SSRG International Journal of Material Science and Engineering
ISSN: 2394–8884 / https://doi.org/10.14445/23948884/IJMSE-V9I3P103

Volume 9 Issue 3, 26-33, Sep-Dec 2023

*Original Article*

# Stochastic Model for Comfort in Dwellings: General Model

Kenya Suárez-Domínguez*[1], Lisbeth A. Brandt-Garcia[1], Elena F. Izquierdo Kulich[2], Yoana Perez-Badell, Rocio R. Gallegos-Villela[1]

[1]*FADU Research Center. Autonomous University of Tamaulipas. Circuito Universitario s/n. Tampico, Tamaulipas. CP. Mexico.*
[2]*Faculty of Architecture, Design and Urbanism. UAT. Circuito Universitario s/n. Tampico, Tamaulipas. CP. Mexico.*
[3]*Physical-Chemistry Department. Faculty of Chemistry. Havana University. Zapata y G, Vedado. Havana, Cuba.*

**Corresponding Author : kenya@cadime.mx*

Received: 26 October 2023 Revised: 03 December 2023 Accepted: 17 December 2023 Published: 29 December 2023

***Abstract** - From an architectural perspective, comfort is essential for well-being, encompassing sensory and emotional aspects. In this context, it focuses on thermal comfort, the feeling of well-being related to the home's internal temperature. Comfort is related to the ability of the home to dampen external fluctuations and keep the interior temperature close to comfort. The present work seeks to develop a stochastic model that describes the behavior of the internal temperature of a house, considering the variability of the exterior temperature. A deterministic model for temperature in the wall and room is proposed. Stochastic variability is then introduced into the outside temperature. The Ito-Stratonovich formalism is used to obtain a stochastic model based on differential equations for heat transfer by conduction in the wall and by convection in the room. A system of stochastic differential equations is obtained that describes the deterministic behaviour of temperature in the house. Variability in outdoor temperature is introduced as a stochastic variable, developing a stochastic differential equation that describes the probability of temperature in the wall. Comfort indices calculated for different construction materials are presented. The stochastic behavior of ambient temperature and housing temperature is simulated for various materials. The results are discussed in terms of the ability of materials to dampen fluctuations. The results indicate that the ability to dampen fluctuations depends on the material selected. The importance of considering thermal and structural properties when choosing materials to ensure thermal comfort and safety is highlighted.*

***Keywords** - House comfort, Heat stochastic model, Thermal in construction materials.*

## 1. Introduction

From an architectural perspective, comfort is a dimension of well-being intricately tied to the visual, olfactory, auditory, and overall architectural experiences of individuals [1,2]. It encompasses emotional, psychological, and environmental elements that harmonize to foster a sense of well-being. Within this framework, thermal comfort refers explicitly to the state of well-being arising from an individual not feeling excessively hot or cold[3-5]. This is achieved when the indoor temperature of a residence falls within a specific range. Evaluating a home's capacity for thermal comfort involves various metrics. In the absence of air conditioning and given a constant external temperature over a sufficient duration, the interior temperature of a residence will inevitably align with the external temperature [5].

The temperature dynamics within a house can be expressed through mathematical models employing phenomenological equations. These equations capture temperature changes by considering two key processes [6]: i) conduction heat transfer, occurring through the wall due to temperature variations on both sides and ii) convection heat transport, originating from the airflow generated naturally by differences between the wall's interior temperature and the average air temperature. This convection is associated with density changes, propelling air from warmer to cooler regions.

However, constructing a model that precisely depicts the internal temperature of a house as a function of external temperature proves practically challenging. This is because the external temperature of the wall exhibits a stochastic behaviour influenced by diverse factors, including sunlight intensity, wind speed, atmospheric conditions, and more.

