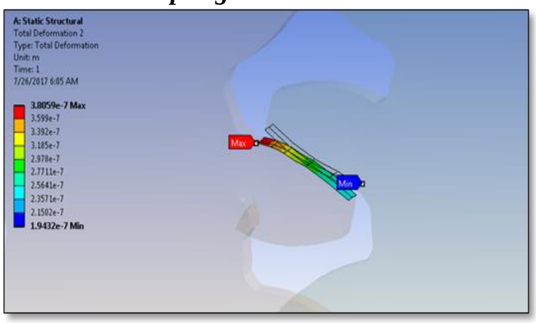

Fig.17: Total Deformation of Cracked Spur Gear Pair (Tooth Face only)

Maximum Total Deformation = 3.81×10^{-7} m

Minimum Total deformation = 1.94×10^{-7} m

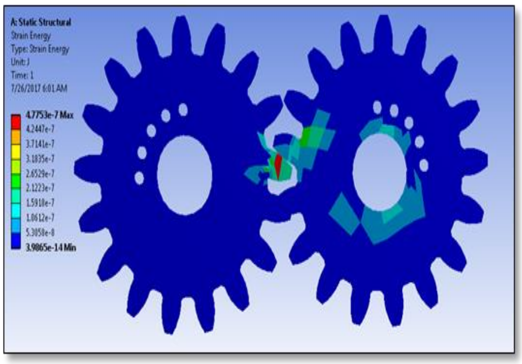

Fig. 18: Strain Energy of Cracked Spur Gear Pair

Maximum Strain Energy = 4.78×10^{-7} J
Minimum Strain Energy = 3.99×10^{-14} J

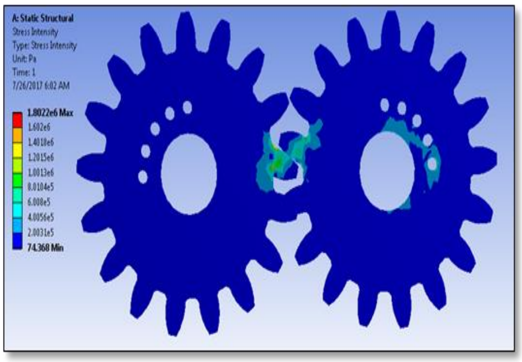

Fig. 19: Strain Intensity of Cracked Spur Gear Pair

Maximum Strain Intensity = 1.8×10^{6} Pa
Minimum Strain Intensity = 74.368 Pa

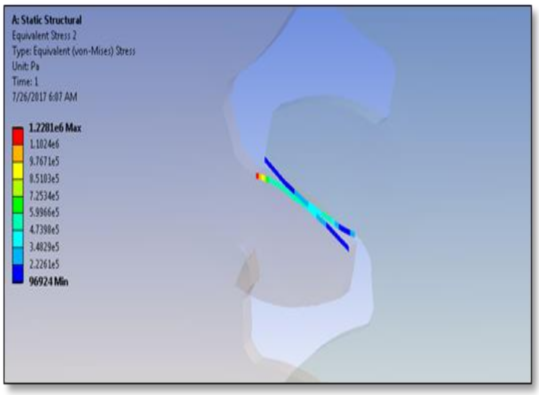

Fig. 20: Equivalent Stress of Cracked Spur Gear Pair (Tooth Edge Only)

Maximum Equivalent Stress = 1.23×10^{6} Pa
Minimum Equivalent Stress = 96924 Pa

7.2 Experimental Result

Table II: Spur Gear Strain and SIF calculated from different parameters

Sr. No.	Force	Theoretical	FEA	Strain Gauge Method
1	0	0	0	0
2	49	1.02×10^{7}	8.06×10^{7}	7.01×10^{7}
3	98	1.02×10^{7}	8.06×10^{7}	7.43×10^{7}
4	147	1.02×10^{7}	8.06×10^{7}	7.52×10^{7}
5	196	1.02×10^{7}	8.06×10^{7}	7.13×10^{7}
6	245	1.02×10^{7}	8.06×10^{7}	7.41×10^{7}
7	294	1.02×10^{7}	8.06×10^{7}	7.09×10^{7}

VIII. CONCLUSION

1. The mesh stiffness of healthy and cracked spur gear pairs is calculated using the potential energy method in theoretical calculation. A single un cracked spur gear pair and single cracked spur gear pair is 1.5×10^{7} m and 1.2×10^{7} m respectively.
2. In FEA, mesh stiffness for single un cracked spur gear pair is 1.42×10^{8} Pa and single cracked spur gear pair is 8.06×10^{7} Pa, 8.1×10^{7} Pa, 8.31×10^{7} Pa, 8.7×10^{7} Pa and 9.3×10^{7} Pa for an angle 0°, angle 22.5°, angle 45°, angle 67.5°, and angle 90° respectively.
3. In the Strain Gauge technique, the mesh stiffness for single cracked spur gear pair is 7.52×10^{7} Pa, 7.47×10^{7} Pa, 7.9×10^{7} Pa, 8.6×10^{7} Pa and 8.77×10^{7} Pa for an angle 0°, angle 22.5°, angle 45°, angle 67.5°, and angle 90°, respectively.
4. The above points show that a spur gear's deflection is linearly proportional to forces applied on it for both healthy and cracked condition.

ACKNOWLEDGMENT

I thanks Prof. A. N. Patil and Prof A. B. Gaikwad for their valuable guidance and providing all the necessary facilities, which were indispensable in completing this work. Also, I sincerely thank all the authors for their work regarding the strain gauge technique used to measure gear stiffness.

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