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Review on Dynamics, Control and Stability of Two Wheeled Vehicle

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Abstract *- This paper aims to give an extended state of the art of the dynamics and the stability of Two-Wheeled Vehicles from 2000 to 2018. The study is divided into three different categories. The review extends to cover in details three aspects which are the Dynamic Modelling that different concern approach to generate the equation of motion, Stability Analysis that concerns some works about the stability and behaviour of Two-Wheeled Vehicles, and different strategies of control conducted in this field. The paper ends up with the conclusion that guides for prospective works in this field.*

Index Terms *- Two Wheeled Vehicle; Lagrange Modelling; Holonomic Constraints; Self-Stability; Stability; Control*

I. INTRODUCTION

The modelling and stability analysis of two-wheeled vehicles is a salient field. It is an area that has attracted the attention of many researchers who aimed at securing the comfortability and safety of the rider. Among the researchers who have dedicated their time and energy to this field are the following: W. Rankine, P.Appell, E. Carvallo, F. Whipple, A. Somerfield, and F.Klein. Those researchers have contributed to the development of the theory and dynamic of a two-wheeled vehicle. In addition, they participated in the description of the critical factors and parameters that influence the safety and comfort efficiency of driving of the two-wheeled. J. Whipple [1] was the first one who used a formal analysis in the literature of self-stability. Consequently, he developed the famous fourth-order model of a straight running bicycle exactly in 1889. Afterwards, the second class of analysis has appeared with some researcher's A ̈ström et al. [6], J.Limebeer and R.Sharp [37]. Those researchers studied some relevant basic studies of rider control that used models with geometry and mass distribution to allow self-stability and to use rules for the control of the steer for uncontrolled bicycle dynamics such simple and steer-controlled approaches.

II. METHODOLOGY

The modelling and stability analysis of two-wheeled vehicles is a salient field. It is a field that has attracted the attention of many researchers who aimed at securing the comfortability and safety of the rider. The purpose review extends to cover in details three aspects which are the Dynamic modelling that concern various approach to generate the dynamic equation, Stability Analysis that concerns some works about the stability and mode of instability, and different strategies of control theory conducted in this field. The literature review of this article is mostly based on some materials extracted from Google Scholar, conference proceedings of the driving simulation conference, the international motorcycle conference, Padua University, and the TU Delft University, and conference on the international of Multimode System Dynamics. The literature review of this article is collected using more than 56 articles.

III. LITERATURE SURVEY

A. Dynamic modelling

The dynamic modelling of two-wheeled vehicles is a salient field. It is an area that has attracted the attention of many researchers. Firstly. We can differentiate between two main approaches to develop the dynamic equation: the first generates the motion equations using Newton's laws [35]. The second approach studies the system from a Lagrangian approach [5, 19, 27, 50,56]. The main steps to derive the equation of dynamic by the two approaches are depicted in figure 1:

