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Original Article

Analysis of the Deformation and of the Stress Intensity Factor on a Simplified Cylinder Head of a Variable Compression Rate Engine

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Abstract** - This work contributes to evaluating the risk of propagation of cracks on a cylinder head subjected to the pressure of the combustion chamber of an engine with variable compression rate, whose numerous scientific researches demonstrated its energy advantages compared to the conventional engine. The analytical approach is characterized by using the Love-Kirchhoff theory, the Navier method to determine the expression of the deformation and the integral J of Rice to determine the expression of the intensity factor of constraints (FIC). In addition, the numerical approach is characterized on the one hand by simulations on the COMSOL multiphysics software to determine the numerical values of the deformation and all the associated curves, by simulations on MATLAB from the analytical expressions of the deformation and the FIC to obtain their numerical values and the associated curves, on the other hand employing the least-squares method (MMC), which by simulations on MATLAB at starting from the numerical values of the FIC makes it possible to obtain another expression of the latter. Thus this work allows the appreciation of the life of the cylinder head, the mastery of the dimensioning of the cylinder head to avoid its rupture, a technological mastery in the design of the engines of the machines of the Cameroonian automotive sector, the presentation of another approach of engineering in the theory of linear elasticity and fracture mechanics concerning a thin plate simply supported and requested in simple bending.

***Keywords** - Love-Kirchhoff, Rice J integral, FIC, COMSOL multiphysics, MATLAB, MMC.*

1. Introduction

Modelling is the theoretical representation of a physical model, and damage is the appearance of damage caused by wear or a chemical-physical attack on a material. This damage leads to the degradation of the capacities of this material which can lead to rupture. The variable compression ratio engine is a heat engine whose compression ratio varies. The latter is the ratio of the cylinder volumes when the piston is at the bottom dead centre (minimum volume) and when it is at the top dead centre (maximum volume). Naturally, the efficiency of the thermal engine also depends on the proper functioning of the elements that constitute it. Hence, the malfunction of a part such as a cylinder head, the role of which is to withstand the shocks due to the explosion and to reduce leaks in this engine, will certainly lead to its failure. Many damages have already been observed around the world. In January 1943, the day-old Tanker T2 SS Schenectady cracked at sea. In January 1998, a ship sank in Newfoundland after cracking. During the conference cycle in March 2005 on the rational use of energy in internal combustion engines, Adrian CLENCI and Pierre PODEVIN [2], knowing that the automobile engine mainly uses partial loads, that the controlled ignition engine at a maximum efficiency of 30, which does not exceed 10% to 15% at low partial loads and that the latter case constitutes 80% to 90% of the time of use of vehicles in urban areas, have looked for constructive solutions that in this interval of time or operation allow a significant increase in performance. They concluded that the variable volumetric ratio gives hope for a consumption gain of more than 35% at partial loads for supercharged engines with small displacements and high specific powers despite the advantages and disadvantages of the engine with a variable compression ratio.

MERABET ABDERREZAK [1] has also contributed to studying exchange phenomena in an atmospheric diesel engine, essentially based on the modelling and calculating factors influencing engine performance. He concludes that the variable compression ratio improves engine performance. The importance of the variable compression ratio engine being demonstrated, then studying its damage through the cylinder head can prove useful for an improved engine design. Jérémie LASRY (2010) devoted himself to numerical development to simulate cracked plates and shells subjected to bending. He presented a contribution to the calculation of the FIC (Stress Intensity Factor), a quantity indicating the crack's propagation risk.

Cameroon, an independent country since 1960, located in the heart of Central Africa, has seen a rapid increase in its demography and an increase in its car fleet in recent years, creating traffic congestion, particularly in cities such as Douala and Yaoundé. In addition, Prime Minister Joseph DION GUTE, Head of the Government of the Republic of Cameroon, received in an audience on Wednesday, July 10, 2019, the Lebanese Group Mikano International Ltd, which wants to launch a car assembly plant. Then the study of the impact of the variability of the compression ratio on the propagation of a crack on the cylinder head of the engine presented on the TD43 test bench of the work of MERABET ABDEREZZAK [2] can become an important line of research for the optimization of a variable compression ratio engine in countries like Cameroon whose demography of car users is constantly growing day by day. A study will also be valid for countries with socio-economic similarities to Cameroon.

2. Materials and Methods

2.1. Analytical Method

2.1.1. Modeling the Problem

Physical Model

The cylinder head is a very important part of the engine's operation. Given its shape, its physical model can be likened to that of a plate subjected to combustion pressure in a chamber of said engine and the figure above illustrates this modelization :

Mathematical Model

i) Deformation Determination Model

The problem previously mentioned requires us to consider the kinematics of the cylinder head of the TD43 test bench. The Love-Kirchhoff theory states that a and b are the sides, and h is the thickness of the cylinder head. The combustion chamber has a radius of 92 mm, and the cylinder head will take a square parallelepipedal shape with a side of 92 mm. By trying to take a limit value on one of the conditions presented in the Love-Kirchhoff theory, the chosen thickness is $max(a,b) > 20h$ $h = 4\ mm$.

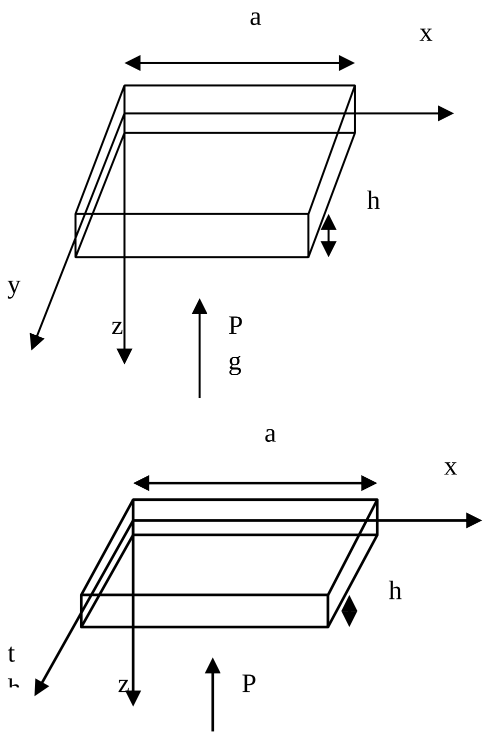

Fig. 1 Sketch of a bending plate

The assumptions of this theory are as follows:

- The Material Linearity Assumption (Hooke's Law)
- The assumption of geometric linearity (Small deformations, small rotations)
- The assumption that the plate is made of an isotropic material

As additional hypotheses, we have

- The stresses normal to the average lamination coming from the load applied transversely are negligible in comparison to the stresses in the plane of the plate.$\sigma_z \sigma_x, \sigma_y, \tau_{xy}$
- The distortions of the cross-sections of the plate under the effect of shear forces are negligible.

