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Contribution of Constraints In Isotropic And Anisotropic Models

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Abstract
In the paper, we compare the stress distribution in a thick tube. On the one hand, we consider isotropic material and, on the other hand, an anisotropic structure. These materials are subjected to elongation and compression. Different models like those of Holzapfel, Delfino, and Fung are used to compare them at the level of radial and azimuthal stresses and their influence depending on whether the material is isotropic or anisotropic.

Keywords: *Hyperelasticity, (in) compressible, anisotropy, pressure, stresses, tubular structure, elongation*

I. Introduction

The modeling of mechanical systems has long been of interest to the scientific world in recent years. The study's purpose was to establish parametric relationships between the geometry, rigidity, pressures, and corresponding forces to which a material can be subjected. These studies made it possible to apprehend certain mechanical properties of the structures [1]. Each mechanics problem can be formulated from the fundamental equilibrium relations of the mechanics of continuous media, specifying three characteristics: the geometry, the relation of behavior, and the loading applied. Pseudoelastic, viscoelastic, or poroelastic behavioral relationships have been used to describe the behavior of certain materials such as tubular structures. Pseudoelasticity dissociates loading and unloading by considering them perfectly elastic. For their part, viscoelastic formulations include mechanical responses to model creep or relaxation phenomena.

To consider the hyper-elastic, incompressible or compressible, homogeneous, or heterogeneous nature of a material, an energy function is introduced [2].

Certain mathematical criteria such as convexity, ellipticity, and objectivity must be satisfied when using strain energy functions. These must take into account the principle of material indifference [2].

The energy potential depends on the gradient tensor of the deformation and the model's various parameters [3]. For example, among the energy functions commonly used to describe a tube's mechanical behavior, we have pseudoelastic, randomly elastic, poroelastic, or viscoelastic energy functions [3].

Fung et al. [4] developed a small diameter soft tube's pseudoelastic model after observing physical changes during repeated loading/unloading.

Holzapfel and Weizsacker [5] developed an incompressible nonlinear viscoelastic 3D model of a thick-walled and fiber-reinforced tubular structure. Their energy function has been decoupled into an elastic part and a viscoelastic part.

Delfino et al. [6] have proposed a finite element pseudoelastic model that uses anisotropic energy function in incompressible exponential form to describe a cylindrical tube's behavior.

Despite the abundant literature on tubular structures' mechanical properties, many models have been developed, not without criticism. They provide ratios between deformation, pressure, and stress fields in equations [7, 8].

In this paper, after a mathematical formulation, we have solved semi-analytically a problem of elongation and compression of tubular material, hyperelastic, and incompressible.

As an application, we compared the influence of elongation and compression on radial and azimuth stresses across the Holzapfel, Delfino, and Fung models.

II. Mathematical considerations

In this study, the chosen coordinate system is cylindrical. A material point is identified by its coordinates (R,Θ,Z) in the undistorted configuration and (r,θ,z) in the deformed configuration. We consider a hollow cylindrical tube of the inner radius A and a respectively before and after deformation. The following deformation kinematics describes the deformation:

$$r=r(R)\ ,\ \ \theta=\Theta+\tau\lambda_z Z\ \ ,\ \ z=\lambda_z Z \qquad (2\text{-}1)$$

The gradient of the transformation is given by:

$$\mathbf{F}=\begin{bmatrix}\lambda_r & 0 & 0\\ 0 & \lambda_\theta & \gamma\lambda_z\\ 0 & 0 & \lambda_z\end{bmatrix} \qquad (2\text{-}2)$$

