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Improving quality characteristics through combined control charts for Weibull distributed time between events

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***Abstract* — *The progress of the quality control techniques and new technological developments have led to high-quality processes in which small defects occur. However, when dealing with high-quality processes, the existing control charting schemes may face some difficulties. In this article, I have designed a mixed cumulative sum-exponentially weighted moving average control chart (MCE) for monitoring Weibull distributed time between events (TBE) with individual measurements and compare it with Weibull cumulative sum, Weibull exponentially weighted moving average, and mixed exponentially weighted moving average-cumulative sum (MEC) by transforming the Weibull data to the exponential data. A control chart's performance is evaluated by analyzing the Average run length (ARL) and the standard deviation of the run length (SDRL). The relative mean index (RMI) is also utilized to measure the proposed control chart's overall performance and three existing control charts. From real data, two illustrative examples show the application of existing control charts and the proposed control charts for monitoring Weibull distributed TBE.*

Keywords — *MCE, MEC, the time between events, WCUSUM, Weibull distribution, WEWMA.*

I. INTRODUCTION

When the production process is continuous, time is measured by units of produced quantities, and an event is the appearance of a defect in the product. So that time-between-events (TBE) is the number of products between two consecutively observed defects. According to [1], the TBE charts are based on the assumption that a homogeneous Poisson process can model the occurrence of events. Thus the time between two successive events follows an exponential distribution. Many researchers paid much attention to high-quality control charts because of technological advancement [2].

Presented the design and implementation procedures for both Poisson CUSUM and exponential CUSUM and for detecting either an increase or a decrease in the event occurrence rate [3]. Studied the design of TBE CUSUM and introduced the exponential CUSUM chart [4]. He presented the relative performance of exponential CUSUM and Poisson CUSUM charts for monitoring the rate of occurrences of events. Subsequently, Gan studied designs of one-and two-sided exponential EWMA methods based on the inter-arrival times of events, independent and identically distributed exponential random variables. Based on his comparison, the CUSUM chart is optimal for detecting the intended mean, and the EWMA chart is slightly less sensitive [5].

Proposed the application of CUSUM and CCC charts for the control of a high-quality process [6]. Gan and Chang [7] provided a FORTRAN program for computing both in-control and out-of-control ARL of the exponential EWMA chart. Presented the robustness of the CUSUM control chart for continuous TBE [8]. They reported an ARL study for the Weibull (fixed scale parameter) and the lognormal (fixed sigma parameter) distributions. Found that the simple, cumulative quantity control chart for monitoring several consecutive defects between r successive events (CQC-r) is more robust for small-sized shifts than EWMA and CUSUM in the two-sided case [1]. Both EWMA and CUSUM charts are used to detect a small-sized shift; however, it is unclear which one outperforms for TBE monitoring. For this purpose, compared to the TBE charts and concluded that for the processes with small improvements, exponential EWMA charts are slightly better than the exponential CUSUM charts [9]. Proposed the robustness of the EWMA chart with transformed exponential data for monitoring Weibull-distributed TBE data [10]. Studied the EWMA TBE charts with transformed Weibull data [11]. They found that the EWMA TBE chart with transformed Weibull data performs well in detecting the shift in scale parameter when the shape parameter is fixed.

Motivated by the estimation of parameter and its effect on the control chart detection ability, studied properties of the TBE EWMA chart based on the exponential distribution [12]. Studied the robustness of the exponential EWMA chart, which is very robust to departures from the exponential distribution to Weibull distribution regardless of the value of shape parameter [13]. Introduced monitoring time-between-events for health management and presented c charts, t charts, and exponential EWMA charts to show the control charts [14]. Addressed the problem of estimation in the CUSUM chart based on the exponential TBE to show that estimation error gets larger when the CUSUM chart is designed to detect smaller shifts in the mean [15]. Proposed Weibull cumulative sum (WCUSUM) control charts for monitoring the Weibull-distributed time between events [16]. Proposed a mixed EWMA-CUSUM (MEC) chart for monitoring the small shifts in the process mean for normal data [17]. Studied a hybrid EWMA (HEWMA) chart based on mixing two EWMA control charts for monitoring the small shifts in the process mean for normal data [18]. The results show that the HEWMA control chart can detect smaller shifts quicker than the CUSUM, EWMA, MEC control charts.

Introduced a mixed CUSUM-EWMA chart for monitoring the process location for normal data [19]. Proposed three control charts such as combined mixed EWMA-CUSUM (CMEC) control chart, combined mixed double EWMA-CUSUM (CMDEC) control chart, and combined CUSUM (CC) control chart for monitoring the normal process mean and variance simultaneously [20]. Studied a mixed EWMA–CUSUM control chart for monitoring the Weibull mean shift under a fixed shape parameter [22]. Recently, we proposed the WEWMA and MCE charts to monitor decreases in the mean of Weibull-distributed TBE observations by transforming the Weibull data to the exponential data [21].

In this article, I proposed the Mixed CUSUM-EWMA (MCE) chart for Weibull-distributed time between events. After investigating the literature, we realized that no comparative study is conducted on designing the (MCE) control chart for Weibull-distributed time between events. The proposed chart's URLs and SDRLs are evaluated through the Monte Carlo simulation approach, and the performance of the proposed chart is compared with the existing control charts. Real data is used to illustrate the application of the proposed control chart.

II. EXISTING CONTROL CHARTS

Nowadays, the advancement of quality control techniques and new technology results in high-quality processes in which fewer non- conforming items occur. In a TBE monitoring chart, a decrease in the mean of TBE can be monitored with a downward TBE and while an increase in the mean of TBE can be monitored with an upward TBE. Even though our prime priority is to detect increases in the defect rate, this article also considers decreases in the control charts' defect rate. Because as long as the process does not defect-free, all the assignable causes should be sought out and maintained to bring the process back to target or in control. For this reason, this article was conducted on monitoring both downward and upward TBE mean shifts.

Weibull-distributed TBE dataset is an individual measurement. Assume that Y_1, Y_2..., Y_t denotes a sequence of independent and identically distributed lifetime data, the random variable (Y) follows a Weibull distribution with the scale parameter θ and the shape parameter β, that is, $Y \sim W(\theta > 0, \beta > 0)$. The probability density function (pdf) and cumulative distribution function (CDF) of the Weibull distribution are given by

$$f(y|\beta,\theta) = \frac{\beta}{\theta}\left(\frac{y}{\theta}\right)^{\beta-1} \exp\left(-\left(\frac{y}{\theta}\right)^{\beta}\right), y > 0 \quad (1)$$

$$\text{F(y)} = \Pr(Y \le y) = 1 - \exp\left\{-\left(\frac{y}{\theta}\right)^{\beta}\right\}; y > 0 \quad (2)$$

where $y > 0, \theta > 0$, and $\beta > 0$. Investigated that monitoring the change of θ is identical to monitoring the mean shift of Weibull-distributed TBE for a fixed value of β and the scale parameter θ, has the same measurement units as y. [16]

$$X = Y^{\beta} \quad (3)$$

Then y is also Weibull-distributed with scale parameter θ^{β} and shape parameter 1.0 as an exponential variable. Therefore, if the shape parameter β can be assumed to be constant, the Weibull random variable's monitoring can be easily done by transforming to exponential first utilizing the formula depicted in (3).