  Thus we have
- Cylinder head displacement field

$$\{u = -z\frac{\partial w}{\partial x} \quad v = -z\frac{\partial w}{\partial y} \quad w = w(x,y) \tag{1}$$

Deformation field

$$\begin{cases} \varepsilon_x = \frac{\partial u}{\partial x} \\ \varepsilon_y = \frac{\partial v}{\partial y} \\ \varepsilon_z = \frac{\partial w}{\partial z} \end{cases} \tag{2}$$

$$\begin{cases} \varepsilon_x = z(-\frac{\partial^2 w}{\partial x^2}) \\ \varepsilon_y = z(-\frac{\partial^2 w}{\partial y^2}) \\ \varepsilon_z = \frac{\partial w}{\partial z} \end{cases} \tag{3}$$

$$\begin{cases} \varepsilon_x = z\chi_x \\ \varepsilon_y = z\chi_x \\ \varepsilon_z = \frac{\partial w}{\partial z} \end{cases} \tag{4}$$

With being respectively the curvature along (O,x) and along (O,y).$\chi_x = -\frac{\partial^2 w}{\partial x^2} \chi_y = -\frac{\partial^2 w}{\partial y^2}$
According to Hooke's law, we have:

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) \quad (5)$$

$$\sigma_x = \frac{Ez}{1-\nu^2}(\chi_x + \nu\chi_y)\sigma_y = \frac{Ez}{1-\nu^2}(\chi_y + \nu\chi_x) \quad (6)$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x z dy dz = M_x dy \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y z dx dz = M_y \quad (7)$$

By replacing the relation Eq (6) with the relation Eq (7), we have:

$$M_x = D(\chi_x + \nu\chi_y)M_y = D(\chi_y + \nu\chi_x) \quad (8)$$

With :

$$D = \frac{E}{1-\nu^2}\int_{-\frac{h}{2}}^{\frac{h}{2}} z^2\, dz = \frac{Eh^3}{12(1-\nu^2)} \quad (9)$$

*D*being the bending stiffness of the plate.
Using the expressions in the relation Eq (8), we have:$\chi_x = -\frac{\partial^2 w}{\partial x^2},\ \chi_y = -\frac{\partial^2 w}{\partial y^2}$

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \quad M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2}{\partial x^2}\right) \quad (10)$$

Fig. 2 above shows the torsor of the efforts of our yoke:

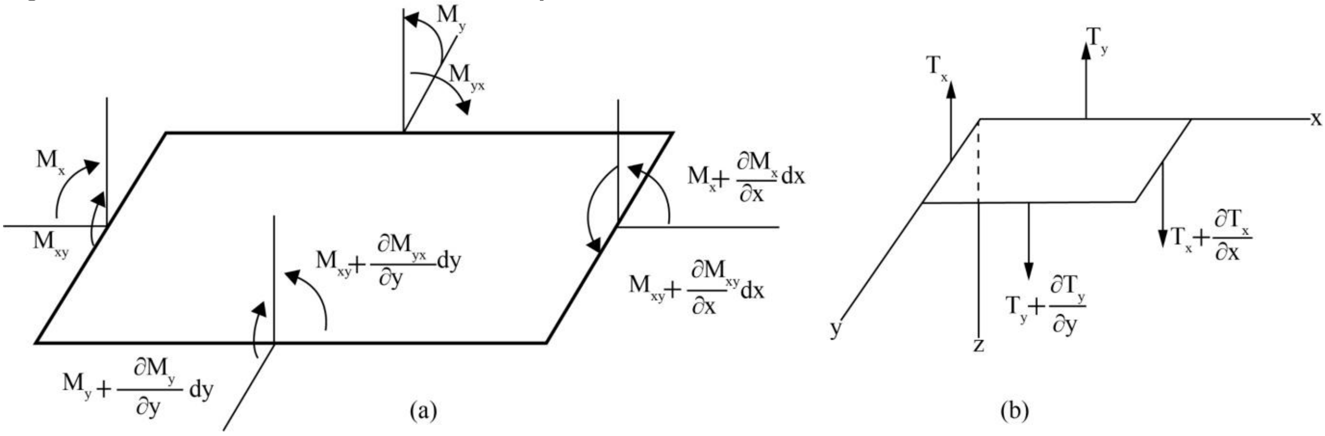

Fig. 2 Reduction elements [3]

From the reduction elements, the balance of the plate gives:

$$\frac{\partial T_x}{\partial x} dx dy + \frac{\partial T_y}{\partial y} dy dx + p dx dy = 0 \quad (11)$$

$$\frac{\partial T_x}{\partial x} + \frac{\partial T_y}{\partial y} + p = 0 \quad (12)$$

With p=p (x,y), the combustion pressure.
From the reduction elements, taking the moments around the axes (O, x), (O, y) of all the forces acting on the cylinder head, neglecting the second-order terms, we obtain respectively:

$$\frac{\partial M_{xy}}{\partial x} - \frac{\partial M_y}{\partial y} + T_y = 0 \quad (13)$$

$$\frac{\partial M_{yx}}{\partial y} + \frac{\partial M_x}{\partial x} - T_x = 0 \quad (14)$$

By drawing the expressions of, respectively, in the relations Eq (13) Eq (14), the relation Eq (12) becomes a balance equation according to the expressions: $T_x\ T_y\ M_x\ M_y\ M_{xy}$

$$\frac{\partial^2 M_x}{\partial x^2} - 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = p \quad (15)$$

According to Kirchhoff's hypothesis, the shear forces have the same negligible effect on the curvatures. Thus the equations Eq (10) in bending are also written as follows:

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) M_{xy} = -M_{yx} = D(1-\nu)\frac{\partial^2 w}{\partial x \partial y} \quad (16)$$

By drawing in the relation Eq (14), in the relation Eq (13) and by using the expressions of the moments of the relation Eq (16) in the relations Eq (14) and Eq (13), we have: $T_x T_y$

$$T_x = \frac{\partial M_{yx}}{\partial y} + \frac{\partial M_x}{\partial x} = -D\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2}\right) = -D\frac{\partial}{\partial x}(\nabla^2 w) \quad (17)$$

$$T_y = -\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} = -D\frac{\partial}{y}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = -D\frac{\partial}{\partial x}(\nabla^2 w) \quad (18)$$

By replacing the expressions and relations Eq (14) and Eq (13) in the equilibrium relation Eq (12), we have this Lagrange equation: $T_x T_y$

$$\nabla^4 w = \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{P}{D} \quad (19)$$

Clearly, we can say that our mathematical model for the deformation of our cylinder head is this Lagrange equation above.