The incompressibility condition is defined by:

$$J=\det(\mathbf{F})=\lambda_r\lambda_\theta\lambda_z=1 \qquad (2\text{-}3)$$

with as boundary condition [9]:

$$r(A)=a \qquad (2\text{-}4)$$

according to equations (2-1) and (2-2), the condition of incompressibility is given by:

$$R=[A^2+\lambda_z(r^2-a^2)]^{1/2} \qquad (2\text{-}5)$$

or

$$r=[a^2+\lambda_z^{-1}(R^2-A^2)]^{1/2} \qquad (2\text{-}6)$$

with $\gamma=\tau r$ and $\lambda_i\,(i=r,\theta,z)$ defined by:

$$\begin{cases}\lambda_r=r'=R\lambda_z^{-1}[a^2+\lambda_z^{-1}(R^2-A^2)]^{-1/2}\\ \lambda_\theta=r/R=[a^2+\lambda_z^{-1}(R^2-A^2)]^{1/2}/R\\ \lambda_z=l/L\end{cases} \qquad (2\text{-}7)$$

where L and l are respectively the lengths of the tube before and after deformation.
The tensors of deformations of Cauchy Green left $\mathbf{B}=\mathbf{F}\mathbf{F}^T$ and right $\mathbf{C}=\mathbf{F}^T\mathbf{F}$ give respectively:

$$\mathbf{C}=\begin{bmatrix}\lambda_r^2 & 0 & 0\\ 0 & \lambda_\theta^2 & \gamma\lambda_z\lambda_\theta\\ 0 & \gamma\lambda_z\lambda_\theta & \lambda_z^2(1+\gamma^2)\end{bmatrix} \quad \text{and}$$

$$\mathbf{B}=\begin{bmatrix}\lambda_r^2 & 0 & 0\\ 0 & \lambda_\theta^2+\gamma^2\lambda_z^2 & \gamma\lambda_z^2\\ 0 & \gamma\lambda_z^2 & \lambda_z^2\end{bmatrix} \qquad (2\text{-}8)$$

The invariants of the Cauchy deformation tensor, in isotropy $(I_1\,,\,I_2\,,I_3)$ and in anisotropy $(I_4\,,\,I_5)$ are given by:

$$\begin{aligned}
I_1&=tr(\mathbf{B})=\lambda_r^2+\lambda_\theta^2+\lambda_z^2(1+\gamma^2)\\
I_2&=tr(\mathbf{B}^*)=\lambda_r^2\lambda_\theta^2+\lambda_r^2\lambda_z^2(1+\gamma^2)+\lambda_\theta^2\lambda_z^2\\
I_3&=\det(\mathbf{B})=\lambda_r^2\lambda_\theta^2\lambda_z^2=1\\
I_4&=\mathbf{m}.\mathbf{m}=\lambda_r^2M_R^2+(\lambda_\theta M_\Theta+\gamma\lambda_z M_Z)^2+\lambda_z^2M_z^2\\
I_5&=\mathbf{m}.\mathbf{Bm}=\lambda_r^4M_R^2+\lambda_\theta^2(\lambda_\theta^2+\gamma^2\lambda_z^2)M_\Theta^2+2\gamma\lambda_\theta\lambda_z(\lambda_\theta^2+\lambda_z^2+\gamma^2\lambda_z^2)M_\Theta M_Z\\
&\quad+\lambda_z^2\left[\gamma^2\lambda_\theta^2+(1+\gamma^2)^2\lambda_z^2\right]M_Z^2
\end{aligned} \qquad (2\text{-}9)$$

Where $\mathbf{B}^*$ is the adjoint of $\mathbf{B}$, the vector $\mathbf{m}$ defined by $\mathbf{m}=\mathbf{FM}$ gives the orientation of the fibers in the deformed configuration, and the vector $\mathbf{M}$ gives the orientation of the fibers in the undistorted configuration.
Using (2-10) in (2-9), we obtain:

$$\begin{aligned}
I_1&=(1/\lambda_\theta\lambda_z)^2+\lambda_\theta^2+\lambda_z^2(1+\gamma^2)\\
I_2&=(1/\lambda_z)^2+(1/\lambda_\theta)^2(1+\gamma^2)+\lambda_\theta^2\lambda_z^2\\
I_4&=(1/\lambda_\theta\lambda_z)^2M_R^2+(\lambda_\theta M_\Theta+\gamma\lambda_z M_Z)^2+\lambda_z^2M_z^2\\
I_5&=(1/\lambda_\theta\lambda_z)^4M_R^2+\lambda_\theta^2(\lambda_\theta^2+\gamma^2\lambda_z^2)M_\Theta^2+2\gamma\lambda_\theta\lambda_z(\lambda_\theta^2+\lambda_z^2+\gamma^2\lambda_z^2)M_\Theta M_Z\\
&\quad+\lambda_z^2\left[\gamma^2\lambda_\theta^2+(1+\gamma^2)^2\lambda_z^2\right]M_Z^2
\end{aligned} \qquad (2\text{-}11)$$

