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MHD Flow of Casson Fluid Due to Quadratic Stretching of the Sheet

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Abstract
The study on steady Non-Newtonian incompressible magnetohydrodynamic Casson fluid flow over a quadratic stretching sheet stretched with a velocity $u_s = ax + bx^2$ *is conducted. The resulting equations, along with the suitable boundary conditions, are solved, and numerical values are tabulated. The velocity fields are studied through graphs, and the numerical values of parameters like Magnetic parameter, Suction parameter, Casson parameter are varied, and their effect on the velocity profiles are discussed. Also, the effect of these parameters on the value of the Skin friction coefficient is analyzed.*

Keywords - *Quadratic, Casson, MHD, incompressible.*

I. INTRODUCTION

The boundary layer flow and heat transfer due to linear/nonlinear stretching have enticed many researchers over the decades due to its extensive applications in many of the industrial processes. A lots of research have been carried out, and many research papers have been published on the study of steady/unsteady flow and heat, mass transfer behavior of fluid-induced by linear stretching in the presence of different parameters like Magnetic effect, suction/blowing, presence of thermal radiation, chemical effect, slip parameter, etc. The boundary layer formation due to fluid flow due to linear stretching was first analyzed by Crane [1]. Crepeau J. C et al. [2] derived a similar solution for natural convection in the presence of internal heat generation. An analytical study of boundary layer flows on a continuous stretching surface was carried out by Bararnia H et al. [3]. The thermal radiation effect on an exponentially stretching sheet was studied by Bidin Biliana [4]. N.Ahmad and M.Mishra conducted a study on unsteady boundary layer flow and heat transfer over a stretching

Sheet [5]. R.C.Bataller [6] conducted a study on the MHD flow and heat transfer of an upper-convicted Maxwell fluid due to a stretching sheet. An analytic study on the heat and mass transfer of a mixed hydrodynamic MHD slip flow of viscous stretching sheet was conducted by M. Turkyilmazoglu [7] .R.Cortell [8] considered a porous medium over a stretching surface to its flow and heat transfer behavior. G.M.Pavithra and B.J.Gireesha [9] have conducted a study on the unsteady flow and heat transfer of a dusty fluid over an exponentially stretching sheet. Unsteady flow and heat transfer of a second-grade fluid due to linear stretching was studied by M.Sajid et al. [10]

Awang and Hashim [11] derived the series solution for flow over a nonlinearly stretching sheet in the presence of chemical reaction and magnetic field. Raftari Behrouz et al. [12] performed an analysis on the solution to the MHD flow over a nonlinearly stretching sheet by a homotopy perturbation method. The Nanofluid boundary layer laminar flow over a nonlinearly stretching sheet was explored by P.Rana et al. [13]. The boundary layer viscous flow of a Nanofluid and heat transfer over a nonlinearly stretching sheet with radiation effect was analyzed by Fekry M Hady et al. [14]. Later Khader.M.M [15] et al. conducted a numerical study on the boundary layer flow due to nonlinear stretching by considering slip velocity. Later, Zaimi .K.A et al. [16] considered nonlinear stretching/shrinking sheet in a nan-fluid and studied the fluid flow and heat transfer behavior.

Many researchers have spotted the light on Quadratic stretching of the surface was analyzed by many researchers, and a number of studies were carried out in this regard. Kumaran and Ramanaiah [17] studied a viscous incompressible flow over a stretching sheet where the velocity of the sheet was considered as a quadratic polynomial, and the sheet was subjected to a linear mass flux. The MHD flow through a porous quadratic stretching sheet with a viscoelastic boundary layer was analyzed by S.K. Khan et al. [18]. Later N.A.Kelson [19] investigated the similarity solutions for viscous flow over an impermeable quadratic stretching sheet. This work was further carried out by Rafael Cortell [20] derived an analytic solution for MHD boundary layer flow and heat transfer by considering permeable sheet with quadratic stretching. In this journal paper, the effect of the suction parameter on the velocity and mass transfer profiles was discussed.

In the present study under consideration, an emphasis is given on the effect of introducing

Magnetic parameter M, Casson parameter, Suction parameter on the velocity fields and on the Skin friction coefficient due to MHD boundary layer flow of a Casson fluid over a quadratic stretching with the application of equal and opposite forces along the x-axis.