The monitoring statistic of the WCUSUM control chart is as follows:

$$C_{w,\text{i}} = \max(0, C_{w,\text{i}-1} - y_i^{\beta} + k_w), C_{\text{w},0} = 0 \quad (4)$$

Where k_w is the reference parameter, which is determined by:

$$k_w = \frac{\beta \ln(\frac{\theta_1}{\theta_0})}{(\theta_0^{-\beta} - \theta_1^{-\beta})} \quad (5)$$

That θ_0 and θ_1 are in control parameter and out-of-control parameter, respectively. If $\text{C}_{w,t} \ge h_w$, it signals an out-of-control situation, where the control limit h_w is determined based on the desired in-control ARL a given value of k_w. According to Lucas[3], the control limit h_w is derived by:

$$h_w = h_t \theta_0^{\beta} \quad (6)$$

Where h_t is a normalized value from an ARL table? Normalized reference value k_t can also be obtained as follows: $k_t = k_w / \theta_0^{\beta}$ (7)

Weibull EWMA chart can be very effective for TBE monitoring when the event's occurrence rate is not constant. The monitoring statistics of the WEWMA control chart is expressed as follows:

$$E_{w,i} = \lambda y_i^{\beta} + (1-\lambda) E_{w,i-1}, E_{w,0} = \theta_0^{\beta} \quad (8)$$

The smoothing constant $0 < \lambda \leq 0.2$. The starting value is the target value $E_{w,0}$,i.e., the mean of the exponential distribution. Hence, the expectation and variance y^{β} can be obtained as $\mu_0 = E(y^{\beta}) = \theta_0^{\beta}$ and $\sigma_0 = \sqrt{Var(E_{w,i})} = \theta_0^{\beta}$. The fixed upper and lower control limits of the WEWMA control chart are obtained as: $UCL = \mu_0 + L\sigma_0 \sqrt{\frac{\lambda}{2-\lambda}}$,

$CL = \mu_0$

$$LCL = \mu_0 - L\sigma_0 \sqrt{\frac{\lambda}{2-\lambda}} \quad (9)$$

Where L_w is the design parameter which influences the width of the control limits? An out of control signal arise when $E_{w,i} > UCL_w$ or $E_{w,i} < LCL_w$, otherwise the process is in control. The recommended optimal design procedure of the WEWMA charts with a fixed shape parameter is utilized to compute WEWMA charts [10].

Proposed mixed EWMA–CUSUM (MEC) chart is stated as follows: [22].

Step-1: Draw a random sample from the Weibull distribution. Compute the EWMA statistic E_i using a transformed variable y_i^{β} as follows:

$$E_i = \lambda y_i^{\beta} + (1-\lambda) \mathrm{E}_{i-1}, \mathrm{E}_0 = \theta_0^{\beta} \quad (10)$$

where λ is the smoothing constant $0 < \lambda \leq 0.2.$

Step-2: Compute the CUSUM statistic $C_{\mathrm{E,i}}$ as follows:

$$C_{\mathrm{E,i}} = \max(0, C_{\mathrm{E,i-1}} - E_i + k_{EC}), C_{\mathrm{E,0}} = 0 \quad (11)$$

A reference parameter is determined by that and is in the control parameter and out-of-control parameter, respectively.

Step-3: If $C_{\mathrm{E,i}} > h_{EC}$, it signals an out-of-control

(7 situation, the control limit is determined based on the desired in-control ARL for a given value k_{EC} .

III. COMPUTATION OF PERFORMANCE METRICS

A control chart's performance is usually evaluated by analyzing the properties of its run length (RL) distribution. Describes that ARL0 denotes the ARL of the chart when the process is in-control (i.e. a false alarm performance). ARL1 denotes the out-of-control run length, and a chart with a small ARL_1 is preferred [23].

In this study, I have utilized the Monte Carlo simulation approach to evaluate the *proposed control chart's ARLs and SDRLs*. We have designed an algorithm in *R* language to calculate the value of *ARLs* and *SDRLs*. This algorithm is run 10,000 iterations to calculate the average of run lengths. In each iteration, one hundred thousand (10^4) data are generated from Weibull distribution. Based on these simulation results, the comparisons and

(9 discussions are studied for described control charts. Investigated that SDRL can be utilized to address the variation in RL values, unlike ARL and MDR, which provides information on RL's central tendency [24]. It can be obtained as follows:

$$SDRL = \sqrt{E(RL)^2 - \{(E(RL))^2\}} \; or \; SDRL = \sqrt{E(RL)^2 - ARL^2} \quad (12)$$

Han and Tsung Studied the relative mean index (RMI) to assess certain control charts' overall performance, and it can be computed by using the following formulas: [25].

$$RMI = \frac{1}{m} \sum_{i=1}^{m} \frac{ARL_{c_i} - ARL_{cs_i}}{ARL_{cs_i}} \quad (13)$$

The number of the shifts utilized ARL_{c_i} is the ARL_1 for certain control chart when the monitored parameter changes too c_i and ARL_{cs_i} is the smallest ARL_1 among all the charts scale changes of the four control charts. The smallest the RMI result, the better the control chart.

A. Design Parameters for Proposed Control Chart

A mixed CUSUM-EWMA control chart is described as follows:

Step-1**:** Draw a random sample from the Weibull distribution. Compute the statistic C_w based on the transformed data y_i^β as follows:

$$C_{w,i} = \max(0, C_{w,i-1} - y_i^\beta + k_{CE}), C_{w,0} = 0 \quad (14)$$

Where k_{CE} is a reference parameter, which is determined by

$$k_{EC} = \frac{\beta \ln(\frac{\theta_1}{\theta_0})}{(\theta_0^{-\beta} - \theta_1^{-\beta})} \quad (15)$$

Step-2**:** Compute the EWMA statistic $E_{C,i}$ as follows:

$$E_{C,i} = \lambda C_i + (1-\lambda)\,\mathrm{E}_{C,i-1}, \mathrm{E}_{C,0} = \theta_0^\beta \quad (16)$$

where λ is the smoothing constant $0 < \lambda \leq 0.2.$ The expectation and variance of C_i can be obtained as $\mu_{0;CE} = \text{mean}\,(C_i)$ and $\sigma_{0;CE} = \sqrt{Var(\mathrm{C}_i)}$? The fixed upper and lower control limits of the MCE control chart are obtained as:

$$UCL_{CE} = \mu_{0;CE} + L_{CE}\sigma_{0;CE}\sqrt{\frac{\lambda}{2-\lambda}},$$

$$CL = \mu_{0;CE}$$

$$LCL_{CE} = \mu_{0;CE} - L_{CE}\sigma_{0;CE}\sqrt{\frac{\lambda}{2-\lambda}} \quad (17)$$

Where L_{CE} is the design parameter that influences the control limits' width and determined based on the desired in-control ARL? An out of control signal arise when $E_{C,i} > UCL_{CE}$ or $E_{C,i} < LCL_{CE}$, otherwise the process is declared to be in control.

The design procedures of the mixed CUSUM-EWMA chart can be described as follows:

Step 1**:** Specify desired in-control *ARL*, and estimate the out of control scale shift (θ_1 / θ_0) to be detected swiftly;

Step 2: Choose λ a value starting from the smaller one such as 0.05,0.10, or 0.2 according to the out-of-control scale shift (θ_1 / θ_0). Because the smaller value λ is recommended for smaller process shifts.

Step 3: Obtain the corresponding L_{CE} value according to the shape parameter and control *ARL*.

Step 4**:** The *ARL* value for the mixed CUSUM-EWMA chart for Weibull distributed TBE process can be obtained using the Monte Carlo simulation approach. By generating a random sample of size 1 at each subgroup (denoted by Y_t) from the Weibull distribution with the specified parameters scale and shape for the control and control processes.

Two properties of RL values of the corresponding proposed control chart cases for both downward and upward TBE mean shifts are presented according to fixed-shaped parameters and scale changes . In this article, two performance metrics of RL distributions are computed under $ARL_0 = 370.$

B. Performance evaluation

Standardized downward shifted scales considered in this article are: These shifts also correspond to the standardized upward scale shift (defect rates).

For example, if the defects are Poisson distributed with a mean rate , a shift in the defect rate corresponds to the standardized exponential mean shift (TBE mean shift). More specifically, considered λ values are $0.05, 0.10, 0.20, 0.40, 0.50, 0.6, 0.8$ and 1.00 . The time between events distribution is assumed exponential. In the following sections, we investigate how upward and downward TBE control charts are affected when the chart is designed under the assumption of the exponential distribution. Still, the observations follow a Weibull distribution. For this robustness study, Weibull distributions with various fixed shape parameter (β) values are considered. The values of shape parameters (β) are determined $0.50, 0.75, 1.00, 1.20, 1.50$ and 2.00.