The constraint tensor of Cauchy in incompressible [8] is given by:

$$\boldsymbol{\sigma}=-p\mathbf{Id}+2W_1\mathbf{B}+2W_2(I_1\mathbf{Id}-\mathbf{B})\mathbf{B}+2W_4\mathbf{m}\otimes\mathbf{m}+2W_5(\mathbf{m}\otimes\mathbf{Bm}+\mathbf{Bm}\otimes\mathbf{m}) \qquad (2\text{-}12)$$

where $W_i=\partial W/\partial I_i\ (i=1,2,3)$ and W the energy function, p a multiplier of Lagrangeassociated with the constraint of incompressibility and $\mathbf{Id}$ the identity tensor.
In cylindrical stress tensor gives:

$$\boldsymbol{\sigma}=\begin{bmatrix}\sigma_{rr} & \sigma_{r\theta} & \sigma_{rz}\\ \sigma_{\theta r} & \sigma_{\theta\theta} & \sigma_{\theta z}\\ \sigma_{zr} & \sigma_{z\theta} & \sigma_{zz}\end{bmatrix} \qquad (2\text{-}13)$$

whose non-zero components are:

$$\begin{aligned}
\sigma_{rr}&=-p+2W_1(1/\lambda_\theta\lambda_z)^2+2W_2(\lambda_\theta^2+\lambda_z^2(1+\gamma^2))(1/\lambda_\theta\lambda_z)^2\\
&\quad+2W_4(M_R/\lambda_\theta\lambda_z)^2+4W_5(M_R/(\lambda_\theta\lambda_z)^2)^2\\
\sigma_{\theta\theta}&=+2W_1(\lambda_\theta^2+\gamma^2\lambda_z^2)+2W_2((1/\lambda_\theta\lambda_z)^2+\lambda_z^2)(\lambda_\theta^2+\gamma^2\lambda_z^2)+2W_4(\lambda_\theta M_\Theta+\gamma\lambda_z M_Z)^2\\
&\quad-p+4W_5((\lambda_\theta^2+\gamma^2\lambda_z^2)(\lambda_\theta M_\Theta+\gamma\lambda_z M_Z)+\gamma\lambda_z^3M_Z)(\lambda_\theta M_\Theta+\gamma\lambda_z M_Z)\\
\sigma_{zz}&=-p+2W_1\lambda_z^2+2W_2((1/\lambda_\theta\lambda_z)^2+\lambda_\theta^2+\gamma^2\lambda_z^2)\lambda_z^2+2W_4(\lambda_z M_Z)^2\\
&\quad+4W_5(\gamma\lambda_z^3M_Z(\lambda_\theta M_\Theta+\gamma\lambda_z M_Z)+\lambda_z^4M_Z^2\\
\sigma_{\theta z}&=-p+2W_1\gamma\lambda_z^2+2W_2(-\gamma\lambda_z^2)+2W_4(\lambda_z M_Z\lambda_\theta M_\Theta+\gamma\lambda_z^2M_Z^2)\\
&\quad+2W_5\begin{bmatrix}(\gamma\lambda_z^2(\lambda_\theta M_\Theta+\gamma\lambda_z M_Z)+\lambda_z^3M_Z)(\lambda_\theta M_\Theta+\gamma\lambda_z M_Z)\\+\lambda_z M_Z(\lambda_\theta^2+\gamma^2\lambda_z^2)(\lambda_\theta M_\Theta+\gamma\lambda_z M_Z)+\gamma\lambda_z^4M_Z^2)\end{bmatrix}
\end{aligned} \qquad (2\text{-}14)$$