II. PROBLEM FORMATION

Consider a steady, incompressible flow of Casson fluid over a quadratic stretching sheet with the application of uniform magnetic field along the y-axis and application of equal and opposite forces along X-axis.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho} \qquad (2)$$

The boundary conditions of the governing flow are

$$u_s = ax+bx^2,\ v=-v_w(x) \text{ at } y=0 \qquad (3)$$

As $y \to \infty$ $u \to 0$

The following transformations are used in order to reduce the above equations (1) and (2), which are the Partial differential equations arising due to quadratic stretching of a sheet over a Non-Newtonian Casson fluid

$$\eta=(a/\upsilon)^{1/2}y \qquad (4)$$

$$U=axf'(\eta)+bx^2g'(\eta) \qquad (5)$$

$$V=-(a\upsilon)^{1/2}f(\eta)-2bx(a/\upsilon)^{-1/2}g(\eta) \qquad (6)$$

After using the transformations (4), (5), (6) into the equations (1) and (2), we get the following Ordinary differential equations,

$$\left(1+\frac{1}{\beta}\right)f'''(\eta)-\left[f'(\eta)\right]^2+f(\eta)f''(\eta)-Mf'(\eta)=0 \qquad (7)$$

$$\left(1+\frac{1}{\beta}\right)g'''(\eta)-3f'(\eta)g'(\eta)+2gf''+fg''-Mg'=0 \qquad (8)$$

$$gg''-(g')^2=0 \qquad (9)$$

The exact solution of equation (7) is

$$f(\eta)=f_e+\frac{1-e^{-\delta\eta}}{\delta(1+1/\beta)}.$$

Where $\delta=\dfrac{f_e+\sqrt{f_e^2+4(1+M)}}{2}$

The exact solution of equation (8) is

$$g(\eta)=-\frac{1-e^{-\delta\eta}}{\delta}. \qquad (10)$$

Where $\delta=\dfrac{f_e+\sqrt{f_e^2+4(1+M)}}{2}$

The transformed boundary conditions for f (η) and g(η) are

$$f(0)=f_e,\ f'(0)=1,\ f'(\infty)=0 \qquad (11)$$
$$g(0)=-\frac{1}{\delta},\ g'(0)=1,\ g'(\infty)=0$$

III. RESULTS AND DISCUSSION

In order to check the accuracy of the result, the value of the skin friction coefficient $f''(\theta)$ for Newtonian fluid (β=0) is compared with the result obtained by Rafael Cortell, and it was found that the present result is in good agreement with the result obtained by Rafael Cortel[20].

Table.1 Correlation of Skin friction coefficient $f''(\theta)$ for β=0.0, S=0.0, M= 0, 0.5.

	M	$f''(\theta)$
Rafael Cortell[20]	0.0	-1.0
Present study	**0.0**	**-1.0**
Rafael Cortell[20]	0.5	-1.22
Present study	**0.5**	**-1.224**

The equations (7) and (8) along with the boundary conditions are solved using the 4th order Runge-Kutta method with the help of MAT-LAB software, and the numerical values are tabulated. The following are the plots obtained for the velocity fields f' (η) and $g'(\eta)$ for the skin friction coefficient when the sheet is stretched quadratic manner in the presence of Magnetic Parameter (M), Casson Parameter (β), and Suction Parameter (S).

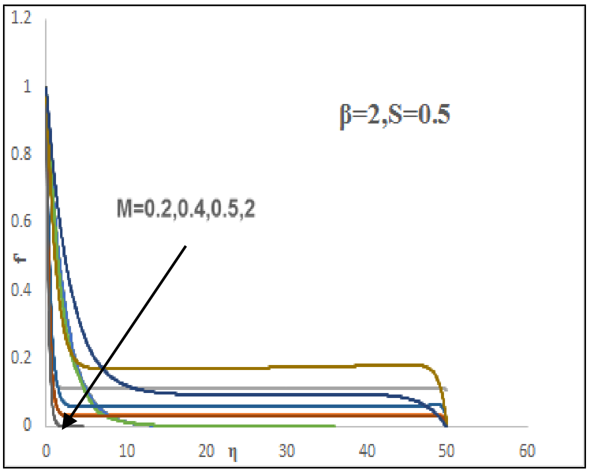

Fig 1.Effect of Magnetic Parameter (M) on the Velocity profile (f') for β=2, S=0.5