In general mechanics, the so-called light alloy aluminium alloy can be used concerning its very low density compared to common metals in the automobile field. We can take the aluminium alloy AS12U, whose chemical designation is AlSi12Cu, the density, the modulus of elasticity, the mechanical resistance, the Poisson's ratio $2.70\ g/cm^3$ $74\ 000\ MPa$ $300\ MPa$ 0.33.

ii) Model for Determining the SCF (Stress Intensity Factor)

The displacement in the cracked domain breaks up into two parts:

- A regular part u^r [4]
- A singular part u^8[4]

The singular part comprises a reduced number of modes, defined up to a multiplicative constant. These multiplicative constants are called noted stress intensity factors for the Love-Kirchhoff model. They indicate the presence or not of the singularity and its amplitude. The Love-Kirchhoff singularities belong to ([4], [5]) and include two singular modes expressed in polar coordinates [6]: $H^{5/2-\eta}(\Omega)\forall\eta > 0$

$$u_3^{s,KL}(r,\varphi) = A_{KL} r^{\frac{3}{2}}\left[K_1\left(\frac{\nu+7}{3(\nu-1)}cos\frac{3}{2}\varphi + cos\frac{\varphi}{2}\right) + K_2\left(\frac{3\nu+7}{3(\nu-1)}sin\frac{3}{2}\varphi + sin\frac{\varphi}{2}\right)\right]$$

$$\text{With } A_{KL} = \frac{\sqrt{2}}{2}\frac{1-\nu^2}{E\varepsilon(3+\nu)} \quad (20)$$

Fig. 3 represents in the plan the position around the point of crack:

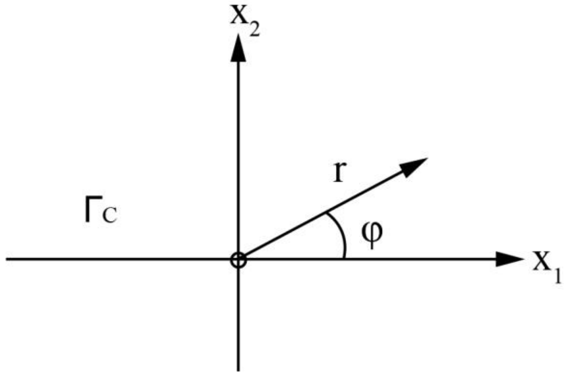

Fig. 3 Polar marker with the crack in bold line [6]

Fig. 4 represents in the theory of Love-Kirchhoff the mode I of cracking:

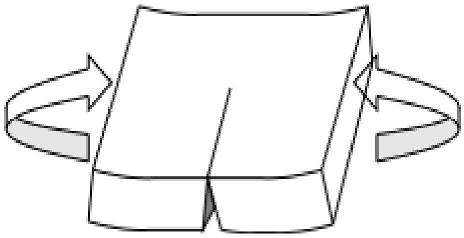

Fig. 4 Mode I in the Love-Kirchhoff model [6]

Fig. 5 represents the Love-Kirchhoff theory mode II of cracking:

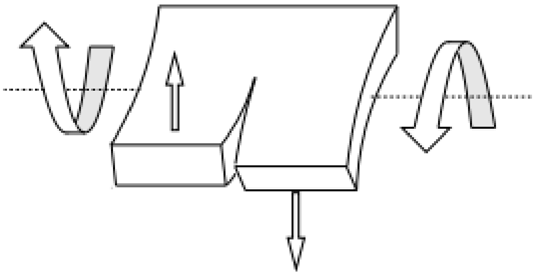

Fig. 5 Mode II in the Love-Kirchhoff model [6]

Fig. 6 above represents the type of stress undergone by a plate or a thin plate in particular and the mode appearing:

Fashion	Types of solicitation
1	Symmetric bendingK_1, K_I
2	Anti-symmetrical bendingK_2, K_{II}
3	ShearK_2, K_{II}

Fig. 6 Mechanical stress and FIC concerned

Assuming that the bending is symmetric, therefore the model of FIC is:

$$K_1 = \frac{u_3^{s,KL}(r,\varphi)}{A_{KL} r^{\frac{3}{2}} \left(\frac{\nu+7}{3(\nu-1)} cos\frac{3}{2}\varphi + cos\frac{\varphi}{2}\right)} \text{With } A_{KL} = \frac{\sqrt{2}}{2}\frac{1-\nu^2}{E\varepsilon(3+\nu)} \quad (21)$$

So :

$$K_1 = \frac{w(x,y)}{A_{KL} r^{\frac{3}{2}} \left(\frac{\nu+7}{3(\nu-1)} cos\frac{3}{2}\varphi + cos\frac{\varphi}{2}\right)} \text{With } A_{KL} = \frac{\sqrt{2}}{2}\frac{1-\nu^2}{E\varepsilon(3+\nu)} \quad (22)$$

Resolution Methods

i) Method for Solving the Deformation or Lagrange Equation

According to the Love-Kirchhoff theory, the mathematical model of the deformation presents an equation with two variables, which are the position variables of a point on our cylinder head. Thus the appropriate method for solving such an equation is none other than the method of separation of variables.

Thus, recalling the equation of the deformation, we have:

$$\nabla^4 w = \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{P}{D} \quad (19)$$

We also have the following notation:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = \nabla^2\nabla^2 w \quad (23)$$

It is necessary to remember that the volumetric load acts in the opposite direction to the transverse axis, so we have taken into account the relation Eq (22), and the relation Eq (23) becomes:

$$\nabla^2\nabla^2 w = -\frac{P}{D} \tag{24}$$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = -\frac{P}{D} \tag{25}$$

The expression Eq (25) is Lagrange's equation. The latter was solved in 1820 by Navier for the case of a rectangular plate simply supported on its four edges. Since as part of the operation of the internal combustion engine, our cylinder head is simply pressed on its four edges.

As an approximate solution, Navier assumed that this deformation could be put in the form of a Fourier series:

$$w(x,y) = \sum_{m=1}^{+\infty}\sum_{n=1}^{+\infty} w_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{a}\right) \tag{26}$$

This field checks the boundary conditions imposed by simple support, i.e.:

$$w(0,y) = 0, w(a,y) = 0, w(x,0) = 0, w(x,a) = 0 \tag{27}$$

Similarly, the uniformly distributed maximum combustion pressure on the cylinder head, which is a uniformly distributed load, this load can be approximated as follows:$q(x,y)$

$$q(x,y) = \sum_{m=1}^{+\infty}\sum_{n=1}^{+\infty} q_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{a}\right) \tag{28}$$

Having obtained the equations of the expressions Eq (26) and Eq (27), Navier calculated and.q_{mn} w_{mn}
By multiplying the two members of the equation of the expression Eq (28), we obtain: $\sin\left(\frac{l\pi x}{a}\right)\sin\left(\frac{k\pi y}{a}\right), l,k \in h$

$$\int_0^a\int_0^a q(x,y)\sin\left(\frac{l\pi x}{a}\right)\sin\left(\frac{k\pi y}{a}\right)dxdy = \int_0^a\int_0^a \sin\left(\frac{l\pi x}{a}\right)\sin\left(\frac{k\pi y}{a}\right)\sum_{m=1}^{+\infty}\sum_{n=1}^{+\infty} -q_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)dxdy \tag{29}$$

$$\int_0^a\int_0^a q(x,y)\sin\left(\frac{l\pi x}{a}\right)\sin\left(\frac{k\pi y}{a}\right)dxdy = \sum_{m=1}^{+\infty}\sum_{n=1}^{+\infty} -q_{mn}\int_0^a \sin\left(\frac{l\pi x}{a}\right)\sin\left(\frac{m\pi x}{a}\right)dx\int_0^a \sin\left(\frac{k\pi y}{a}\right)\sin\left(\frac{n\pi y}{a}\right)dy \tag{30}$$