With the relations (2-10), the invariants $I_1\,,I_4\,,I_5$ defined in (2-11) become functions of $\lambda_\theta\,,\lambda_z\,,\gamma$.
As a result, we can write the energy function as a function of these three variables by asking:

$$\widehat{W}(\lambda_\theta\,,\lambda_z\,,\gamma)=W(I_1\,,I_4\,,I_5) \qquad (2\text{-}15)$$

We can then express the components of the stress tensor as follows:

$$\begin{cases} \sigma_{\theta\theta} - \sigma_{rr} = \lambda_\theta \dfrac{\partial \widehat{W}}{\partial \lambda_\theta} + \gamma \dfrac{\partial \widehat{W}}{\partial \gamma} \\ \sigma_{\theta z} = \dfrac{\partial \widehat{W}}{\partial \gamma} \\ \sigma_{\theta\theta} + \sigma_{zz} - 2\sigma_{rr} = \lambda_\theta \dfrac{\partial \widehat{W}}{\partial \lambda_\theta} + \lambda_z \dfrac{\partial \widehat{W}}{\partial \lambda_z} \end{cases} \quad (2\text{-}16)$$

Because of the deformation kinematics, equilibrium equations in a cylindrical system are reduced to:

$$\begin{cases} r \dfrac{d}{dr}(\sigma_{rr}) + \sigma_{rr} - \sigma_{\theta\theta} = 0 \\ \dfrac{d}{dr}(r^2 \sigma_{r\theta}) = 0 \\ \dfrac{d}{dr}(r \sigma_{rz}) = 0 \end{cases} \quad . \quad (2\text{-}17)$$

Using the $\sigma_{rr}(a) = p$ and $\sigma_{rr}(b) = 0$, boundary conditions, equation (2-17)$_1$ gives:

$$p = -\int_a^b (\sigma_{\theta\theta} - \sigma_{rr}) \frac{dr}{r} \quad (2\text{-}18)$$

With the radial component and the circumferential component of the tensor of the stresses defined in (2-14), the difference between these two expressions gives:

$$\begin{aligned} \sigma_{\theta\theta} - \sigma_{rr} = {} & 2W_1(\lambda_\theta^2 + \gamma^2\lambda_z^2 - (1/\lambda_\theta\lambda_z)^2) + 2W_4((\lambda_\theta M_\Theta + \gamma\lambda_z M_Z)^2 - (M_R/\lambda_\theta\lambda_z)^2) \\ & 2W_2\left(((1/\lambda_\theta\lambda_z)^2 + \lambda_z^2)(\lambda_\theta^2 + \gamma^2\lambda_z^2) - (\lambda_\theta^2 + \lambda_z^2(1+\gamma^2))(1/\lambda_\theta\lambda_z)^2\right) \\ & + 4W_5\begin{pmatrix} ((\lambda_\theta^2 + \gamma^2\lambda_z^2)(\lambda_\theta M_\Theta + \gamma\lambda_z M_Z) + \gamma\lambda_z^3 M_Z)(\lambda_\theta M_\Theta + \gamma\lambda_z M_Z) \\ -(M_R/(\lambda_\theta\lambda_z)^2)^2 \end{pmatrix} \end{aligned} \quad (2\text{-}19)$$

III. Results and applications

A. Holzapfel model

Holzapfel et al. [10] proposed an extension of their hyperelastic model, taking into account the tubular structure's viscosity. The strain energy function has been decoupled into an elastic part and a viscoelastic part. The potential of energy that these authors proposed is written:

$$W = \frac{c}{2}[I_1 - 3] + \beta_1[\exp(\beta_2(I_4 - 1)^2) - 1] \quad (3\text{-}1)$$

Where c, β_1 and β_2 are constants related to the material.
The derivatives concerning the invariants of the energy function, defined in (2.12), are given by:

$$\begin{cases} W_1 = c/2 \\ W_4 = 2\beta_1\beta_2(I_4 - 1)\exp\left(\beta_2(I_4 - 1)^2\right) \end{cases} \quad (3\text{-}2)$$