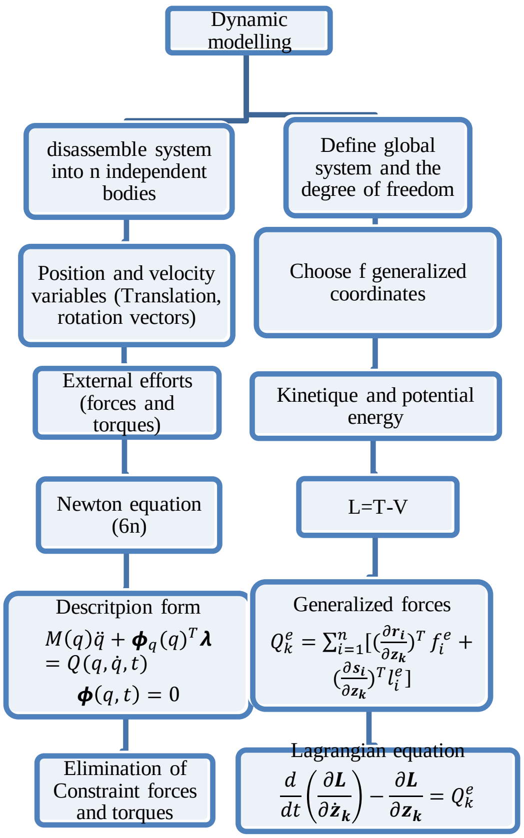

Figure 1: The two main steps to generate the equations of motion

E. Carvallo [57] was the first to present a correct model of linearisation equations of motion for a bicycle with some simplifying assumptions on the mass distribution of the front fork. Firstly we can found some basic model of the equation that has not enough complexity. Also, L. Keo and M. Yamakita [33] treated the simplified dynamic model of a bicycle with a balancer by using Lagrange dynamic equations.

Some other has been treated a complex form of the nonlinear equation. So we can cite the works conducted by S. Timoshenko and D.H. Young [54], they derived a nonlinear lean equation for a Basic bicycle model having just a point mass in the rear part of the bicycle, and a steering angle controlled by the rider. Their model ignored the wheel inertias, steering axis tilt, trail and the front-mass offset from the steering axis. Therefore, the lean equation has been linearised. D. J. Limebeer has introduced another development and R.Sharp [37], they developed the nonlinear lean equations with assuming a zero-radius front wheel. Whereas an important development in the theoretical analysis of motorcycles was achieved by R. Sharp [50], he constructed, and the developed the theoretical analysis of motorcycles with two rigid frames, he developed the Lagrangian equation that describes the motions of a motorcycle with a passive rider.

The scientific community has faced dynamic modelling by linearisation of fully nonlinear equations [54]. On the other hand, J.Meijaard and A.Schwab [45] and J.Manning [42], and J.Papadopoulos [46] are conducted a series of research which are developed the complete Linearised dynamic model with investigated the Whipple bicycle model that contains four rigid bodies which are rear wheel, rear frame with a passive rider attached to it, front frame composing of a handlebar and fork assembly and a front wheel.

B. Stability analysis

Modelling and analysis of the stability of Two-Wheeled vehicles have been an attractive area of research, T.Foale [14] introduced an explanation of some factors that affect the motorcycle handling and performance. One of the earliest and the most famous bicycle stability studies can be dated back to 1869 by W.Rankine [49] but, since it was the beginning, this study was purely qualitative; thus they examined the balancing and the steering without giving any mathematical modelling. Subsequently, the scholar F. J. W. Whipple [1] was the first one to use a formal analysis in the literature of bicycle self-stability. Consequently, he developed the famous fourth-order model of a straight running bicycle exactly in 1889. JA ̈ström et al. [6] created a simplified model that presents a basic explanation of simplified model of the bicycle (the wheels have no spin momentum, the front assembly has no mass or inertia, and non-zero head angle) on control, equilibrium, and self-stability. Also, another work conducted by. J.Kooijman et al. [29] measured dynamic responses on an instrumented bicycle, and they confirmed Whipple's model by comparing the experimentally measured eigenvalues and the eigenvalues predicted by the formulae for a range of speeds.

We can make considerate the works of J.Meijaard [44] J.Meijaard, and A.Schwab [45], A.Schwab, et al.[48] and A.Schwab et al. [49] and R. Sharp [50], and J.Kooijman et al. [29] which produced the equations of dynamic, which are definitive for stability analysis. They have established the necessary formulae for the linearized equation and the theoretical analysis of the two-wheeled vehicle. They have identified the main modes of instability such as capsize, weave and wobble by calculating the eigenvalues, they have been performed widely the interpretation of the eigenvalue that illustrates some mode of instability so, The lateral dynamics of the bicycle in the configuration can be expressed by:

$$M\ddot{q} + v\dot{q}_1 + (gK_0 + v^2K_2)q = f \qquad (1)$$

$\boldsymbol{q}$=[φ, δ] and $\boldsymbol{f}$ =[T_φ, T_δ]. The constant entries in matrices
Where:

- **M** is the mass moment of the inertia matrix, brings the kinetic energy into the model.
- $\boldsymbol{C_1}$is the damping matrix that captures asymmetric gyroscopic torques by turning the steering wheel and changing the roll angle. It also captures inertial reactions due to the speed of orientation.
- $\boldsymbol{K_0}$is the stiffness matrix independent of the speed, bringing the potential energy into the model.
- $\boldsymbol{K_2}$is the velocity-dependent stiffness matrix, bringing the gyroscopic and centrifugal forces to the model.

The eigenvalues are calculated by assuming an exponential solution of the for $q = q_0\exp(\lambda t)$.

For the homogeneous equations. This later leads to the characteristic polynomial equation:

$$\det(M\lambda^2 + vC_1\lambda + (gK_0 + v^2K_2) = 0 \qquad (2)$$

For the case of two-second order differential equation, it is common to write the equations in matrix form as follows:

$$\begin{bmatrix} M_{\varphi\varphi} & M_{\varphi\delta} \\ M_{\varphi\delta} & M_{\delta\delta} \end{bmatrix}\begin{bmatrix} \ddot{\varphi} \\ \ddot{\delta} \end{bmatrix} + v\begin{bmatrix} 0 & C_{\varphi\delta} \\ C_{\varphi\delta} & C_{\delta\delta} \end{bmatrix}\begin{bmatrix} \dot{\varphi} \\ \dot{\delta} \end{bmatrix} + \left\{g\begin{bmatrix} K_{0\varphi\varphi} & K_{0\varphi\delta} \\ K_{0\delta\varphi} & K_{0\delta\delta} \end{bmatrix} + v^2\begin{bmatrix} 0 & K_{2\varphi\delta} \\ 0 & K_{2\delta\delta} \end{bmatrix}\right\}\begin{bmatrix} \varphi \\ \delta \end{bmatrix} = 0 \qquad (3)$$

The behaviour of the system is examined by plotting the eigenvalues for the linearised system and explaining the relevant bicycle modes.
To illustrate this concept, we are going to analyze of the eigenvalue, and produce some graph to determine the different mode of instability also the self-stabilizing area of an uncontrolled two-wheeled vehicle. For this reason, the state variables of the vehicle with 2 degrees of freedom. The mathematical models used to generate the results obtained in Figure 2 are described below.

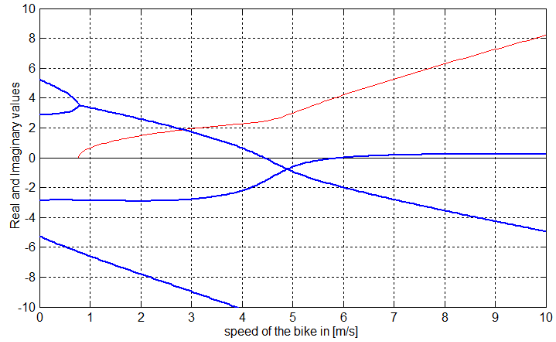

Figure 2: Eigenvalues and stability of the benchmark system, with positive real parts of eigenvalues, negative real parts of eigenvalues, and imaginary parts of eigenvalues.