This paper aims to characterize the temperature dynamics within a house by incorporating the unpredictable nature of external temperature. The emphasis is on establishing a connection between thermal comfort and the house's ability to dampen fluctuations in external temperature, ensuring that the interior temperature closely aligns with the comfort zone. To achieve this objective, a stochastic model is sought to describe the house's temperature behaviour, guided by the following principles:

1. Heat transfer occurs through conduction in the wall and natural convection within the room.
2. At time zero, the house's temperature is assumed to be equivalent to the ambient temperature, with the initial value falling within a temperature range deemed comfortable for individuals.
3. The external temperature is treated as a stochastic variable, and the time interval for simulating the system's behavior is considered nearly independent of time. This allows for the assumption that, despite temporal fluctuations, the outside temperature is in a steady state from a stochastic perspective.

The organization of this study is as follows. In Section 2, a deterministic dynamic model is introduced to characterize the house's temperature behaviour. This model is derived by incorporating phenomenological equations describing temperature changes and a macroscopic heat balance. Section 3 outlines the development of a stochastic model constructed from the deterministic dynamic model. This is achieved by employing stochastic methods to account for external fluctuations, utilizing the formalism of Ito Stratonovich's differential calculus. Section 4 is dedicated to presenting and discussing the theoretical outcomes, and the paper concludes with a summary of findings in the final section.

## 2. Model Development

### *2.1. Deterministic Method*

To derive the deterministic model, we initiate the process by applying the equation of temperature change to heat transport in the wall. This consideration involves a non-steady state, conduction-based heat transfer, and the use of a Cartesian coordinate system. Consequently, the result is the formulation of a partial differential equation [7]:

$$\tilde{n}_1 C_1 \frac{\partial T_1(x;t)}{\partial t} = \hat{e}_1 \frac{\partial^2 T_1(x;t)}{\partial x^2} \tag{1}$$

Where 1, $\rho_{C1}$ and *1 are the density [kg.m-3*], the heat capacity [κKcal.kg-1. K-1] and the coefficient of thermal conductivity [W.m-1. $^{K-1}$], respectively, which characterize the material of which the wall is constructed, t *is the time [s]*, *x* is the distance [m] taking the outside of the wall as a reference, and *T1* is the temperature [*0C* $^{]}$. Based on the assumptions set out in Section 1, the initial and boundary conditions are defined:

$$T_1(0,0) = T_0$$

$$T_1(x,0) = T_{1,0}$$

$$T_1(h,\infty) = T_0 \tag{2}$$

Where T1.0 is the temperature for a time equal to zero, and *T0* is the outside temperature. The partial differential equation (1) can be solved exactly by the method of separation of variables, obtaining:

$$T_1(x;t) = \left(\left(T_{1,0} - T_0\right)\frac{x}{h}\right) exp\left(-\frac{t}{\hat{o}_1}\right) + T_0 \tag{3}$$

Where *h* is the thickness of the wall and $\tau_1$ is the time constant that characterizes the temporal dynamics of the temperature in the wall:

$$\hat{o}_1 = \frac{\tilde{n}_1 C_1 h^2}{\hat{e}_1} \tag{4}$$

Doing x = h in equation (3) shows that the temporal differential equation describing the behaviour of the temperature $T_{1,h}$ on the inner side of the wall can be written as follows:

$$\frac{dT_{1,h}}{dt} = \grave{E}_1\left(T_0 - T_{1,h}\right) \tag{5}$$

Where

$$\grave{E}_1 = \frac{\hat{e}_1}{\tilde{n}_1 C_1 h^2} \tag{6}$$

The indoor temperature $T_2$ of the room is contingent on the natural convection heat transfer of the air. By conducting a macroscopic energy balance in a non-steady state, the result is the derivation of the following differential equation:

$$\frac{dT_2}{dt} = \grave{E}_2\left(T_{1,h} - T_2\right) \tag{7}$$

Equation (7) involves the parameter:

$$\grave{E}_2 = \frac{U_2 A}{\tilde{n}_2 C_2 V} \tag{8}$$

Moreover, it is subject to the initial conditions:

$$T_2(0) = T_{1,0} \tag{9}$$

Where $\rho_{2,}$ C2 *and* U *are the density [kg.m-3*], the heat capacity [$^{Kcal.kg-1}$. $^{K-1}$], and the natural convection coefficient of air [W.m-2. K-1], respectively, *A* is the wall area [m2], and V *is the volume [*m3] of the room.