For the fixed value β, the parameter θ must be adjusted to give the desired control and out-of-control TBE scale c_0 and c_1 considered in the study, respectively. The scale parameter values needed to give the TBE mean for each value β used in this study. For example, when $\beta = 1.00$ to find the in-control value $c_0 = 1.00$ and the out-of-control value $c_1 = 0.50$, we can calculate by using the following formula for in-control (θ_0) and out-of-control (θ_1) mean values. Upward TBE means can also be computed similarly.

$$c_0 = \theta_0 \, \Gamma(1.00 + \tfrac{1.00}{\beta}) \quad ,$$

$$if \quad \theta_0 = 1.00, \; c_0 = 1.00 * \Gamma(1.00 + \tfrac{1.00}{1.00}) = 1.00$$

$$c_1 = \theta_1 \Gamma(1.00 + \tfrac{1.00}{\beta}) \quad ,$$

$$if \quad \theta_1 = 0.5, \; c_1 = 0.5 * \Gamma(1.00 + \tfrac{1.00}{1.00}) = 0.50$$

Table 1: Downward and upward in control (θ_0) and out-of-control (θ_1) scale parameter for different combinations of shape parameters (β) to obtain the desired TBE mean values respectively.

β	Time-between-event mean: $c = (\theta_1 / \theta_0)$									
	1.00	0.90	0.80	0.70	0.60	**0.50**	0.40	0.30	0.20	0.10
0.5000	**0.5000**	0.4500	0.4000	0.3500	0.3000	**0.2500**	0.2000	0.1500	0.1000	0.0500
0.7500	**0.8399**	0.7559	0.6719	0.5879	0.5039	**0.4199**	0.3360	0.2520	0.1680	0.0840
1.0000	**1.0000**	0.9000	0.8000	0.7000	0.6000	**0.5000**	0.4000	0.3000	0.2000	0.1000
1.2000	**1.0631**	0.9568	0.8505	0.7442	0.6379	**0.5315**	0.4252	0.3189	0.2126	0.1063
1.5000	**1.1077**	0.9970	0.8862	0.7754	0.6646	**0.5539**	0.4431	0.3323	0.2215	0.1108
2.0000	**1.1284**	1.01554	0.9027	0.7899	0.6770	**0.5642**	0.4514	0.3385	0.2257	0.1128

β	Time-between-event mean: $c = (\theta_1 / \theta_0)$									
	1.0	1.11	1.25	1.43	1.67	**2.00**	2.50	3.33	5.00	10.00
0.5000	**0.5000**	0.5550	0.6250	0.7150	0.8350	**1.0000**	1.2500	1.6650	2.5000	5.0000
0.7500	**0.8399**	0.9323	1.0499	1.2010	1.4026	**1.6798**	2.0997	2.7968	4.1994	8.3988
1.0000	**1.0000**	1.1100	1.2500	1.4300	1.6700	**2.0000**	2.5000	3.3300	5.0000	10.0000
1.2000	**1.0631**	1.1800	1.3289	1.5202	1.7754	**2.1262**	2.6577	3.5401	5.3154	10.6309
1.5000	**1.1077**	1.2296	1.3847	1.5841	1.8499	**2.2155**	2.7693	3.6887	5.5387	11.0773
2.0000	**1.1284**	1.2525	1.4105	1.6136	1.8844	**2.2568**	2.8209	3.7575	5.6419	11.2838

In the first section, performance results obtained using the Monte Carlo simulation approach are displayed. Computed in-control and out-of-control performance metrics (ARLs and SDRLs) for Weibull distributed time-between-events are presented in several tables. In the second section, these computed performance metrics are analyzed and compared to investigate the robustness of the upward and downward proposed and existing control charts.

Table 2: Downward ARL with different scale and shape parameters for the proposed control chart and other three control charts

β	θ_0, θ_1	Design Parameters	Chart type	c										
				1.00	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10	RMI
0.50	0.50, 0.25	h_t=7.035	WCUSUM	370.14	233.45	146.30	94.45	62.93	43.88	32.02	24.08	18.49	14.08	0.10
		L=2.498,λ=0.05	WEWMA	370.19	554.08	528.93	307.56	161.33	88.99	53.87	35.65	24.84	17.68	1.31
		h_t=1.047,λ=0.05	MEC	370.14	211.22	125.82	79.07	53.25	38.47	28.78	22.65	18.35	15.00	0.01
		L=4.656, λ=0.1	MCE	370.04	231.72	144.43	93.34	63.24	44.73	33.33	25.96	20.81	16.82	0.14
0.75	0.8399,0.4199	h_t=5.105	WCUSUM	370.17	203.34	112.81	64.31	40.29	26.87	19.42	14.73	11.57	9.26	0.12
		L=2.498,λ=0.05	WEWMA	370.19	581.82	341.09	146.86	73.72	44.16	29.90	22.00	17.04	13.53	1.14
		h_t=0.16012,λ=0.05	MEC	370.04	168.31	86.83	50.67	33.33	23.80	18.19	14.48	11.94	10.04	0.01
		L=5.351,λ=0.05	MCE	370.04	193.51	105.30	61.14	39.55	27.58	21.05	17.08	14.35	12.31	0.18
1.00	1.00, 0.50	h_t=3.859	WCUSUM	370.22	183.47	92.45	47.95	28.47	18.70	13.47	10.35	8.37	7.05	0.09
		L=2.498,λ=0.05	WEWMA	370.19	540.88	207.93	84.11	45.40	29.55	21.68	17.07	14.06	11.98	0.97
		h_t=0.480,λ=0.05	MEC	370.62	154.25	72.92	39.46	24.54	17.28	13.25	10.85	9.27	8.17	0.03
		L=5.604,λ=0.05	MCE	370.04	166.64	81.48	44.30	27.59	19.44	15.01	12.45	10.77	9.54	0.15
1.20	1.0631,0.5315	h_t=3.144	WCUSUM	370.49	172.41	80.79	40.33	22.75	14.75	10.66	8.30	6.86	6.03	0.12
		L=2.498,λ=0.05	WEWMA	370.19	481.13	146.38	60.57	34.78	23.98	18.37	14.99	12.82	11.29	0.98
		h_t=0.1687,λ=0.1	MEC	370.14	134.72	58.49	31.22	19.70	14.01	10.89	9.04	7.86	7.04	0.04
		L=5.623,λ=0.05	MCE	370.04	152.62	68.97	36.19	21.86	15.45	12.09	10.18	8.95	8.07	0.17
1.50	1.1077,0.5539	h_t=2.359	WCUSUM	370.61	159.15	68.63	31.82	17.37	11.02	8.00	6.37	5.39	5.00	0.15
		L=2.498,λ=0.05	WEWMA	370.19	369.63	95.36	42.36	26.27	19.28	15.50	13.23	11.79	11.00	1.02
		h_t=0.0059,λ=0.1	MEC	370.15	110.62	43.98	23.39	15.08	11.12	8.89	7.57	6.73	6.01	0.08
		L=5.111,λ=0.05	MCE	370.02	132.79	55.23	27.46	16.36	11.53	9.17	7.86	7.12	6.93	0.18
2.00	1.1284,0.5642	h_t=1.521	WCUSUM	370.66	145.58	56.31	23.82	12.12	7.57	5.59	4.57	4.03	4.00	0.14
		L=2.498,λ=0.1	WEWMA	370.19	231.00	56.43	28.53	19.58	15.41	13.14	11.79	11.01	10.99	1.07
		h_t=0.087,λ=0.2	MEC	370.70	102.74	36.71	17.63	10.73	7.79	6.36	5.57	5.03	5.00	0.09
		L=4.265,λ=0.05	MCE	370.07	113.15	41.63	19.45	11.28	7.90	6.44	5.65	5.06	5.00	0.13

Table 3: Downward SDRL with different scale and shape parameters for the proposed and other three control charts