So Eigenvalues from the linearised stability analysis for the benchmark bicycle where red line represents the imaginary part of the eigenvalues and blue lines presents the real part of the eigenvalues, in the forward speed range 0 - 10 m/s. The speed range for the stability of the bicycle is from v_w< v <v_c.
There are two main velocities indicated in the figure. One is the velocity at which the weave mode becomes stable,v_w, and the other is the velocity at which the capsize mode becomes unstable, v_c. So, the weave mode begins at zero velocity.
Consequently, the motion is unbalanced but at v_w≈ 4.5 m/s, and for the capsize motion at v_c ≈ 6.0 m/s, the capsize speed, and the bicycle becomes unstable. This motion is still unstable but becomes stable as soon as the real eigenvalues cross the real axis. This happens at a weave speed of about v_w≈ 4.5 m/s, and for the capsize motion at capsizing speed v_c ≤ 6m/s and oscillations emerge at the real double root at v_d≤ 0.7m/s. At near-zero speeds, 0< v < 0.5 m/s, there are two pairs of real eigenvalues. From our observation, we noticed that when speed is increased to 0.7 m/s; two real eigenvalues incorporate. The speed range for which the uncontrolled bicycle displays asymptotically stable behaviour, with all eigenvalues having negative real parts, is v_w< v <v_c. So, we can observe from the graphs plotted, when velocity is in the range between 0 - 1m/s, the bike is unstable, between 1 - 5,277m/s, the eigenvalue still positive, so, the instability decrease, between 5,27 - 7,98 m/s the bike became stable up velocity 7,98 m/s, a few positive eigenvalues appear, and the bike became unstable, at a forward speed v=5m/s; the bicycle becomes stable to a steering torque that begins as an impulse and then remains constant.
D. Limebeer and R. Sharp [37] represented the equations of Schwab et al. [48], The Stability and steering characteristics of a motorcycle are discussed by utilizing a linearization approach with taking into consideration a constant velocity has been presented, the effect of passive rider steering impedance on vehicle stability has been investigated.

On the other hand, R.Sharp has published a series of studies that concerns numerous problems of motorcycle dynamics, stability and control. R.Sharp [18] showed that a wide wheelbase and handlebar angle improved the damping of the Weave mode. Also, he proposed an improvement of his model [50] by adopting a model to four rigid bodies.

Also in the domain of Powered two-wheeled vehicle, V. Cossalter et al. [52, 34, 35, 31] conducted much research in the quantitative modelling work on motorcycle handling, they published a substantial book that deals with the all what is qualitative information and explanations of the stability of motorcycle for different vibration modes - Capsize, wobble and weave as presented in Figure 3. They are used a simple mathematical model of a motorcycle under steady cornering to evaluate those factors that influence the steering torque, relating to vehicle manoeuvrability. They introduced a new approach for the evaluation of vehicle handling and manoeuvrability; also, they used modal analysis to model the stability of the motorcycle for both straight running and cornering behaviour. The modal analysis consisted of calculation of the steady-state conditions, linearization of the equations, and finally, the solution of the eigenvalue problem.

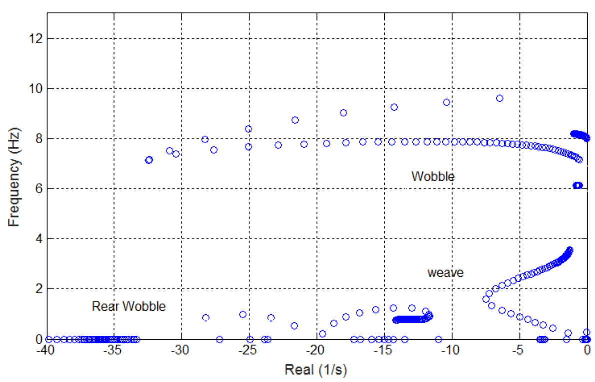

Fig 3. Root locus of the weave, rear weave and wobble modes

D. Limebeer et al. [30] has been performed widely the interpretation of the eigenvalue that illustrates the main mode of the instability of Powered two vehicles, so, Figure 3 shows the main mode of instability which are; weave, and wobble modes and is a plot of the speed-dependent eigenvalues of the linearised model of Powered two-wheeled vehicles. The wobble mode covers the frequency range 6.4 - 8.0 Hz, and weave mode frequency is nearly 3.8 Hz. the weave mode begins at a low velocity as two different real unstable modes.