Equations (5) and (9) form a system of stochastic differential equations that represent the deterministic dynamical model that describes the behavior of the temperature of the house as a function of the ambient temperature.

### *2.2. Stochastic Model*

To incorporate the stochastic nature of the outside temperature, we begin with the deterministic differential equation (5), assuming that the outside temperature is a random variable. Consequently, $T_1$, $T_{1,h}$ becomes a stochastic variable. Equation (5) is then reformulated as follows.

:

$$\frac{dT_{1,h}{}^{s}}{dt} = \Theta_1\left(T_0{}^{s} - T_{1,h}{}^{s}\right) \tag{10}$$

Where the supra script indicates that it is a stochastic variable.

The stochastic character of the outside temperature will be taken into account by expressing this variable as the sum of its deterministic value *T0* and a random variable ξ representing the fluctuations [7-9]:

$$T_0^s = T_0 + \hat{\imath} \tag{11}$$

Where the expected value and the temporal covariance of the fluctuations meet the conditions:

$$\langle \hat{\imath}(t) \rangle = 0 \tag{12}$$

$$\sqrt{\langle \hat{\imath}(t)\hat{\imath}(t+\ddot{A}t) \rangle} = \acute{o}_0 \ddot{a}(\ddot{A}t) \tag{13}$$

Where $\sigma_0$ is the standard deviation of external fluctuations and δ is the Dirac delta function, defined as:

$$\ddot{a}(x) = \begin{matrix} 1 & : & x = 0 \\ 0 & : & x \neq 0 \end{matrix} \tag{14}$$

Condition (14) implies that the coefficient of self-correlation of external fluctuations is equal to zero, a necessary consideration in order to assume the Markov property and obtain an analytical solution of the stochastic model. From this consideration, the stochastic differential equation is obtained:

$$dT_{1,h} = \grave{E}_1\left(T_0 - T_{1,h}\right)dt + \grave{E}_1\acute{o}_0 dW \tag{15}$$

Where *W* is a stochastic variable whose probability distribution function is known as the Weiner process. According to the Ito-Stratonovich differential calculus, the Fokker-Planck equation (EFP) that describes the behaviour of the probability $P(T_{1,h};t)$ of which the temperature on the inside of the wall has a value of *T1,h* at time t is:

$$\frac{\partial P\left(T_{1,h};t\right)}{\partial t} = -\frac{\partial}{\partial T_{1,h}}\grave{E}_1\left(T_0 - T_{1,h}\right)P\left(T_{1,h};t\right) + \frac{1}{2}\frac{\partial^2}{\partial T_{1,h}^2}\grave{E}_1^2\acute{o}_0^2 P\left(T_{1,h};t\right) \tag{16}$$

Subject to the initial condition:

$$P\left(T_{1,h};0\right) = 1 \tag{17}$$

Which implies that for a time equal to zero, the temperature is known with absolute certainty. The PFS (16) is linear, so its solution is a Gaussian distribution function whose expected value and variance 12 σ are described through the system of temporal differential equations:

$$\frac{dT_{1,h}}{dt} = \grave{E}_1\left(T_0 - T_{1,h}\right)$$

$$\frac{d\acute{o}_1^2}{dt} = -2\grave{E}_1\acute{o}_1^2 + \grave{E}_1^2\acute{o}_0^2 \tag{18}$$

If the fluctuating behaviour of the outside temperature is such that its time averages are constant over time, then the probability distribution function that describes the stochastic behaviour of the temperature on the inner side of the wall is time-independent and has as its expected value and variance:

$$T_{1,h} = T_0 \tag{19}$$

$$\acute{o}_1^2 = \grave{E}_1\acute{o}_0^2 \tag{20}$$

To determine the temperature inside the house, we start from the differential equation (7), where the temperature of the inner side of the wall and the air temperature are stochastic variables, from which the stochastic differential equation is obtained:

$$dT_2 = \grave{E}_2\left(T_{1,h} - T_2\right)dt + \grave{E}_2\acute{o}_1 dW \tag{21}$$