β	θ_0, θ_1	Design Parameters	Chart type	c										
				1.00	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10	RMI
0.50	0.50, 0.25	h_t=7.035	WCUSUM	349.26	211.49	123.36	72.39	41.92	24.45	14.10	8.27	4.67	2.23	0.19
		L=2.498,λ=0.05	WEWMA	359.13	542.63	515.18	276.50	131.46	60.78	30.08	14.67	7.20	3.05	1.86
		h_t=1.047,λ=0.05	MEC	359.00	193.60	106.76	59.33	33.91	20.21	11.68	6.71	3.68	1.76	0.00
		L=4.656, λ=0.1	MCE	343.60	208.19	120.63	69.00	40.36	23.44	13.23	7.54	4.17	1.98	0.12
0.75	0.8399,0.4199	h_t=5.105	WCUSUM	357.71	189.15	97.74	49.23	25.87	13.44	7.38	4.06	2.11	0.95	0.27
		L=2.498,λ=0.05	WEWMA	359.13	572.22	312.29	118.48	46.98	21.59	10.66	5.45	2.76	1.16	1.39
		h_t=0.16012,λ=0.05	MEC	360.88	152.94	68.44	34.22	18.40	10.11	5.82	3.28	1.76	0.81	0.00
		L=5.351,λ=0.05	MCE	349.60	176.36	87.04	43.85	22.83	11.67	6.31	3.48	1.84	0.84	0.13
1.00	1.00, 0.50	h_t=3.859	WCUSUM	355.61	171.68	80.09	36.35	17.38	8.70	4.47	2.35	1.17	0.48	0.25
		L=2.498,λ=0.05	WEWMA	359.13	529.76	178.69	56.36	22.88	10.35	5.26	2.77	1.38	0.60	0.91
		h_t=0.480,λ=0.05	MEC	356.52	141.81	59.66	27.46	13.08	6.76	3.47	1.86	0.95	0.39	0.00
		L=5.604,λ=0.05	MCE	357.58	152.52	67.36	30.55	14.51	7.30	3.70	1.94	0.99	0.54	0.10
1.20	1.0631,0.5315	h_t=3.144	WCUSUM	361.00	163.24	70.48	30.69	13.54	6.46	3.20	1.61	0.81	0.19	0.28
		L=2.498,λ=0.05	WEWMA	359.13	460.17	118.07	35.67	14.06	6.67	3.42	1.79	0.89	0.46	0.84
		h_t=0.1687,λ=0.1	MEC	357.87	122.36	46.79	20.12	9.57	4.80	2.47	1.29	0.69	0.20	0.01
		L=5.623,λ=0.05	MCE	356.48	140.31	56.52	24.44	11.03	5.38	2.60	1.34	0.70	0.26	0.13
1.50	1.1077,0.5539	h_t=2.359	WCUSUM	368.78	152.51	60.88	24.15	10.13	4.49	2.08	1.03	0.53	0.03	0.51
		L=2.498,λ=0.05	WEWMA	359.13	344.76	67.00	20.11	8.16	3.91	2.03	1.05	0.57	0.02	0.60
		h_t=0.0059,λ=0.1	MEC	359.52	98.52	32.80	13.10	6.03	3.04	1.58	0.84	0.51	0.12	0.55
		L=5.111,λ=0.05	MCE	360.68	122.37	46.20	18.08	7.86	3.53	1.68	0.84	0.33	0.25	1.31
2.00	1.1284,0.5642	h_t=1.521	WCUSUM	371.29	142.27	51.25	18.16	6.93	2.76	1.22	0.62	0.18	0.00	0.45
		L=2.498,λ=0.1	WEWMA	359.13	203.37	32.12	9.58	4.08	1.99	1.01	0.57	0.11	0.05	0.15
		h_t=0.087,λ=0.2	MEC	365.31	93.55	28.60	10.46	4.40	1.91	0.91	0.55	0.18	0.00	0.08
		L=4.265,λ=0.05	MCE	373.25	105.13	34.23	12.47	5.11	2.07	0.96	0.56	0.25	0.00	0.23

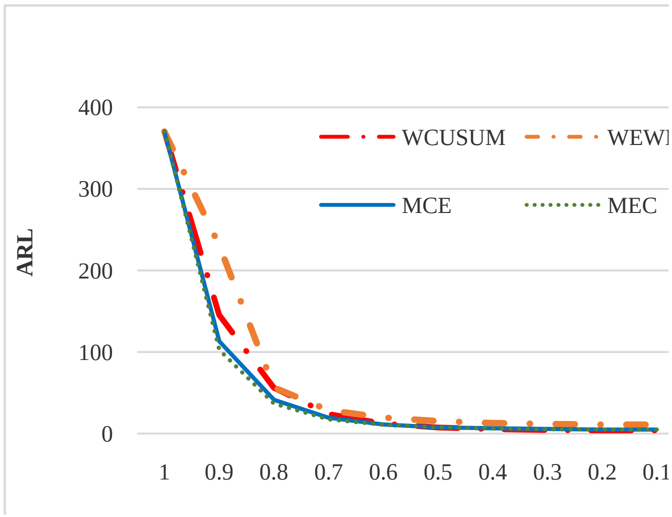
IV. PERFORMANCE COMPARISONS

Figure 1 (a) and (b) shows the ARL and SDRL comparison of the proposed and other three control charts respectively. The proposed control chart is more sensitive or robust than the WCUSUM chart in detecting small to moderate scale changes and better than the WEWMA chart for all scale changes. However, the WCUSUM chart performs better than the proposed chart for large scale changes and the WEWMA chart is biased at $(c = 0.90 \, and \, 0.80)$ for shapeparameters $(\beta = 0.50, 0.75, 1.00 \, and \, 1.20)$ and the worst chart among others. MEC chart performs better than the MCE chart.

From the downward SDRL Table 3, it can be observed that, the proposed control chart outperforms the WCUSUM and the WEWMA charts in detecting small to large scale changes. The MEC chart is better than the proposed charts. The WEWMA chart outperforms the WCUSUM, and MCE charts when the shape parameter $(\beta = 2.0)$ and $(c = 0.80 - 0.50)$. However, it's biased at $(c = 0.90 \, and \, 0.80)$ for shape parameters $(\beta = 0.50, 0.75, 1.00 \, and \, 1.20)$ and the worst chart under comparison.

From the RMI value in Table 2, it can be observed that the RMI value of the WCUSUM chart is 0.14, the proposed chart is 0.13, the MEC is 0.09 and the WEWMA is 1.07 when the shape parameter is 2.0. Based on this, the proposed chart is the best chart in detecting small scale changes quickly next to MEC chart for decreasing of time between events mean. Moreover, from the tables and the figures, it can be observed that the ARL of the WEWMA chart with $\lambda = 0.05$ is biased for different values shape parameters. It's ARL_1 are greater than ARL_0. For example, when the scale change $(c = 0.90, 0.80 \; for \; \beta = 0.50)$ and $(c = 0.90 \; for \; \beta = 0.75 - 1.20)$, the calculated $ARL_1 > ARL_0$ valuesare: $(ARL_1 = 554.08, 528.93 \; for \; \beta = 0.50)$, and $(ARL_1 = 581.82, 540.88, 481.13)$ for $(\beta = 0.75, 1.0, 2.0)$ respectively. From the RMI value in Table 3, we can see that the RMI value of the WCUSUM chart is 0.28, the proposed chart is 0.13, the MEC is 0.01 and the WEWMA is 0.84 when the shape parameter is 1.2. According to this results, the proposed chart is the performs better than the WCUSUM and the WEWMA charts in detecting small scale changes quickly next to MEC. The worst control chart for process deterioration is the WEWMA chart with ARL RMI value 1.30 and SDRL RMI value 1.86.

(a)



(b)

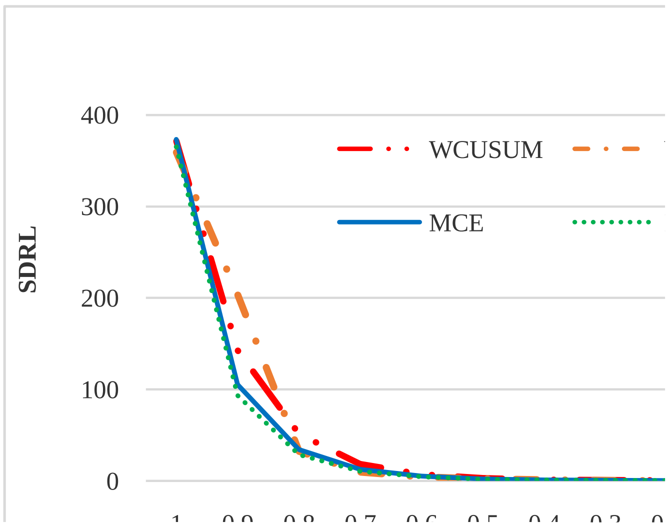


Figure 1. **(a)** ARL, **(b)** SDRL comparison of the existing and the proposed control charts for downward TBE mean with $(\beta = 2.0, \theta_0 = 1.1284, \theta_1 = 0.5642)$ respectively.

Section two is about monitoring the increasing of the scale parameter of Weibull-distributed time between events. The ARL and SDRL metrics are depicted in Table 4 and 5 for upward TBE respectively. Figure 2 shows that ARL and SDRL comparison of proposed control chart with other three control charts for upward TBE scale parameterchanges.