The tensor components of the constraints defined in (2-14) and relative to the potential (3.1) are written:

$$\begin{aligned} \sigma_{rr} &= -p + 2W_1(1/\lambda_\theta\lambda_z)^2 + 2W_4(\lambda_r M_R)^2 \\ \sigma_{\theta\theta} &= -p + 2W_1(\lambda_\theta^2 + \gamma^2\lambda_z^2) + 2W_4(\lambda_\theta M_\Theta + \gamma\lambda_z M_Z)^2 \\ \sigma_{zz} &= -p + 2W_1\lambda_z^2 + 2W_4(\lambda_z M_Z)^2 \\ \sigma_{\theta z} &= -p + 2W_1\gamma\lambda_z^2 + 2W_4(\lambda_z M_Z \lambda_\theta M_\Theta + \gamma\lambda_z^2 M_Z^2) \end{aligned}$$

(3-3)

we deduce the difference

$$\sigma_{\theta\theta} - \sigma_{rr} = 2W_1(\lambda_\theta^2 + \gamma^2\lambda_z^2 - (1/\lambda_\theta\lambda_z)^2) + 2W_4((\lambda_\theta M_\Theta + \gamma\lambda_z M_Z)^2 - (M_R/\lambda_\theta\lambda_z)^2) \quad (3\text{-}4)$$

Consider the invariants as functions of the eigenvalues $I_i = I_i(\lambda_\theta, \lambda_z, \gamma), i = 1,4.$
We get another expression of the system (3.2):

$$\begin{aligned} W_1 &= c/2 \\ W_4 &= 2\beta_1\beta_2((1/\lambda_\theta\lambda_z)^2 M_R^2 + (\lambda_\theta M_\Theta + \gamma\lambda_z M_Z)^2 + \lambda_z^2 M_z^2 - 1)D \end{aligned} \quad (3\text{-}5)$$

with

$$D = \exp\left[\beta_2\left((1/\lambda_\theta\lambda_z)^2 M_R^2 + (\lambda_\theta M_\Theta + \gamma\lambda_z M_Z)^2 + \lambda_z^2 M_z^2 - 1\right)^2\right]$$

Thus, the difference between the radial stress and the circumferential stress in equation (3.4) gives:

$$\sigma_{\theta\theta} - \sigma_{rr} = c\alpha_1 + 4\alpha_2\alpha_3 D \quad (3\text{-}6)$$

where

$$\begin{aligned} \alpha_1 &= \lambda_\theta^2 + \gamma^2\lambda_z^2 - (1/\lambda_\theta\lambda_z)^2 \\ \alpha_2 &= \beta_1\beta_2((1/\lambda_\theta\lambda_z)^2 M_R^2 + (\lambda_\theta M_\Theta + \gamma\lambda_z M_Z)^2 + \lambda_z^2 M_z^2 - 1) . \\ \alpha_3 &= (\lambda_\theta M_\Theta + \gamma\lambda_z M_Z)^2 - (M_R/\lambda_\theta\lambda_z)^2 \end{aligned}$$

We arrive at the expression of the pressure defined in (2.18), in the model of Holzapfel:

$$p = \int_a^b (c\alpha_1 + 4\alpha_{21}\alpha_{22}D)\frac{dr}{r} \quad (3\text{-}7)$$

B. Delfino model

Delfino et al. [11] chose an isotropic deformation energy function of the exponential two-parameter form:

$$W=\frac{\beta_1}{2}\left[\exp(\frac{\beta_2}{2}(I_1-1))-1\right] \qquad (3\text{-}8)$$

the non-zero components of the constraint whose general expressions have been given in (2.14) give with this model:

$$\begin{aligned}\sigma_{rr}&=-p+2W_1(1/\lambda_\theta\lambda_z)^2\\ \sigma_{\theta\theta}&=-p+2W_1(\lambda_\theta^2+\gamma^2\lambda_z^2)\\ \sigma_{zz}&=-p+2W_1\lambda_z^2\\ \sigma_{\theta z}&=-p+2W_1\gamma\lambda_z^2\end{aligned} \qquad (3\text{-}9)$$

and

$$\sigma_{\theta\theta}-\sigma_{rr}=2W_1(\lambda_\theta^2+\gamma^2\lambda_z^2-(1/\lambda_\theta\lambda_z)^2) \qquad (3\text{-}10)$$