Let's say:

$$I = \int_0^a \sin\left(\frac{l\pi x}{a}\right)\sin\left(\frac{m\pi x}{a}\right)dx, II = \int_0^a \sin\left(\frac{k\pi x}{a}\right)\sin\left(\frac{n\pi x}{a}\right)dy \tag{31}$$

Gold:

$$I = \begin{cases} 0\ sim \neq l \\ \frac{a}{2} sim = l \end{cases}, II = \begin{cases} 0\ sin \neq k \\ \frac{a}{2} sin = k \end{cases} \tag{32}$$

So :

$$\int_0^a\int_0^a q(x,y)\sin\left(\frac{l\pi x}{a}\right)\sin\left(\frac{k\pi y}{a}\right)dxdy = \frac{a}{2}\frac{a}{2}(q_{mn}) \tag{33}$$

That is :

$$q_{mn} = \frac{4}{aa}\int_0^a\int_0^a q(x,y)\sin\left(\frac{l\pi x}{a}\right)\sin\left(\frac{k\pi y}{a}\right)dxdy \tag{34}$$

Navier to find has simply developed Lagrange's expression, taking into account the approximate solution cited above. Thereby :w_{mn}

$$\frac{\partial^4 w(x,y)}{\partial x^4} = \sum_{m=1}^{+\infty}\sum_{n=1}^{+\infty} w_{mn}\left(\frac{m\pi}{a}\right)^4 \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right) \tag{35}$$

$$\frac{\partial^4 w(x,y)}{\partial y^4} = \sum_{m=1}^{+\infty}\sum_{n=1}^{+\infty} w_{mn}\left(\frac{n\pi}{a}\right)^4 \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right) \tag{36}$$

$$\frac{\partial^4 w(x,y)}{\partial x^4 \partial y^4} = \sum_{m=1}^{+\infty}\sum_{n=1}^{+\infty} w_{mn}\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{a}\right)^2 \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right) \tag{37}$$

)

By introducing these developed expressions into the Lagrange equation, we have:

$$\sum_{m=1}^{+\infty}\sum_{n=1}^{+\infty}\left\{w_{mn}\left[\left(\frac{m\pi}{a}\right)^4+2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{a}\right)^2+\left(\frac{n\pi}{a}\right)^4\right]+\frac{q_{mn}}{D}\right\}\times\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)=0 \quad (38)$$

That is :

$$\sum_{m=1}^{+\infty}\sum_{n=1}^{+\infty}\left\{w_{mn}\left[\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{a^2}\right)^2\right]+\frac{q_{mn}}{D}\right\}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)=0 \quad (39)$$

$$w_{mn}=-\frac{q_{mn}}{D\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{a^2}\right)^2} \quad (40)$$

The deflection of our cylinder head, taking into account the direction of the combustion pressure vector, is then written:

$$w(x,y)=\sum_{m=1}^{+\infty}\sum_{n=1}^{+\infty}-\frac{q_{mn}}{D\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{a^2}\right)^2}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right) \quad (41)$$

Knowing the expression of our deflection, the different moments related to the flexion of our cylinder head are:

$$M_{xx}=-\frac{1}{\pi^2}\left(\sum_{m=1}^{+\infty}\sum_{n=1}^{+\infty}\frac{q_{mn}}{\left(\frac{m^2}{a^2}+\frac{n^2}{a^2}\right)^2}\left[\left(\frac{m}{a}\right)^2+\nu\left(\frac{n}{a}\right)^2\right]\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)\right) \quad (42)$$

$$M_{yy}=-\frac{1}{\pi^2}\left(\sum_{m=1}^{+\infty}\sum_{n=1}^{+\infty}\frac{q_{mn}}{\left(\frac{m^2}{a^2}+\frac{n^2}{a^2}\right)^2}\left[\left(\frac{n}{a}\right)^2+\nu\left(\frac{m}{a}\right)^2\right]\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)\right) \quad (43)$$

$$M_{xy}=-M_{yx}=\frac{1}{\pi^2}\sum_{m=1}^{+\infty}\sum_{n=1}^{+\infty}-\frac{q_{mn}}{\left(\frac{m^2}{a^2}+\frac{n^2}{a^2}\right)^2}\left(\frac{mn}{a^2}-\nu\frac{mn}{a^2}\right)\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{a}\right) \quad (44)$$

ii) Calculation Flowchart for Determining the Deformation

From the expression of the deformation, the flow chart for calculating the latter on MATLAB is as follows:

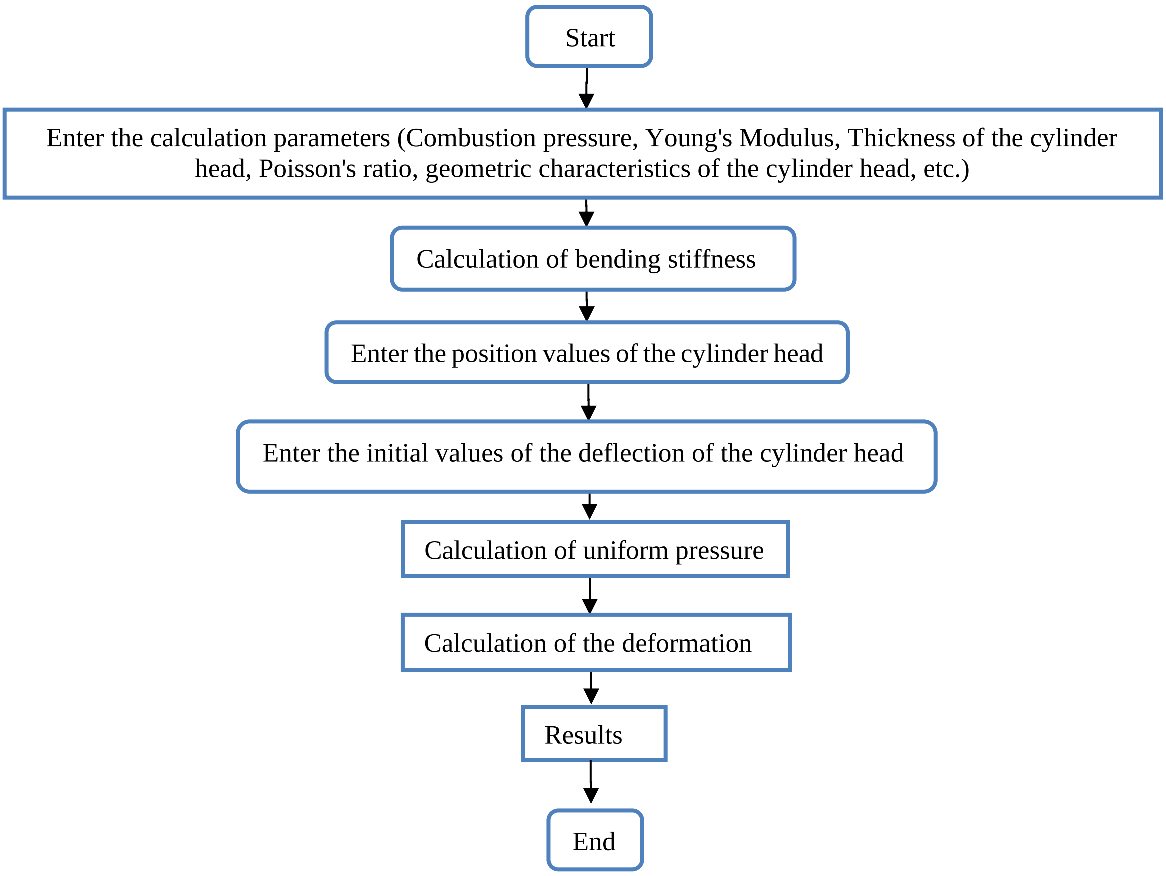

Fig. 7 Flow chart for calculating the deformation in MATLAB

iii) Method for Solving the FIC Equation

Whether it is the determination model by direct estimation or by calculating the contour J integral, the method for determining the FIC is numerically based on developing a subroutine on MATLAB.