Where the corresponding PFE associated with the temperature inside the house is written as follows:

$$\frac{\partial P(T_2;t)}{\partial t} = -\frac{\partial}{\partial T_2}\grave{E}_2\left(T_{1,h} - T_2\right)P(T_2;t) + \frac{1}{2}\frac{\partial^2}{\partial T_2^2}\grave{E}_2^2\acute{o}_1^2 P(T_2;t) \tag{22}$$

In steady state, it is obtained that the expected value and the variance of the interior temperature of the room can be given by:

$$T_2 = T_{1,h} = T_0 \tag{23}$$

$$\acute{o}_2^2 = \grave{E}_2\acute{o}_1^2 = \grave{E}_2\grave{E}_1\acute{o}_0^2 \tag{24}$$

The term:

$$\grave{E}_2\grave{E}_1 = \frac{A}{Vh^2}\frac{U \times \hat{e}_1}{(\tilde{n}_2 C_2) \times (\tilde{n}_1 C_1)} \tag{25}$$

It quantifies the extent to which the dimensions of the house and the properties of the building materials can dampen the fluctuations that take place in the ambient temperature in such a way that as this term decreases, so do the fluctuations in the temperature of the house, which in the context of this work is identified with an increase in the comfort of the home. Therefore, the comfort index is defined as the relationship:

$$\varnothing = \frac{Vh^2}{A}\frac{(\tilde{n}_2 C_2) \times (\tilde{n}_1 C_1)}{U \times \hat{e}_1} \tag{26}$$

Predicted theoretical outcomes and discussion

To analyze the influence of the construction material used on the comfort of the home, different materials were considered, the properties of which are shown in Table 1. For all cases, the air properties and room dimensions shown in Table 2 were considered.

**Table 1. Properties of the building materials considered to obtain the predicted results according to the proposed model [10-12]**

| **Material** | **$\kappa$ [W.m$^{-1}$. K$^{-1}$]** | ***C* [Kcal.kg$^{-1}$. K$^{-1}$]** | **$\rho$ [kg.m$^{-3}$]** |
|---|---|---|---|
| Concrete | 0,8 | 0,8 | 2400 |
| Marble | 2,09 | 0,879 | 2690 |
| Glass | 0,8 | 0,84 | 2500 |
| Brick | 0,6 | 0,85 | 1800 |

**Table 2. Air Properties and Dimensions of the House**

| Properties of the air | ***A* [W.m-2. K-1]** | ***C* [Kcal.kg$^{-1}$. K$^{-1}$]** | **$\rho$ [kg.m$^{-3}$]** |
|---|---|---|---|
| | 5 | 0,24 | 1,2 |
| Dimensions of the house | *A* [m2] | *V* [m3] | *h* [m] |
| | 125 | 125 | 0,15 |

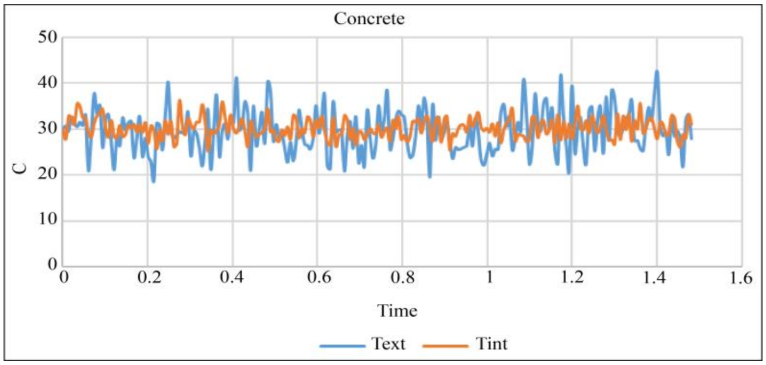
Table 3 shows the values of the comfort index calculated for each of the materials according to the model proposed in this work.