with

$$W_1=\frac{\beta_1\beta_2}{4}\exp\left[\frac{\beta_2}{2}(((1/\lambda_\theta\lambda_z)^2+\lambda_\theta^2+\lambda_z^2(1+\gamma^2)-3)\right]$$

or

$$\sigma_{\theta\theta}-\sigma_{rr}=\frac{\beta_1\beta_2}{4}(\lambda_\theta^2+\gamma^2\lambda_z^2-(1/\lambda_\theta\lambda_z)^2)\exp\left[\frac{\beta_2}{2}(((1/\lambda_\theta\lambda_z)^2+\lambda_\theta^2+\lambda_z^2(1+\gamma^2)-3)\right] \qquad (2.10)$$

From the study made in paragraph (2), the pressure p, with this Delfino model results in:

$$p=-\int_a^b\left(\frac{m_1m_2}{2r}(\lambda_\theta^2+\gamma^2\lambda_z^2-(1/\lambda_\theta\lambda_z)^2)\exp\left[\frac{m_2}{2}(((1/\lambda_\theta\lambda_z)^2+\lambda_\theta^2+\lambda_z^2(1+\gamma^2)-3)\right]\right)dr \qquad (3\text{-}11)$$

C. Fung model

Fung et al. [12] proposed an exponential energy function assuming that the stress tensor's principal directions coincide with the radial, circumferential and axial directions of the tubular structure.

$$W=\frac{c_1}{2}\left[\exp(\frac{c_2}{2}(I_1-3))-1\right]+\beta_1\left[\exp(\beta_2(I_4-1)^2)-1\right] \qquad (3.12)$$

c_1, c_2, β_1 and β_2 are constants related to the material.

The partial derivatives of the energy potential (3.12), concerning the invariants, are given by:

$$\begin{cases}W_1=\dfrac{c_1c_2}{4}(\exp(\dfrac{c_2}{2}(I_1-3)))\\ W_4=2\beta_1\beta_2(I_4-1)\exp\left(\beta_2(I_4-1)^2\right)\end{cases}$$

The system of equations (2.14) then becomes:

$$\begin{aligned}\sigma_{rr}&=-p+2W_1(1/\lambda_\theta\lambda_z)^2+2W_4(\lambda_r M_R)^2\\ \sigma_{\theta\theta}&=-p+2W_1(\lambda_\theta^2+\gamma^2\lambda_z^2)+2W_4(\lambda_\theta M_\Theta+\gamma\lambda_z M_Z)^2\\ \sigma_{zz}&=-p+2W_1\lambda_z^2+2W_4(\lambda_z M_Z)^2\\ \sigma_{\theta z}&=-p+2W_1\gamma\lambda_z^2+2W_4(\lambda_z M_Z\lambda_\theta M_\Theta+\gamma\lambda_z^2M_Z^2)\end{aligned}$$

We deduce from it:

$$\sigma_{\theta\theta}-\sigma_{rr}=2W_1(\lambda_\theta^2+\gamma^2\lambda_z^2-(1/\lambda_\theta\lambda_z)^2)+2W_4((\lambda_\theta M_\Theta+\gamma\lambda_z M_Z)^2-(M_R/\lambda_\theta\lambda_z)^2) \qquad (3.14)$$

with:

$$W_1=\frac{c_1c_2}{4}(\exp(\frac{c_2}{2}((1/\lambda_\theta\lambda_z)^2+\lambda_\theta^2+\lambda_z^2(1+\gamma^2)-3)))$$
$$W_4=2\beta_1\beta_2((1/\lambda_\theta\lambda_z)^2M_R^2+(\lambda_\theta M_\Theta+\gamma\lambda_z M_Z)^2+\lambda_z^2M_z^2-1)D$$

and:

$$D=\exp\left[\beta_2\left((1/\lambda_\theta\lambda_z)^2M_R^2+(\lambda_\theta M_\Theta+\gamma\lambda_z M_Z)^2+\lambda_z^2M_z^2-1\right)^2\right]$$