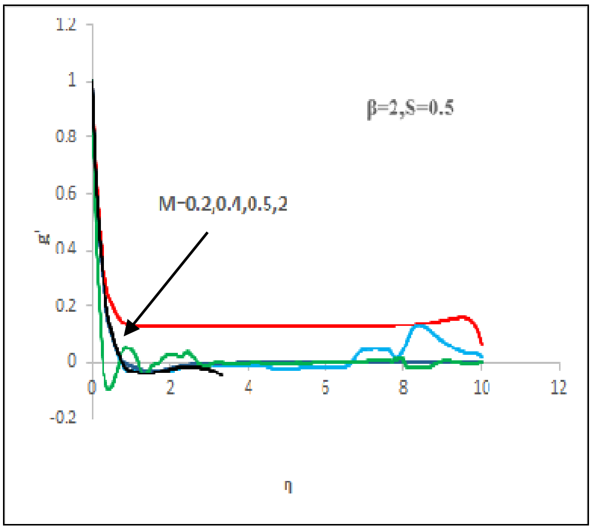

Fig 2.Effect of Magnetic Parameter (M) on the Velocity field (g') for β=2, S=0.5

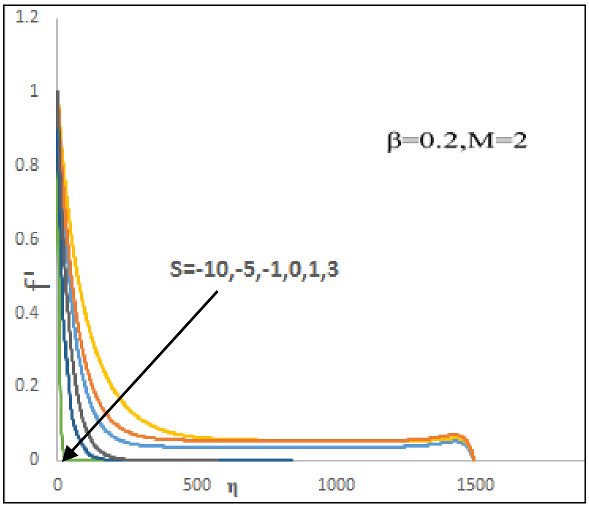

Fig 3.Effect of Suction Parameter (S) on the Velocity profile (f') for β=0.2, M=2.

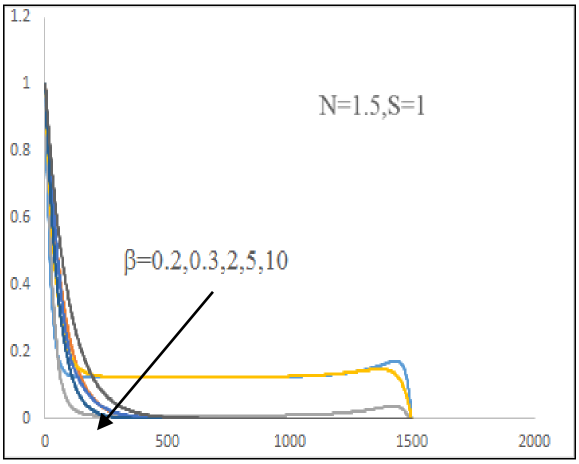

Fig 4.Effect of Casson Parameter (β) on the Velocity field (f') for N=1.5, S=1.

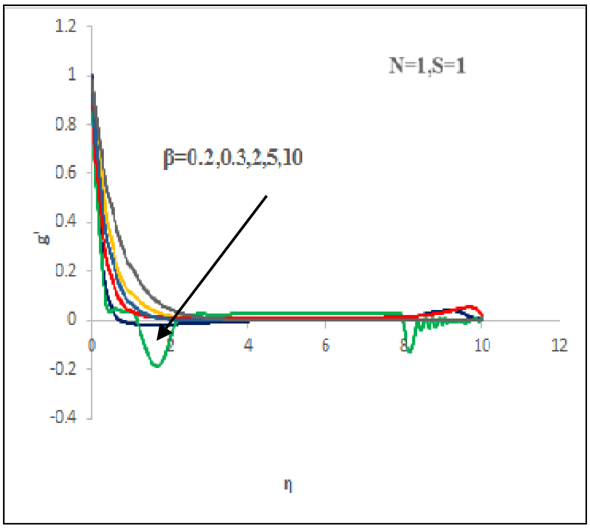

Fig 5.Effect of Casson Parameter (β) on the Velocity field (g') for N=1.5, S=1.