Table 4: Upward ARL with different scale and shape parameters for the proposed control chart and other three control charts

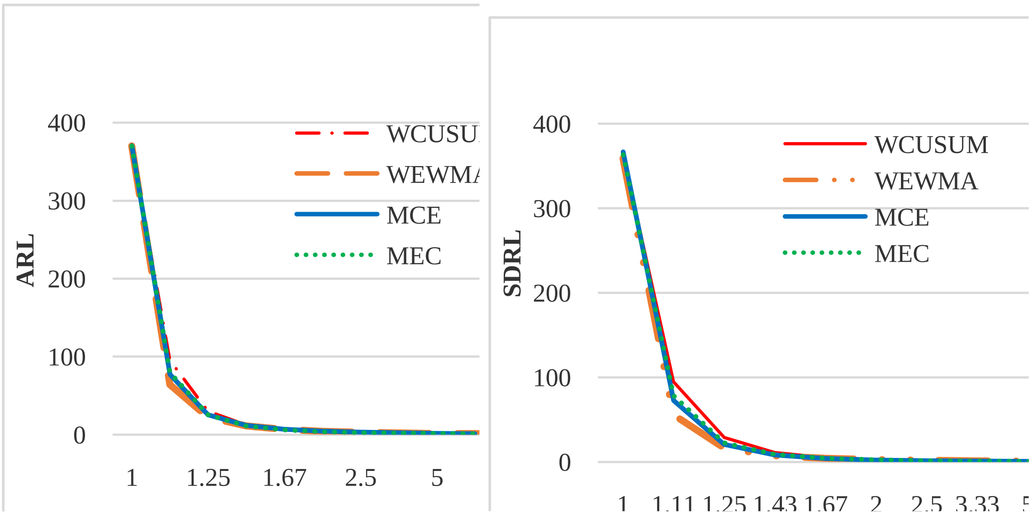
β	θ_0, θ_1	Design Parameters	Chart type	c										
				1.00	1.11	1.25	1.43	1.67	2.00	2.50	3.33	5.00	10.00	RMI
0.50	0.50, 1.0	h_t=9.282	WCUSUM	370.06	216.95	129.40	79.35	51.22	34.01	23.01	15.54	10.38	6.09	0.08
		L=2.498, λ=0.05	WEWMA	370.19	222.39	129.44	77.77	49.02	31.98	21.4	14.32	9.4	5.53	0.03
		h_t=1.3277,λ=0.05	MEC	370.05	202.39	116.56	71.92	47.11	32.51	22.96	16.56	11.84	7.89	0.09
		L=5.326, λ=0.05	MCE	370.06	213.47	127.54	79.74	53.11	36.68	26.67	19.47	14.25	9.73	0.24
0.75	0.75, 1.6798	h_t=7.778	WCUSUM	370.06	179.41	91.30	50.42	29.77	18.56	12.19	8.08	5.21	3.06	0.07
		L=2.498,λ=0.05	WEWMA	370.19	173.96	84.38	45.87	27.62	17.68	11.80	7.91	5.14	3.05	0.03
		h_t=0.269,λ=0.05	MEC	370.05	159.84	77.30	43.36	26.81	17.62	12.06	8.31	5.59	3.41	0.03
		L=5.795, λ=0.05	MCE	370.27	172.33	87.40	49.84	31.28	21.17	15.07	11.09	8.03	5.33	0.27
1.00	1.00,2.0	h_t=6.823	WCUSUM	370.01	154.17	69.60	35.00	19.36	11.89	7.60	4.97	3.22	1.95	0.05
		L=2.498, λ=0.05	WEWMA	370.19	137.63	58.75	30.67	18.03	11.67	7.73	5.17	3.37	2.04	0.02
		h_t=0.8459,λ=0.1	MEC	370.06	138.29	60.62	31.44	18.30	11.87	8.09	5.57	3.77	2.35	0.07
		L=5.837, λ=0.05	MCE	370.17	142.47	63.40	34.04	20.55	13.85	9.88	7.21	5.15	3.32	0.26
1.20	1.0631,2.1262	h_t=6.251	WCUSUM	370..1	137.59	56.90	27.21	14.62	8.81	5.59	3.66	2.41	1.54	0.07
		L=2.498, λ=0.05	WEWMA	370.19	114.73	46.04	23.62	13.88	8.96	5.88	3.93	2.60	1.61	0.03
		h_t=0.387,λ=0.1	MEC	370.26	118.12	47.88	23.93	13.74	8.72	5.77	3.88	2.62	1.63	0.03
		L=5.665, λ=0.05	MCE	370.06	123.46	51.16	26.29	15.56	10.48	7.48	5.41	3.80	2.40	0.26
1.50	1.1077,2.2155	h_t=5.606	WCUSUM	370.06	118.88	44.05	19.47	10.19	6.03	3.85	2.58	1.75	1.24	0.09
		L=2.498, λ=0.05	WEWMA	370.19	90.84	33.58	16.91	10.01	6.42	4.22	2.87	1.91	1.29	0.05
		h_t=0.0634,λ=0.1	MEC	370.26	95.27	34.53	16.59	9.44	5.90	3.88	2.63	1.79	1.26	0.01
		L=5.503, λ=0.05	MCE	370.06	101.86	38.17	18.73	11.11	7.43	5.26	3.74	2.62	1.63	0.25
2.00	1.1284,2.2568	h_t=4.850	WCUSUM	370.40	95.30	30.25	12.42	6.28	3.75	2.47	1.71	1.29	1.07	0.10
		L=2.498, λ=0.05	WEWMA	370.19	63.97	22.26	11.15	6.54	4.17	2.79	1.92	1.38	1.09	0.06
		h_t=0.406,λ=0.2	MEC	370.06	79.72	25.12	10.99	5.99	3.72	2.50	1.75	1.31	1.07	0.04
		L=4.696,λ=0.05	MCE	370.33	77.11	25.38	11.89	6.95	4.59	3.22	2.27	1.58	1.15	0.18

Table 5: Upward SDRL with different scale and shape parameters for the proposed control chart and other three control charts

β	θ_0,θ_1	Design Parameters	Chart type	c										
				1.00	1.11	1.25	1.43	1.67	2.00	2.50	3.33	5.00	10.00	RMI
0.50	0.50, 1.0	h_t=9.282	WCUSUM	355.44	204.68	117.74	67.71	41.28	24.95	15.60	9.76	6.08	3.53	0.11
		L=2.498, λ=0.05	WEWMA	359.13	213.05	122.09	69.84	40.61	24.88	15.47	9.66	5.99	3.36	0.11
		h_t=1.3277,λ=0.05	MEC	358.69	186.84	104.85	60.25	35.44	21.97	13.80	8.58	5.44	3.22	0.00
		L=5.326, λ=0.05	MCE	349.81	195.02	110.84	63.58	38.20	22.79	14.28	8.85	5.64	3.40	0.04
0.75	0.75, 1.6798	h_t=7.778	WCUSUM	358.33	174.54	84.39	43.73	23.57	13.47	8.20	5.10	3.18	1.83	0.12
		L=2.498,λ=0.05	WEWMA	359.13	166.81	76.12	37.65	20.93	12.36	7.75	4.90	3.13	1.82	0.02
		h_t=0.269,λ=0.05	MEC	364.84	151.43	69.08	34.88	19.77	11.74	7.33	4.70	3.03	1.82	0.03
		L=5.795, λ=0.05	MCE	355.62	163.09	75.29	38.85	21.04	12.02	7.27	4.67	3.06	1.92	0.04
1.00	1.00,2.0	h_t=6.823	WCUSUM	356.16	150.37	64.81	30.14	15.18	8.69	5.13	3.21	2.01	1.11	0.13
		L=2.498, λ=0.05	WEWMA	359.13	129.84	50.58	23.68	12.72	7.64	4.76	3.14	2.02	1.15	0.02
		h_t=0.8459,λ=0.1	MEC	361.42	133.35	53.61	25.37	13.06	7.61	4.65	2.98	1.95	1.19	0.03
		L=5.837, λ=0.05	MCE	362.52	136.65	53.97	25.87	13.11	7.45	4.58	2.98	2.00	1.28	0.04
1.20	1.0631,2.1262	h_t=6.251	WCUSUM	362.79	135.31	52.68	23.76	11.59	6.49	3.83	2.38	1.46	0.79	0.16
		L=2.498, λ=0.05	WEWMA	359.13	107.32	37.77	17.50	9.26	5.63	3.61	2.38	1.50	0.84	0.02
		h_t=0.387,λ=0.1	MEC	360.94	113.83	42.19	19.28	9.90	5.72	3.54	2.30	1.47	0.84	0.05
		L=5.665, λ=0.05	MCE	361.41	117.61	43.52	19.61	9.68	5.59	3.43	2.28	1.54	1.02	0.08
1.50	1.1077,2.2155	h_t=5.606	WCUSUM	363.31	117.54	41.29	16.98	8.15	4.36	2.63	1.64	0.99	0.50	0.19
		L=2.498, λ=0.05	WEWMA	359.13	82.65	26.23	11.76	6.43	3.95	2.53	1.69	1.06	0.56	0.03
		h_t=0.0634,λ=0.1	MEC	358.92	91.47	29.88	12.85	6.71	3.97	2.50	1.61	1.79	1.26	0.28
		L=5.503, λ=0.05	MCE	359.19	97.46	32.01	13.40	6.74	3.84	2.43	1.63	1.13	0.73	0.12
2.00	1.1284,2.2568	h_t=4.850	WCUSUM	363.80	94.77	28.82	10.84	4.93	2.71	1.63	0.98	0.56	0.26	0.24
		L=2.498, λ=0.05	WEWMA	359.13	55.50	16.21	7.19	4.03	2.50	1.65	1.06	0.64	0.31	0.15
		h_t=0.406,λ=0.2	MEC	363.34	78.38	22.64	8.81	4.33	2.50	1.57	0.98	0.58	0.28	0.08
		L=4.696,λ=0.05	MCE	366.76	72.60	20.93	8.44	4.14	2.43	1.57	1.08	0.72	0.38	0.16