C. *Control Theory*

Many studies were conducted on the control of two-wheeled vehicles. In this regard, the following parts provide a slight review of the literature which focuses on controllers for two-wheeled vehicles. Thus, many types of controllers are designed to ensure control of the system. Motion control of two-wheeled vehicles was achieved by using various techniques such as a fuzzy controller, LQR, Classical control, and H∞ optimal control, and rider action control.

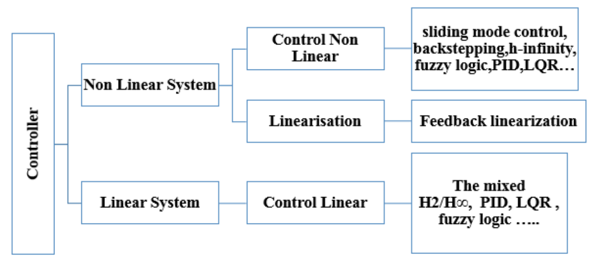

Figure 4 summarises the different control approach for linear and nonlinear systems

- **LQR/LQG optimal control**

LQR is one of the optimal control techniques that's largely used in this field, so, A.Schwab et al. [59] used an LQR controller with full-state feedback which was implemented in two different situations to investigate the effect of a leaned upper body on the control required to stabilize a bicycle. In the first situation, they investigated a rider attached to the frame of the bicycle, and they show that the system can be stabilized easily through steer torque control. In the second situation, the rider is modelled with an inverted pendulum, they find that adding a pivoted upper body does not greatly affect the uncontrolled system eigenvalues.

B. Connors [17] investigated the effect of pedalling on the steering control torque for a recumbent bike. He modelled the rider to control the balance of the bicycle as LQR steer optimal torque control. They concluded that for a recumbent bike, the oscillating legs could radically augment the roll angle sensitivity and the steering torque required to balance the bicycle.

K.Huyge et al. [22] used motorcycle model using the biomechanical approach to represent the rider model. They stabilize the tracking path using LQR control.

R.Sharp [12] extended his LQR control method to the motorcycle with the addition of rider lean torque control. The goal was to implement a control scheme that represents a rider; the controller inputs are the lean angle of the upper body of rider and the path tracking error. He claims that riders control the motorcycle at the weaving frequency at high speeds. He finds that the rider lean torque control is relatively ineffective and, even with high weighting in the LQR formulation, the steer torque input dominates the optimal solution.

Y. Marumo and M.Nagai [16] designed both a PD controller with respect to roll angle and an LQR controller with full state feedback to stabilize the role of

Sharp's basic motorcycle model through steer torque. The intention is to have a steer-by-wire system, so, the rider can determine the desired roll angle with a joystick.

- **H∞ optimal control**

S.Mammar et al. [33] synthesized a PID steer torque controller with feedback capabilities using H∞ optimal for the stabilization. Once again, the roll angle was used as the input for the steer torque controller. The developed controller is shown to stabilize the motorcycle model, to be able to enter a constant radius corner and to be robust to parameter variations for this manoeuvre.
M. Yamakita et al. [23] Implemented a controller modified form with an additional H∞ controller. They showed successful roll stabilization of a robot scooter in which they only implement the roll stabilization control.
B. Thanh and M. Parnichkun [24] designed a controller with H2/H∞ techniques and applied it to a bicycling robot which uses a flywheel for stabilization. He compares it to a PD controller and a genetic algorithm and shows that it is more robust.

- **Fuzzy logic**

Fujii et al. [8] develop a fuzzy PD controller to ensure control of the angle of roullis. The gains of the fuzzy controller were determined using a genetic algorithm applied to the constant speed, constant roll angle situations for forwarding speeds ranging from 1 to 15m/s.
C. Chen and T.Dao [3–5] developed several fuzzy logic-based steer torque controllers of increasing complexity for a bike. First, they developed a PID steer torque controller for stabilizing the bicycle. Then, they investigated roll angle tracking by introducing a second fuzzy logic controller placed in parallel to the stabilizing fuzzy logic controller.
T.Dao, and C. Chen [58] used a genetic-fuzzy control system to control a bicycle to follow the desired roll angle with small tracking error at various speeds, the controller ensures the stability by producing a steering torque to control the roll angle, and another to control a path tracking that generates the reference roll angle.
H.Sharma and N.UmaShankar [25] they stabilized a simple bicycle model using fuzzy control rules to provide the desired roll correction based on the current steer and roll angles.