These three expressions reported in (3.14) make it possible to obtain the equation:

$$\sigma_{\theta\theta}-\sigma_{rr}=\alpha_1+4\alpha_2\alpha_3D$$

where

$$\alpha_1=\frac{c_1c_2}{4}(\lambda_\theta^2+\gamma^2\lambda_z^2-(1/\lambda_\theta\lambda_z)^2)\exp(\frac{c_2}{2}((1/\lambda_\theta\lambda_z)^2+\lambda_\theta^2+\lambda_z^2(1+\gamma^2)-3)))$$
$$\alpha_2=\beta_1\beta_2((1/\lambda_\theta\lambda_z)^2M_R^2+(\lambda_\theta M_\Theta+\gamma\lambda_z M_Z)^2+\lambda_z^2M_z^2-1)$$
$$\alpha_3=(\lambda_\theta M_\Theta+\gamma\lambda_z M_Z)^2-(M_R/\lambda_\theta\lambda_z)^2$$

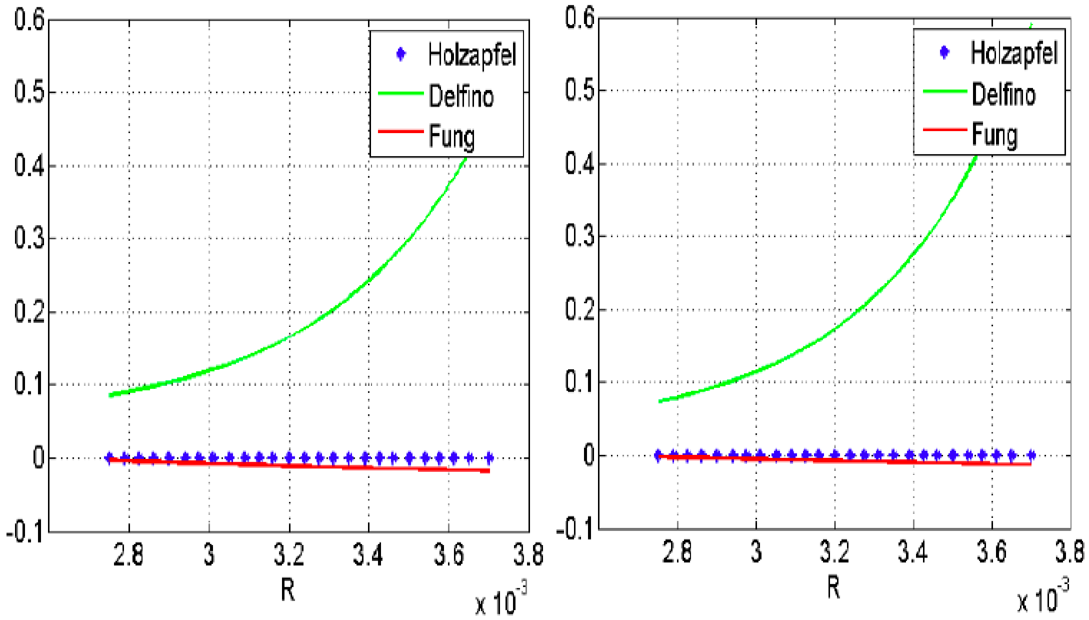
Taking into account the $\alpha_i, (i=1,2,3)$, equations (3.15) and (2.18), we obtain the expression of the intralaminar pressure through the tubular structure:

$$p=\int_a^b\left(\alpha_1+4\alpha_2\alpha_3D\right)\frac{dr}{r} \qquad (3.16)$$

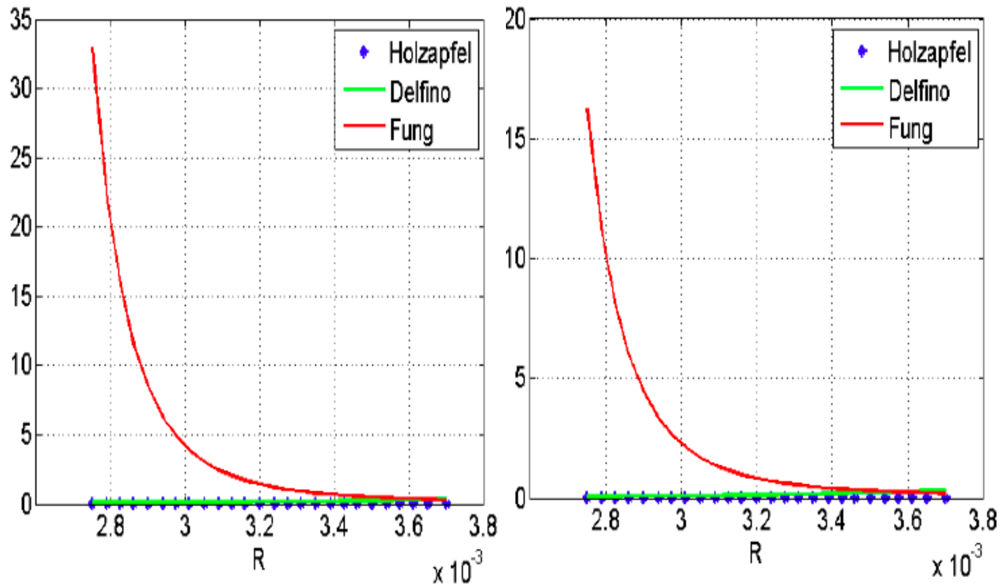
IV. Simulation

In this section, we simulate the influence of an elongation (Fig.1) and a compression (Fig.2) on the distribution of radial and azimuth stresses developed in paragraph 3. The material used parameters are those of a thick tubular structure of a small diameter [13].

On the (Fig.1) case of elongation, we have the radial and circumferential stresses according to the radius (m). The aim is to compare this deformation's influence through the different models described in the previous section.



radial stress (Mpa) vs radius in (m)- Fig 1: $l/L \succ 1$ **- circumferential stress (Mpa) vs radius in (m)**



radial stress (Mpa) vs radius in (m)- Fig 1: $l/L \prec 1$ **- circumferential stress (Mpa) vs radius in (m)**

We find that the constraints from the Delfino model are much higher than those of Holzapfel and Fung. This is explained by the fact that the Delfino model is isotropic, unlike the other two, which are anisotropic models. We can deduce that the fibrous contribution proposed by Holzapfel and Fung does not have a very great influence on the behavior of the tube.

On the other hand, during this elongation, the simulated constraints are increasing in the Delfino model. Because of the kinematics of radial deformation described in paragraph 2, we note that the greater the radius, the higher the stresses if we assume the isotropic material.

The forms of energy potentials also have a strong influence on the stress distribution. Fung and Delfino propose exponential forms, unlike Hozapfel, which develops a model as the sum of a polynomial function (isotropic part) and exponential one (anisotropic contribution).

Anisotropy is not only manifested by a dependence of stiffness, depending on the direction of the stresses. More confusing phenomena may occur when the material is biased in any direction, neither parallel nor perpendicular to the fibers.

The deformation of the volume element is more complicated than an isotropic model would have predicted; not only does the element become longer and narrower, but moreover, it twists. In other words, longitudinal and transverse deformation and shear deformation are obtained while the element is stressed in pure tension.

V. Conclusion

This article has highlighted differences in the distribution of stresses in a small diameter cylindrical tube. These differences are all the greater as the material is isotropic or fibrous. The forces of the energy potentials also have a strong influence on the stresses.

The stresses noted in Fung's anisotropic exponential model are much higher than those from Holzapfel and Delfino. In elongation, it is the isotropic model of Delfino that offers more important constraints.

We have shown that the shape of the energy potential, depending on whether its isotropic part is exponential or polynomial, also plays a no less negligible role in distributing radial and azimuth stresses.

When modeling mechanical problems, the hypothesis of homogeneous medium with isotropic behavior is very commonly adopted for the configurations, which are often considered deterministic. However, in reality, these simplified models are sometimes too ideal.

For some materials, heterogeneity and anisotropy may exist at all scales, and the lack of knowledge of some information may require modeling with some degree of uncertainty.

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