Based on the upward ARL results provided in table 4, the WEWMA control chart is very sensitive than the WCUSUM and MCE charts in detecting small to large scale changes quickly. The WEWMA chart also very sensitive than the MEC in detecting small scale changes quickly when shape parameter increases $(\beta = 1.0 - 2.0)$. However, the MEC chart outperforms the control charts under comparison and the proposed chart is better than the WCUSUM and WEWMA charts for small scale change when shape parameter is $(\beta = 0.50 \, and \, 0.75)$. On the other hand, the WCUSUM chart has better performance than the MCE and WEWMA in detecting large scale changes except for shape parameter $(\beta = 0.50 \, and \, 0.75)$. It can be observed that the WEWMA chart and MEC chart are the best control chart with the smallest RMI, 0.06 and 0.04 respectively to detect the increasing of time between events mean swiftly. Subsequently, from SDRL results provided in the table 5, we can see that, the MEC and the WEWMA charts are better than other control charts under comparison with the smallest RMI, 0.08 and 0.15 respectively to detect the increasing of time between events mean quickly. The worst control charts when the shape parameter becomes large is the WCUSUM chart with RMI value 0.24, for shape parameter $(\beta = 2.0)$.

For the WCUSUM and the proposed control charts, the in-control chart performance (TBE mean, $c = 1$) varies and becomes increasing for values of the fixed shape parameter $(\beta = 0.50 - 2.0)$.



(a) (b)

Figure 2. **(a)** ARL **(b)** SDRL comparison of the existing and the proposed control charts for upward TBE mean with $(\beta = 2.0 \, , \theta_0 = 1.1284 \, and \, \theta_1 = 2.2568)$ respectively.

The main findings for the proposed control chart based on the results are as follows:

The following conclusions can be drawn by summarizing the analysis of results above:

1. Among from the downward Weibull distributed TBE charts, when the TBE mean decreases or the process deteriorates, the proposed chart is outperformed the WCUSUM chart in detecting small to moderate scale changes swiftly and better than the WEWMA for all scale changes based on ARL and SDRL performance metrics. The MEC chart performs better than the proposed control charts. WEWMA chart is more sensitive than other control charts under comparison in detecting small to moderate scale changes $(c = 0.8 - 0.5)$, when $(\beta = 2.0)$. In general view comparison for both charts, i.e., the proposed chart is more sensitive in detecting small to moderate deterioration, while the WCUSUM is suitable for large deterioration except for shape parameter $(\beta = 0.5)$. For the proposed chart, the larger the value of λ , the better the performance of the chart for large deterioration, though at the expense of large ARL_1 probability. However, MEC chart performs better than other control charts under comparison.

2. When the process improvement is small and the shape parameter is moderate to large $(\beta = 1.0 - 2.0)$, the WEWMA chart is most sensitive than all other charts based on ARL and SDRL comparison in detecting small scale changes swiftly. On the contrary, the MEC chart is more robust than the WEWMA and others in terms of ARL and SDRL in detecting small scale changes quickly when shape parameter is $(\beta = 0.5 - 0.75)$.

However, the proposed chart outperforms the WEWMA chart in detecting small scale changes quickly based on ARL and better than the WCUSUM in detecting small to moderate scale changes quickly based on SDRL.

Therefore, the proposed chart is the best control chart with shape parameter $(\beta = 2.0)$ in detecting the process deterioration of Weibull-distributed TBE quickly next to MEC control chart. On the other hand, the MEC and the WEWMA control charts are also the best control chart in detecting process improvement swiftly when the shape parameter is small $(\beta = 0.5 - 0.75)$ and moderate to large $(\beta = 1.0 - 2.0)$ respectively.

V. ILLUSTRATIVE EXAMPLES

The illustrative examples are used to show the applicability and efficiency of the proposed control chart and other three existing control charts for Weibull distributed TBE .

Example 1: The first example is the real data collected from the Table 6 to show the application of the proposed charting schemes [14]. The data consist of the time intervals in days between maintenance record of the failed printers it fixed every month (i.e., the events) during May 2009 (30 observations in total) in a printer company in China. As in [14], I considered the first $m = 10$ observations to be from the in-control process, from which the mean time to failure can be estimated as that 20.562 days. These failure times can be modeled as Weibull-distributed TBE.

The design parameters to calculate the control limits for the different control charts are as follows: for the WCUSUM: $h_t = 4.372,$ for the WEWMA: $L = 2.4975, \lambda = 0.05,$ for the MEC: $h_t = 0.0245, \lambda = 0.05$ and for the proposed chart: $L = 5.4658, \lambda = 0.05$ in downward TBE mean The Weibull parameters of the first 10 failure times data are $\theta_0 = 19.2993$ and $\beta_0 = 0.8844.$ In this example, $\mu_0 = 13.7067$ and $\sigma_0 = 13.7067$. The next 20 failure times data are $\theta_1 = 9.6497$ and $\beta_1 = 0.8844.$, so as to simulate the downward TBE mean with $c = 0.5.$ The downward TBE mean of the failure time decreases to $\mu_1 = 7.4251$ and the standard deviation decreases to $\sigma_1 = 7.4251.$ The plots of these data are depicted in Figure 3. The calculated control limits for the WCUSUM, the WEWMA, MEC and the MCE control charts are: $(h_w = 59.93, UCL = 19.19, LCL = 8.23, h_w = 0.34 \text{ and } UCL = 8.34, LCL = 0.85).$