From the expression of the FIC, the calculation flowchart of the latter on MATLAB is the following:

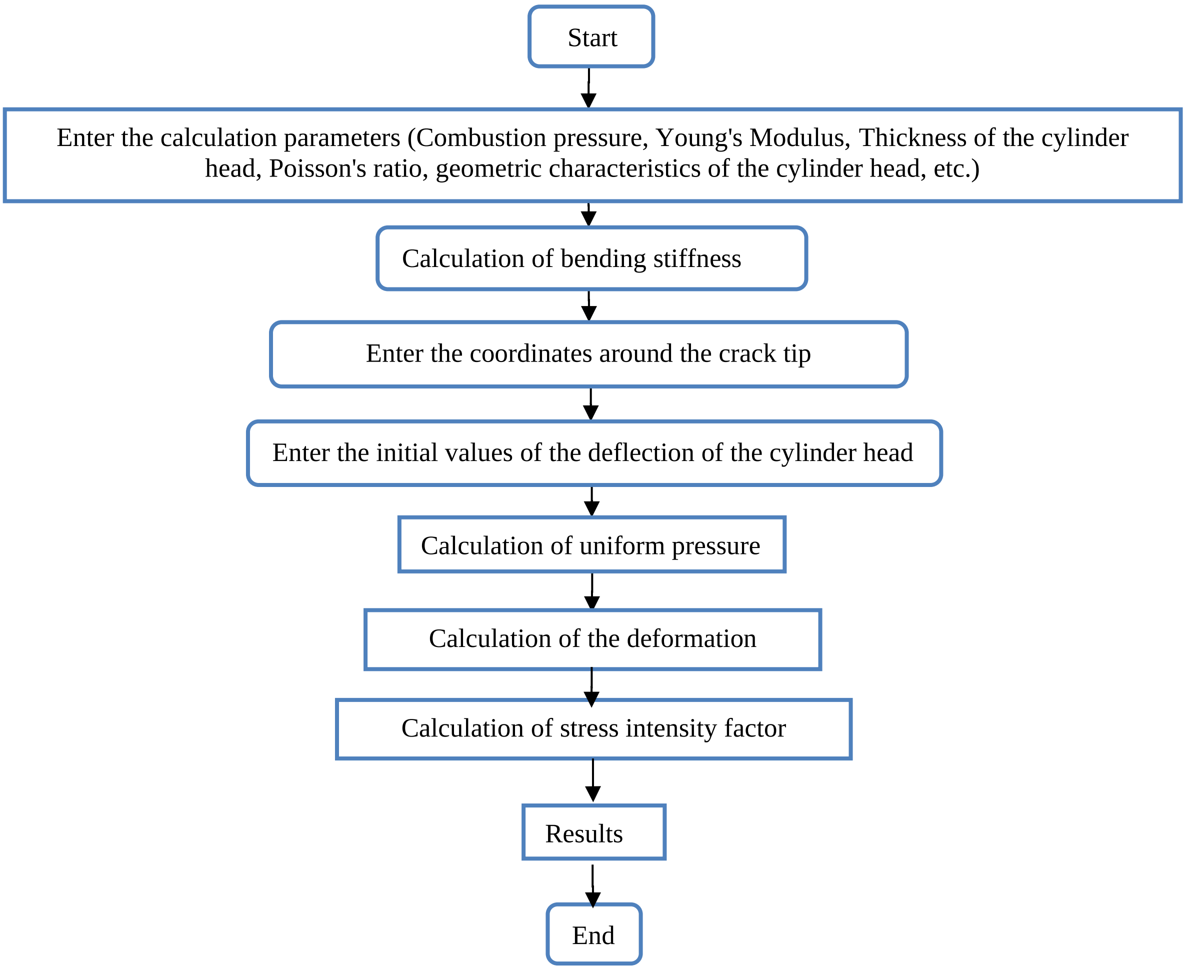

Fig. 8 FIC calculation flowchart in MATLAB

2.1.2. Numerical Method

In this part, the COMSOL multiphysics software is used to simulate the loading of thin plate stress in bending, whose initial and boundary conditions are taken into account by integrating the nature of the material, which constitutes it in a steady state. This part of the modelling results in presenting values such as the deformation of the stress.

Numerical simulation on COMSOL multiphysics comes from the finite element method, which considers scientific hypotheses, particularly mechanical ones, in our study environment. Thus, after integrating the calculation parameters (geometric and mechanical characteristics of our cylinder head, applied load, initial conditions, boundary conditions, study regime, and mesh type), we obtain mechanical results, particularly the deformation. A dynamic that can start again until a valid result is obtained.

In this case, the calculation flowchart is as follows:

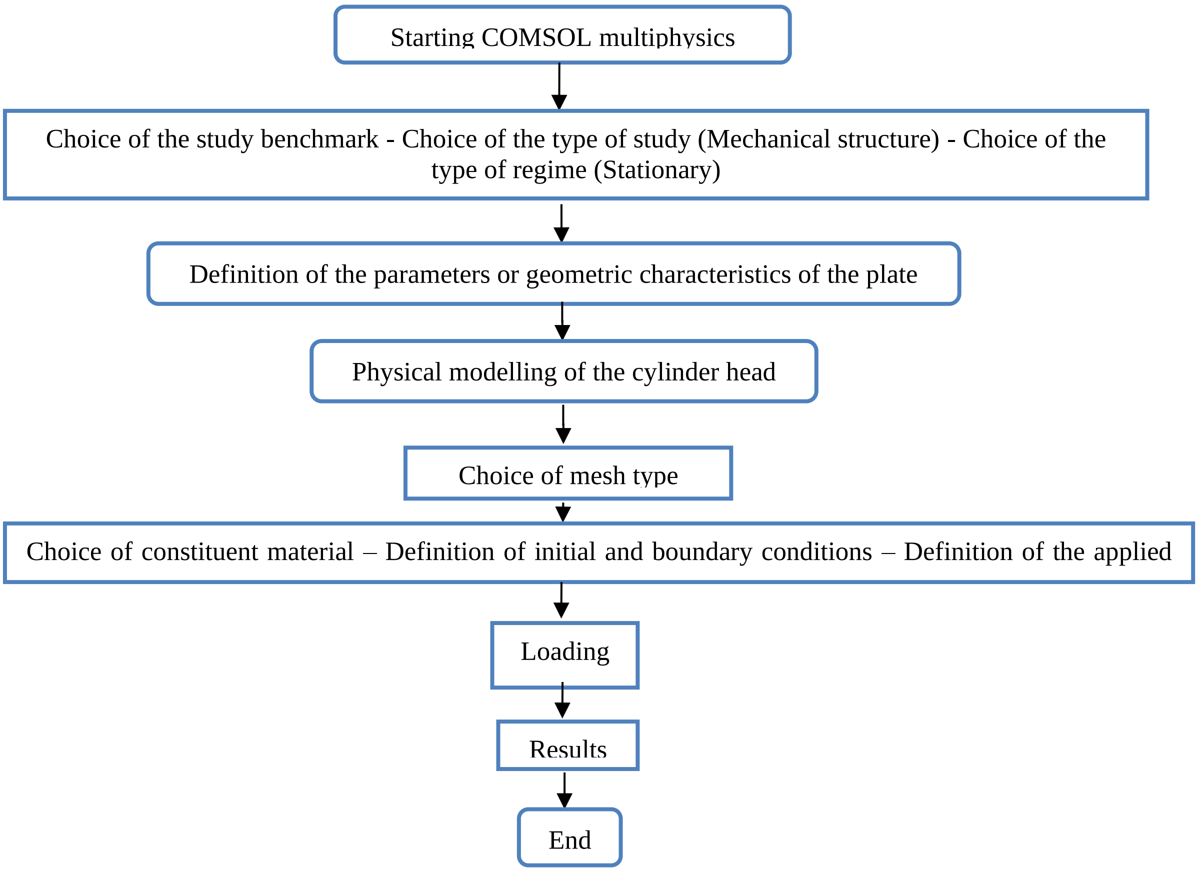

Fig. 9 Calculation flowchart under COMSOL multiphysics

3. Results and Discussions

Fig. 10 illustrates the representation according to the extreme compression ratios of the deformations:

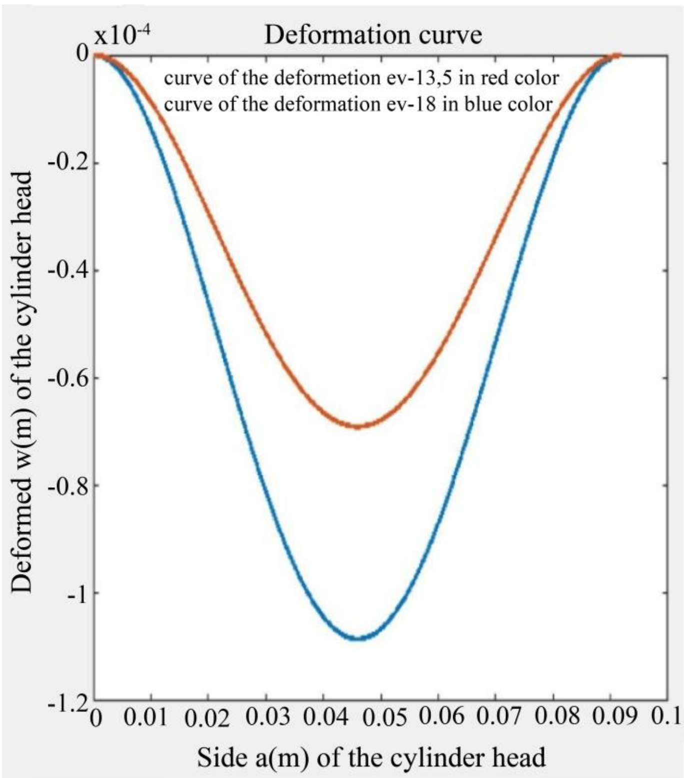

Fig. 10 Curve of the different deflections

from a subroutine developed in MATLAB
B$\varepsilon_v = 13{,}5\ et\varepsilon_v = 18$

After compilation under MATLAB of the developed sub-program, $\varepsilon_v = 13{,}5$, the yoke bent to the position following the Cartesian reference of fig.3. With the breach bent from to the position following the mark previously mentioned.

$$w_{min} = -6{,}902.10^{-5}m\ (0{,}046\ m, 0{,}046\ m)\ \varepsilon_v = 18\ w_{max} = -10{,}85.10^{-5}m(0{,}046\ m, 0{,}046\ m)$$

Timoshenko S and Woinowsky-Krieger S (1959) gave analytical solutions to the deformations of the plates requested in bending according to the various supports they are subjected to. For a supported square plate with a uniformly applied load, they presented the analytical solution as $w_{ref} = 4{,}062.10^{-3}\frac{qa^4}{D}$,q being the uniformly distributed load, the side of the plate, and D the bending stiffness.

$$\text{A with P.}\varepsilon_v = 13{,}5\ w_{ref} = -6{,}8988.10^{-5}m = 105000\ N/m^2$$
Has with
.
$$\varepsilon_v = 18\ w_{ref} = -10{,}841.10^{-5}m\ P = 165000\ N/m^2$$

The previous results show a difference between the deformation resulting from the subroutine and the reference deformation from to0.0464 % $\varepsilon_v = 13{,}5$, of0.0829 % at$\varepsilon_v = 18$. We observe a very rapid convergence with the results. These variations being less than, one can say that the results resulting from the compilation of the subroutine are satisfactory. 0,5 %

The use of the COMSOL multiphysics simulation software after modelling our cylinder head in the form of a thin plate stressed in simple bending made of AlsiCu material gives the following fig. 11 and 12, which represent the deformations according to the positions:

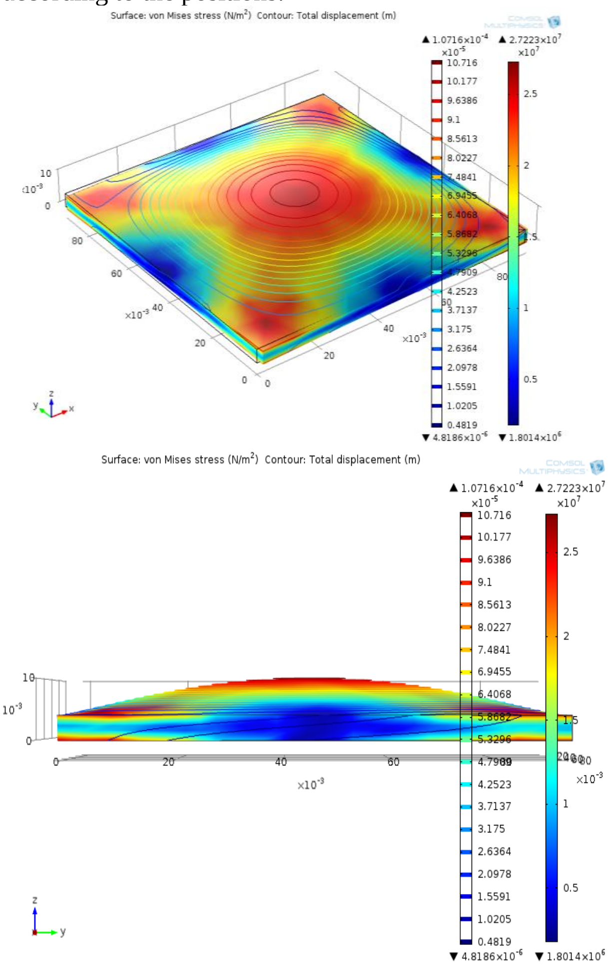

Fig. 11 Deflection curves from simulation under COMSOL multiphysics$\varepsilon_v = 18$