- **Rider control**

D.Weil and W.Zellner were the first authors studied in detail the interaction between rider and motorcycle; they investigated the rider control process. They published a series of papers; the main goal was that the primary objective of the rider was to stabilize the roll angle by using the transfer function.
An advanced model was introduced by T.Nishimi et al. [16]; they put the rider on motorcycle model, the rider can move laterally by their body to the motorcycle: the lower body move on the seat and a rotate relatively to the upper body. Also, the rider can lean on the motorcycle.
R.Lot and V.cossalter [11] developed the rider control law as a function of the curvilinear coordinate, and they developed two PD to stabilize the capsize mode and reduce error with respect the target trajectory and damping terms of rider model to stabilize the wobble mode.
Also, R.lot and M.Massaro [10] presented an improved version of the rider model [9] where the integral term was added to remove the error of roll angle. They also proposed a new methodology where the inner loop includes a PI Controller to ensure the desired roll angle and outer loop to follow the desired path.
T.Chu, and C. Chen [26] introduced a new control model of the bike-rider system based on MPC theory that actively stabilizes steering of the bicycle using the handlebar and leaning of the upper body to follow the desired roll angle.
R.Roland and J.Lynch [53] concentrated on the rider control model for path tracking. In their study, they made experimental tests to determine the effect of design parameters on the stability and manoeuvrability of the bicycle.
A.Schwab and J.Kooijman [55] demonstrated how a Whipple bike with rider model could be controlled by both steer torque or upper body lean torque. A transfer function has been introduced to control inputs around uncontrollable speeds to study and the controllability.
A.Schwab et al. [49] treated the impact of the passive rider on the lateral dynamics of a Whipple model of bike and the controllability of this later using both steer torque control and marginal for upper body lean motions.
R.Sharp [56] made some comparison between the control properties of the motorcycle such as small perturbation from straight-line motion and from cornering equilibrium states; steering control by handlebar torque and by rider upper body lean torque, etc. As a result, this comparison indicated that the significant parameter in the control of stability is the steering torque and the rider lean. He used speed, acceleration, load, rider weight as variables to discuss linear stability; and road unevenness to discuss nonlinear stability.

T.Katayama et al. [17] employed a motorcycle model with a rider model similar to the one used in [38] to study the important aspects of the driver control actions through the linearly linked torque to the roll angle. He compared the simulation results with the experiments that he has done.

IV. FUTURE PERSPECTIVE

During the past several years, the researchers made significant advances in the modelling of the two-wheeled vehicle. So, They have developed many aspects of modelling, stability and control. The papers that we cited before are identified on the three central fields of engineering, dynamics modelling, stability analysis, and control. Those topics are the most discussed by researchers.

We would like to conclude this paper by presenting a perspective of the future research with some of the potential research themes modelling of the rider by biomechanical approaches that are considered to be challenging but promising from the authors' points of view. Therefore future work will be treated the process of modelling and integrating of the active rider using the biomechanical approach, then, his impact in the security and stability in the powered two-wheeled vehicles, and finally to understand and define the concept of handling qualities.

V. CONCLUSION

This paper aims to give an extended state of the art of the dynamics and the stability of Two-Wheeled Vehicles. So, we have classified the existing research into three distinct categories and will investigate them: Dynamic Modelling that different concern approach to generate the equation of motion, Stability Analysis that concerns some works about the stability and mode of instability, and different strategy of control theory conducted in this field.

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