Table 6 Printer failure time dataset for downward TBE mean

Sample no.	Y	$X = Y^{\beta}$	MEC	$WCUSUM$	$WEWMA$	MCE
1	16.55	11.96479	0.00	0.00	13.61961	4.3678
2	1.00	1.0000	0.00	8.93207	12.98863	4.59602
3	21.13	14.8505	0.00	4.01363	13.08172	4.5669
4	12.88	9.585386	0.00	4.36031	12.9069	4.55657
5	16.18	11.72791	0.00	2.56446	12.84795	4.45696
6	1.88	1.747693	0.00	10.74884	12.29294	4.77156
7	44.13	28.48411	0.00	0.00	13.1025	4.53298
8	81.00	48.7376	0.00	0.00	14.88425	4.30633
9	6.90	5.519225	0.00	4.41284	14.416	4.31165
10	3.99	3.400168	0.00	10.94474	13.86521	4.64331
11	7.08	5.646371	0.00	15.23043	13.97907	5.17267
12	5.01	4.158498	0.00	21.004	13.45614	5.96423
13	5.98	4.863115	0.00	26.07295	12.96721	6.96967
14	3.03	2.665556	0.00	33.33946	12.49413	8.28816
15	8.10E-05	0.000241	0.00	43.27128	11.92513	10.03731
16	79.00	47.67178	0.00	5.53156	11.49435	9.81203
17	1.69	1.590534	0.00	13.87309	12.64864	10.01508
18	8.02	6.304517	0.00	17.50064	12.55782	10.38936
19	0.17	0.208646	0.00	27.22406	12.08096	11.23109
20	0.04	0.058031	0.00	37.0981	11.72366	12.52444
21	2.95	2.603218	0.00	44.42694	11.27347	14.11957
22	5.05	4.187848	0.00	50.17116	10.93525	15.92215
23	37.01	24.37931	0.00	35.72392	10.85754	16.91224
24	3.81	3.26415	0.00	42.39183	10.34413	18.18622
25	3.99	3.400168	0.00	48.92373	10.28341	19.72309
26	17.29	12.43672	0.00	46.41907	9.83529	21.05789
27	2.88	2.548512	0.00	53.80262	9.80796	22.69513
28	1.76	1.648661	0.275506	62.08603	9.808	24.66467
29	10.19	7.791647	0.644257	64.22645	9.94071	26.64276
30	34.12	22.68784	0.356782	51.47067	9.98561	27.88416

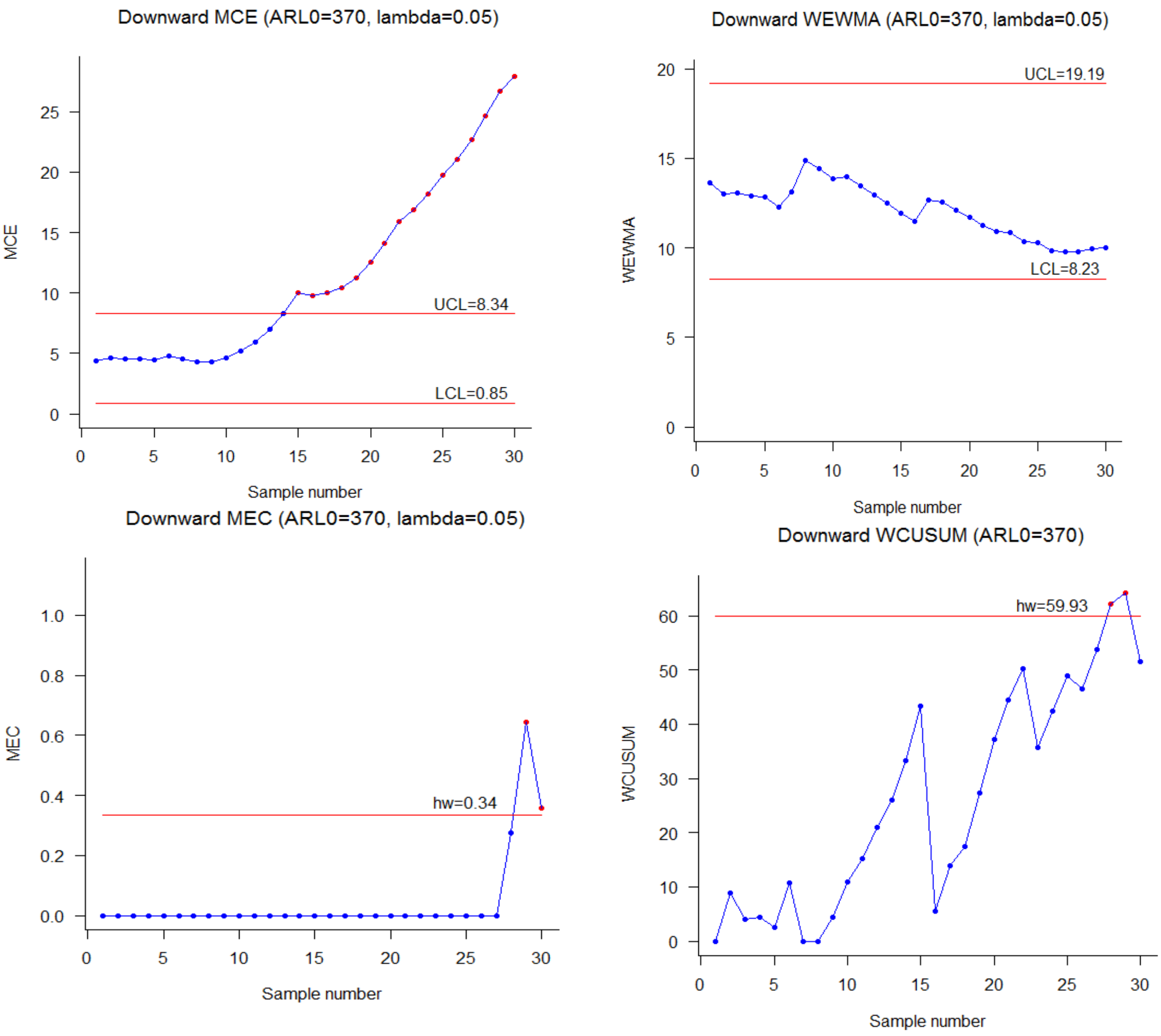


Figure 3. Plot of downward proposed, MEC, WEWMA and WCUSUM control charts $(\lambda = 0.05)$ for printer failure data set.

It can be seen from Figure 3, the proposed, MEC and WCUSUM control charts can detect out-of-control situation on the 15th, 29th and 28th sample, whereas from Figure 3, the WEWMA chart failed to detect the out-of-control situation and declared shifted process as in control. It means that the WEWMA chart is failed to monitor Weibull-distributed TBE of printer failure dataset. Conversely, the proposed control chart raised an alarm faster than the other control charts under comparion.

Example 2: I utilized the data from the Table 7 (also Jarett 1979) in order to illustrate the application of the proposed charting schemes [26]. The data consist of the time intervals in days between explosions in coal mines (i.e., the events) from 15 March 1981 to 22 March 1962 (190 observations in total) in Great Britain. As in [26], we consider the first $m = 30$ observations to be from the in-control process, from which we estimate that $\theta_0 \approx 0.0081$ (or, equivalently, the mean TBE is, approximately,123 days). In the sequel, we assume that this is the true in-control value θ_0. Since our numerical analysis showed that $(r = 4)$ is the best choice, we apply the t_4-chart, with or without run rules for these data. Thus, the remaining 160 observations are first converted by accumulating a set of four consecutive failure times and the corresponding observations. These are the times until the fourth failure, and are used for monitoring the process in order to detect a change in the mean TBE; an increase (which means process improvement).

The design parameters to calculate the control limits for the different control charts are as follows: for the WCUSUM: $h_t = 5.38598$, for the WEWMA: $L = 2.4975, \lambda = 0.05$, for the MEC: $h_t = 4.41659, \lambda = 0.75$ and for the proposed chart: $L = 5.32857, \lambda = 0.05$ in upward TBE mean. The first 30 failure times data Weibull parameters are $\theta_0 = 723.58$ and $\beta_0 = 1.6286.$ In this example, $\mu_0 = 45389.96$ and $\sigma_0 = 45389.96$. The next 10 failure times data are $\theta_1 = 1447.16$ and $\beta_1 = 1.6286.$, so as to simulate the upward TBE mean with $c = 2.0.$

The upward TBE mean of the failure time increases to $\mu_1 = 140351.60$ and the standard deviation increases to $\sigma_1 = 140351.60.$ The calculated control limits for the WCUSUM, the WEWMA, MEC and the MCE control charts for the downward and upward TBE mean are:

$(\text{h}_w = 244469.40, UCL = 63542.25, LCL = 27237.68, h_w = 200468.8 \text{ and } UCL = 142977.90, LCL = 0.00)$