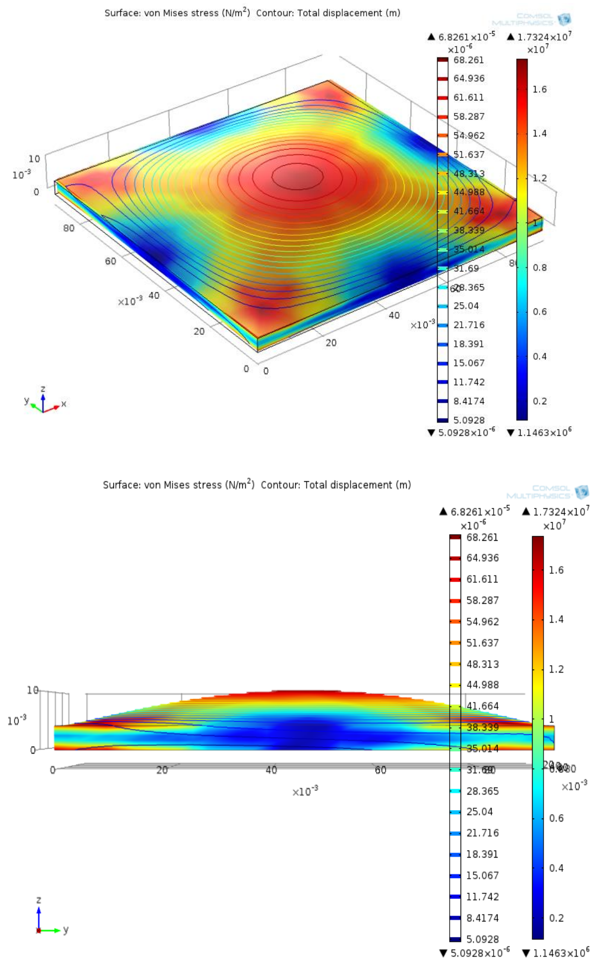

Fig. 12 Deflection curves from simulation under COMSOL multiphysics$\varepsilon_v = 13,5$

The COMSOL multiphysics software, the maximum deformation values give the following results:

A with P.$\varepsilon_v = 13{,}5\ w_{min} = -6{,}8261.10^{-5}m = 105000\ N/m^2$
Has with.$\varepsilon_v = 18\ w_{max} = -10{,}716.10^{-5}mP = 165000\ N/m^2$

In the same way, from the previous results, we observe a difference between the deformation resulting from the simulation and the reference deformation resulting from Timoshenko S and Woinowsky-Krieger S from to$1{,}05380\ \%\ \varepsilon_v = 13{,}5$, of$1{,}1530\ \%$ at$\varepsilon_v = 18$. We observe a very rapid convergence with the results. Since these deviations are not less than, it can be said that the results from the compilation of the simulation are satisfactory, with deviations less than $0{,}5\ \%1{,}5\ \%$

We see the cylinder head bent more and deforms more when the engine has a compression ratio. Although the variable compression ratio engine improves performance, its cylinder head deformation is significant and increases its risk of damage.$\varepsilon_v = 18$

Considering the interval of the compression ratio and choosing a few points in this interval, the maximum combustion pressures are summarized in the following table:

Table 1. Some values of deformations on the interval of the compression rate resulting from a subroutine developed under MATLAB

Some values of the maximum combustion pressure (N/m2)	105000	120000	135000	150000	165000
Maximum deformations (m)	-6, 9016. 10^{-5}	-7.8875. 10^{-5}	-8.8734. 10^{-5}	- 9.8594. 10^{-5}	- 1.0845. 10^{-4}

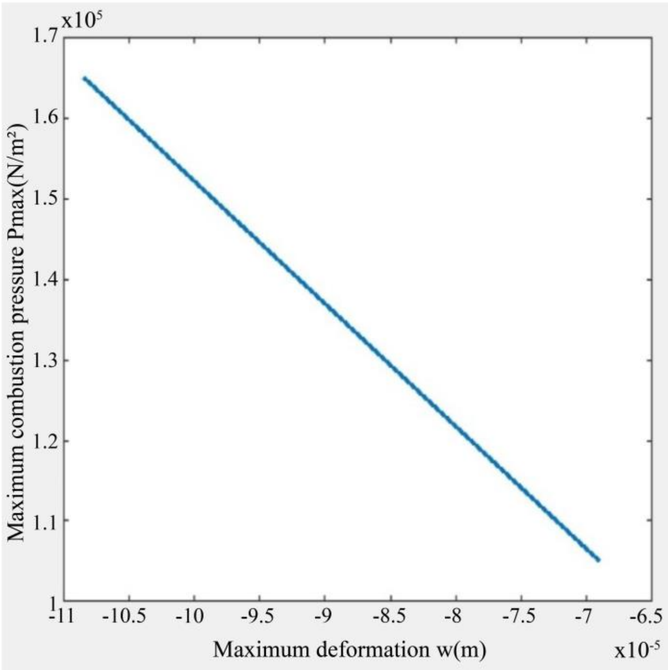

Fig. 13 Curve of maximum pressure according to the maximum deformation resulting from MATLAB

Thus the maximum combustion pressure as a function of the maximum deformation is represented as follows:
The function resulting from the curve is a linear line of expression:

$$P = -1{,}5228.10^9 w - 130 \quad (45)$$

With reversible adiabatic compression (isentropic), we also have:

$$\varepsilon_v = \left[\frac{1}{p_0}(-1{,}5228.10^9 w - 130)\right]^{\frac{1}{\gamma}} avec\ p_0\ la\ pression\ d'air \quad (46)$$

Therefore the maximum combustion pressure is a linear and increasing function according to the maximum deformation. This expression, after study, is the same at minimum combustion pressure.