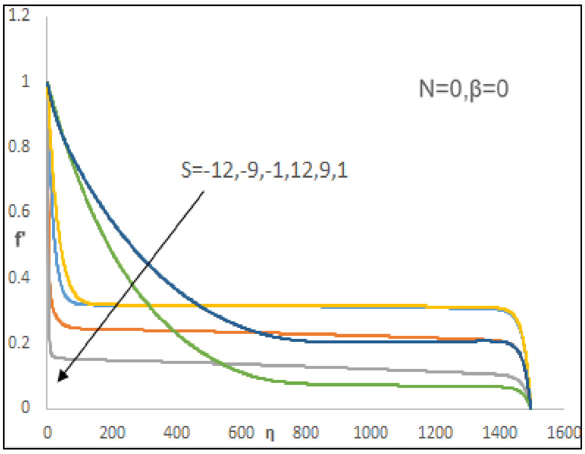

Fig 6.Effect of Suction Parameter (S) on the Velocity profile (f') for N=0, β=0

In order to characterize the behavior of velocity fluids due to quadratic stretching of the sheet, the value of many parameters like Magnetic Parameter, Casson Parameter, a Suction parameter is varied, and effects are noticed.

The magnetic parameter is increased as represented by figs1 and 2 to study its effect on the velocity fields in the presence of fixed values of suction parameter S=0.5 and Casson parameter β=2; as a result, the velocity fields f' and g' drops by decreasing the thickness of the boundary layer. It can be noticed from fig.3 that Variation in the suction parameter S decreases the velocity of the fluid in the presence of β=0.2, M=2. Also, fig.4and fig.5 illustrate the effect of Casson parameter β on the velocity fields for some fixed values of M and S. It is clear that the velocity fields decrease with the increase in the value of the Casson parameter.

The effect of the Suction parameter on the velocity field is represented by fig.6 when M=0 and β=0. The graph clearly describes that the velocity profile drops with Variation in the suction parameter. This effect is more precise when S increases positively.

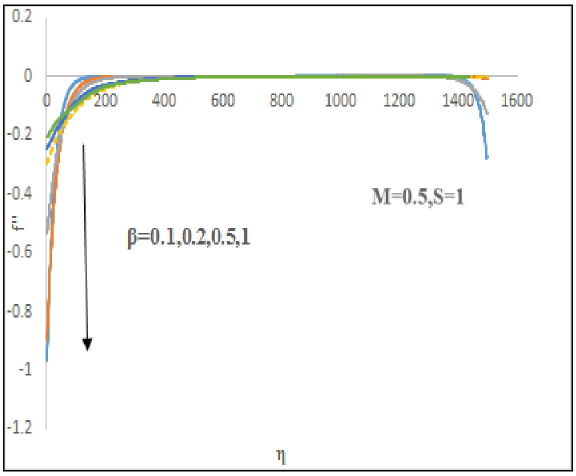

Fig.7.The skin friction coefficient for Variation in β When M=0.5, S=1

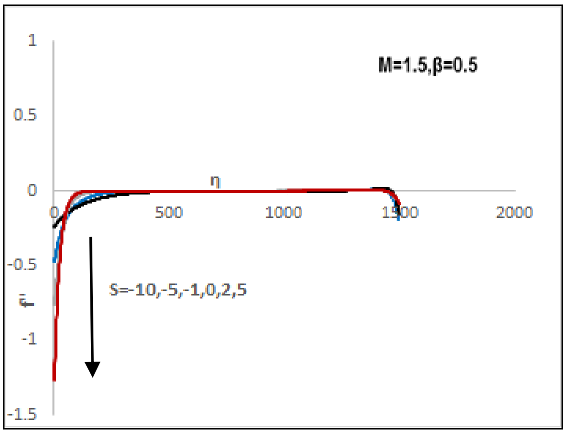

Fig.8.The skin friction coefficient for Variation in S When M=1.5, β=0.5

It is being noticed from fig.7 and fig.8 that the Skin friction coefficient $f''(\theta)$ decreases with an increase in the values of the Casson parameter (β) and the Suction parameter (S), respectively.

IV. CONCLUSION

The results of the above study can be summarized as follows:

- The velocity fields drop boundary when the Magnetic parameter is increased.
- When the Casson parameter (β) is increased, the velocity fields diminishes, thereby decreasing the thickness of the velocity boundary.
- The velocity profile drops with Variation in suction parameter(S) at β=0 and M=0. This effect is more precise when S increases positively.
- The skin friction coefficient decreases when the values of the Casson parameter (β), the Suction parameter, Magnetic parameter are increased, respectively.

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