Table 7 Coal mining data set for upward TBE mean

Sample no.	Y	$X = Y^{\beta}$	MEC	$WCUSUM$	$WEWMA$	MCE
1	426	19153.87	0.00	0.00	44078.16	45949.29
2	605	33913.05	0.00	0.00	43569.9	43651.82
3	795	52910.02	0.00	0.00	44036.91	41469.23
4	315	11715.17	0.00	0.00	42420.82	39395.77
5	373	15427.13	0.00	0.00	41071.13	37425.98
6	414	18282.97	0.00	0.00	39931.73	35554.68
7	300	10820.31	0.00	0.00	38476.16	33776.95
8	274	9335.13	0.00	0.00	37019.1	32088.1
9	222	6626.29	0.00	0.00	35499.46	30483.7
10	392	16727.29	0.00	0.00	34560.86	28959.51
11	842	58098.28	0.00	0.00	35737.73	27511.54
12	504	25187.11	0.00	0.00	35210.2	26135.96
13	416	18427.03	0.00	0.00	34371.04	24829.16
14	272	9224.42	0.00	0.00	33113.71	23587.7
15	228	6920.42	0.00	0.00	31804.04	22408.32
16	390	16588.53	0.00	0.00	31043.27	21287.9
17	527	27085.74	0.00	0.00	30845.39	20223.51
18	494	24378.32	0.00	0.00	30522.04	19212.33
19	548	28865.43	0.00	0.00	30439.21	18251.71
20	501	24943.40	0.00	0.00	30164.42	17339.13
21	557	29641.47	0.00	0.00	30138.27	16472.17
22	260	8570.88	0.00	0.00	29059.9	15648.56
23	872	61507.09	0.00	0.00	30682.26	14866.14
24	684	41416.40	0.00	0.00	31218.97	14122.83
25	1334	122921.76	27322.19	47191.6	35804.11	15776.27
26	1575	161098.36	98178.89	132559.8	42068.82	21615.44
27	2030	243549.95	241757.9	300379.6	52142.88	35553.65
28	1126	93268.59	290806.5	317918	54199.16	49671.87
29	762	49380.09	283306.1	291567.9	53958.21	61766.67
30	1480	145575.48	333814.9	361413.2	58539.07	76749
31	3020	465092.47	638463.9	750775.5	78866.74	110450.3
32	2731	394814.89	953939.7	1069860	94664.15	158420.8
33	632	36412.28	1003320	1030542	91751.55	202026.9
34	730	46047.66	993403.4	1000860	89466.36	241968.6
35	991	75754.00	990942.1	1000884	88780.74	279914.3
36	1323	121275.30	1024486	1046429	90405.47	318240
37	495	24458.74	994418	995157.4	87108.13	352085.9
38	1962	230403.78	1102906	1149831	94272.92	391973.2
39	271	9169.25	1080108	1083270	90017.73	426538
40	5308	1165259.06	1891287	2172442	143761.9	513833.2

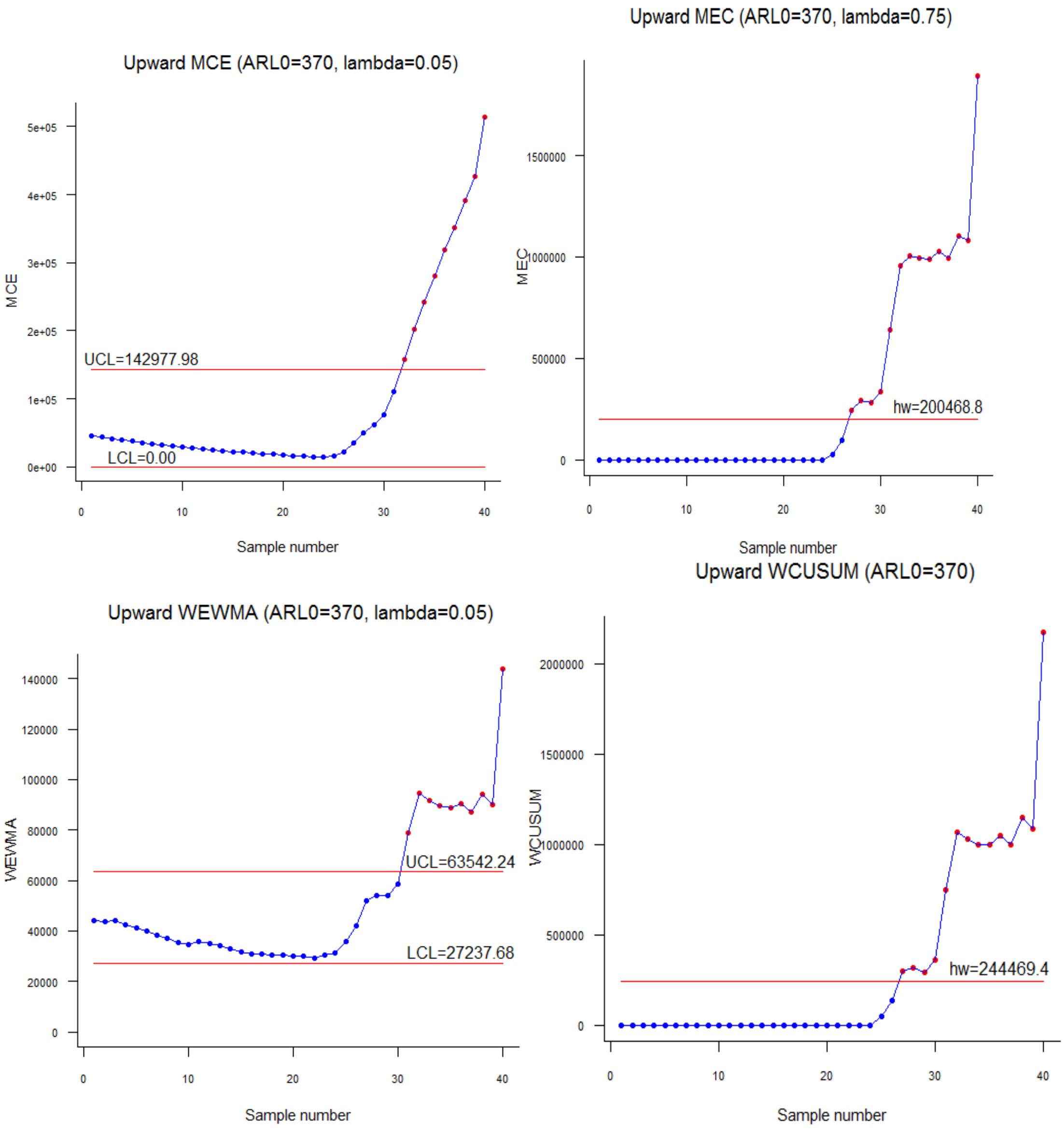


Figure 4. Plot of upward proposed, MEC, WEWMA and WCUSUM control charts for coal mining data set.

From Figure 4, we can see that the proposed and the WEWMA control charts detect an out-of-control alarm on the 32nd and 31st sample respectively, whereas, the MEC and WCUSUM charts shows false alarm rate on the 27th sample. So, the WEWMA and the proposed control charts outperform the other control charts to monitor Weibull-distributed TBE coal mining data set while others are failed to do so.

VI. CONCLUSIONS

In this article, I have proposed MCE chart for Weibull distributed TBE and compared the performance of the proposed chart along with other WCUSUM, WEWMA and MEC charts to monitor decreases and increases in the TBE mean for Weibull distributed TBE observations. For process deterioration, proposed chart is more sensitive than the WCUSUM chart in detecting small scale changes and better than the WEWMA chart for all scale changes. MEC chart is better than MCE chart.

For increasing TBE mean, the WEWMA chart is more sensitive than all others based on ARL and SDRL RMI performance from small to large shape parameters. Conversely, the MEC chart performs better than the others in detecting small scale changes when shape parameter is small based on ARL SDRL. The proposed chart outperforms the WCUSUM chart in detecting small and from small to large scale changes swiftly based on ARL and SDRL respectively. From illustrative examples, it can be concluded that the proposed chart is more sensitive to detect the process deterioration swiftly as compared to the existing control chart for the same values of specified shape and scale parameters, whereas for upward TBE, the WEWMA chart is more efficient than others to detect a shift in the process quickly. The proposed chart, WCUSUM, WEWMA and MEC control charts to monitor decreasing and increasing for Weibull distributed TBE mean with both shape and scale parameter changes can be studied as future research. In addition, the robustness of this chart can be checked with other distributions. Moreover, in this research, individual observations are employed in order to compute the control statistic of the proposed chart for Weibull distributed TBE charts. Instead of individual observations, subgroups of observations may be studied and the effect of sample size on robustness can be analyzed in the future.

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