The centre of our cylinder head is where the deflection is maximum. Considering the centre of our cylinder head as the crack tip, for a circle with a radius of at most one-hundredth of the said crack tip, after simulation on MATLAB, the various maximum values of the FIC are:

Has with.$\varepsilon_v = 13{,}5\ K_1 = 0{,}0503\ Nm^{-3/2}\ P = 105000\ N/m^2$
Has with.$\varepsilon_v = 18\ K_1 = 0{,}1766\ \ Nm^{-3/2}\ P = 165000\ N/m^2$

The associated curve around this crack tip is:

The adjustment option deemed optimal in the sense of the Least Squares Method (MMC) is an optimal polynomial model of order 2. We have:$x \in [0{,}0920.10^{-3}\ m, 0{,}4600.10^{-3}\ m]$

$$\mathrm{AT}\varepsilon_v = 13{,}5 K_1 = -1{,}738.10^5 x^2 + 262{,}9x + 0{,}02794 \quad (48)$$

$$\mathrm{AT}\varepsilon_v = 18\ K_1 = -2{,}726.10^5 x^2 + 412{,}7x + 0{,}04396 \quad (49)$$

The adjustment option deemed optimal in the sense of the Least Squares Method (MMC) is an optimal polynomial model of order 2. We have:$y \in [0{,}0920.10^{-3}\ m, 0{,}4600.10^{-3}\ m]$

$$\mathrm{T}\varepsilon_v = 13{,}5\ \ K_1 = -1{,}738.10^5 y^2 + 262{,}9y + 0{,}02794 \quad (50)$$

$$\mathrm{AT}\varepsilon_v = 18\ K_1 = -2{,}726.10^5 y^2 + 412{,}7y + 0{,}04396 \quad (51)$$

By choosing, for example, any point on our cylinder head, namely (0.0054 m, 0.0084 m) as the tip of the crack, for a circle with a radius located at most one hundredth from the said crack tip, after simulation on MATLAB, the various maximum values of the FIC are:

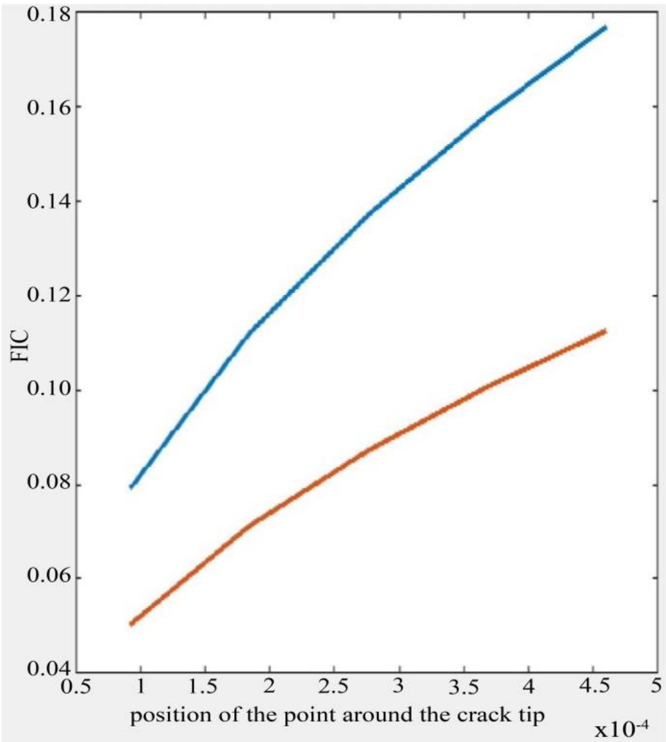

Fig. 14 Curve of the FICs according to the position of the point around the crack tip located in the centre of the cylinder head from MATLAB.

Has with.$\varepsilon_v = 13{,}5\ K_1 = 1{,}8146.10^{-7} Nm^{-3/2}\ \ P = 105000\ N/m^2$
Has with.$\varepsilon_v = 18\ \ K_1 = 2{,}8515.10^{-7} Nm^{-3/2}\ P = 165000\ N/m^2$

The associated curve around this crack tip is:

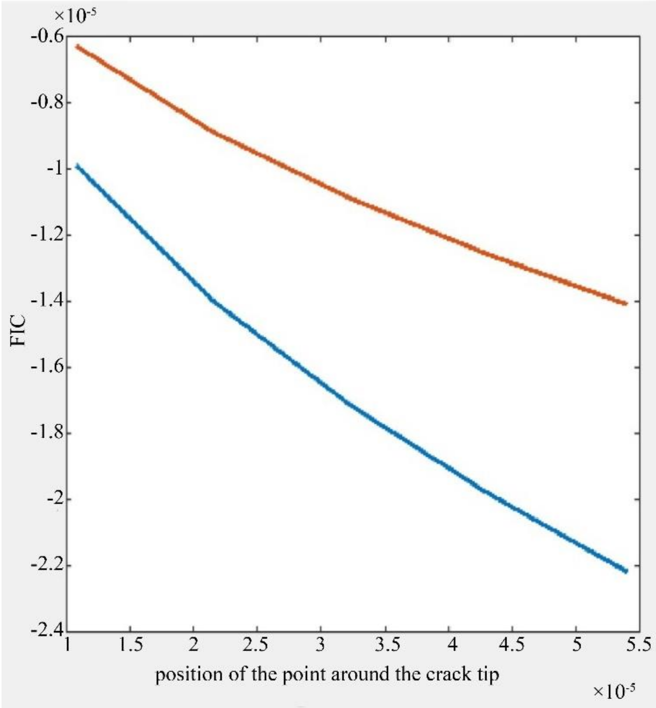

Fig. 15 Curve of the FICs as a function of the position of the point around the arbitrary crack tip chosen from MATLAB

The adjustment option deemed optimal in the sense of the Least Squares Method (MMC) is an optimal polynomial model of order 2. We have:$x \in [0{,}1080.10^{-4}\ m, 0{,}54.10^{-4}\ m]$

$$\text{AT}\varepsilon_v = 13{,}5\ \ K_1 = 1562x^2 - 0{,}2796x - 3{,}516.10^{-6} \quad (52)$$

$$\text{AT}\varepsilon_v = 18\ K_1 = 2456x^2 - 0{,}4398x - 5{,}514.10^{-6} \quad (53)$$

The adjustment option deemed optimal in the sense of the Least Squares Method (MMC) is an optimal polynomial model of order 2. We have:$x \in [0{,}1680.10^{-4}, 0{,}84.10^{-4}]$

$$\text{AT}\varepsilon_v = 13{,}5\ \ K_1 = 645{,}3y^2 - 0{,}1798y - 3{,}516.10^{-6} \quad (54)$$

$$\text{AT}\varepsilon_v = 18K_1 = 1015y^2 - 0{,}2827y - 5{,}514.10^{-6} \quad (55)$$

The above values of the FIC show that the risk of a crack is higher at a high compression ratio. Therefore the damage has a greater magnitude at a high rate.

4. Conclusion

In short, the deflection of our cylinder head, which is the parametric element of the deformation, becomes a central point of the study of rupture within the framework of this research. The variable compression ratio engine brings in our era of modernity a subsequent advantage in energy intake; through numerous studies conducted, its reliability in the spring of mechanical engineering is proven through elaborate scientific studies, so although the increase in the efficiency of the engine with variable compression ratio favours the improvement of efficiency, it presents enormous risks of damage, hence the consideration of the sizing parameters allowing the reduction of this phenomenon degrading our engine. By using fracture mechanics and fatigue approaches, by previously presenting details of linear elasticity associated with the mechanical, Physico-chemical characteristics of the alloy constituting the material made by our cylinder head, the validation of the deflection triggered a wave of questions, reflections, and analyses whose sole purpose is to draw scientific attention to a social scope to inform the scientific community about a possible danger through an analytical, numerical analysis giving computer programming perspectives through a language at the height of possible research in the field of computing and innovation. So from this work, it was thus possible to obtain polynomial forms of the stress intensity factors, functions of the maximum combustion pressure depending on the maximum deformation, and elements that, taken into account in dimensioning, would reduce the risk of damage. Naturally, an experiment should make this study more expensive.

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