**Table 3. Comfort indices calculated for the materials considered**

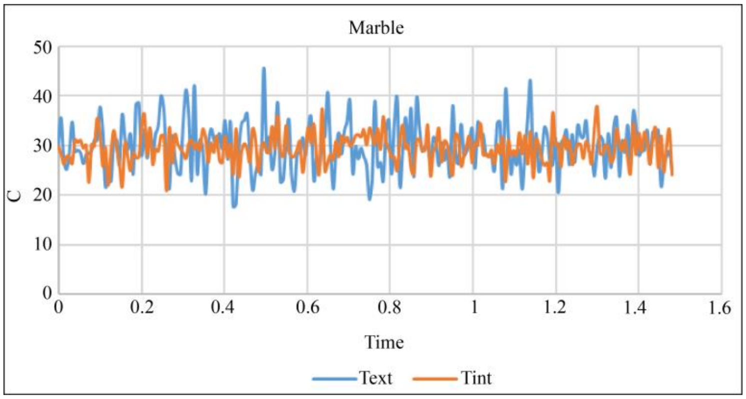
| **Material** | **$\Psi$** |
|---|---|
| Concrete | 2,3062 |
| Marble | 1,5834 |
| Glass | 2,4119 |
| Brick | 2,3772 |

## 3. Results and Discussion

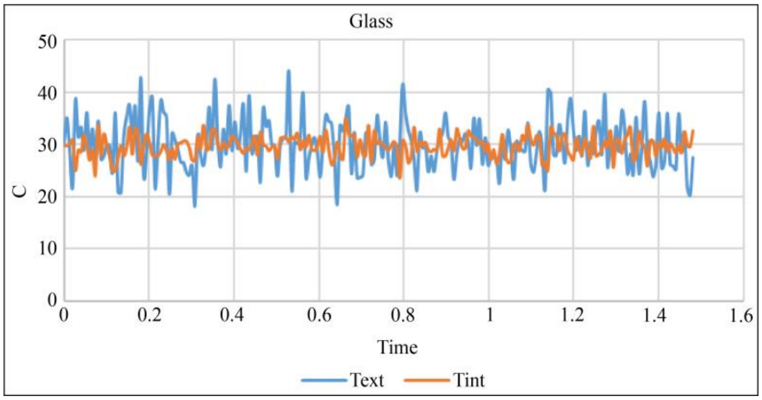
Figure 1 shows the temporal behaviours of the outside temperature and the temperature inside the house obtained by applying the Monte Carlo simulation method and the proposed stochastic model.



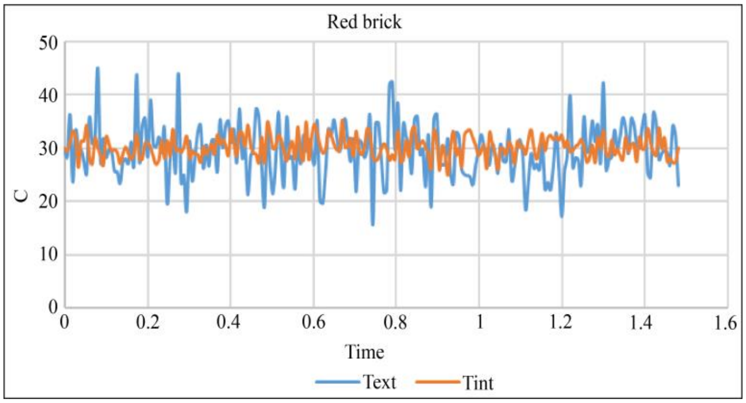
Concrete $\Psi$ = 2.3062



Marble $\Psi$ = 1.5834



Glass Ψ = 2.4119



Ladrillo Ψ = 2,3772

**Fig. 1 Simulation of the stochastic behavior of the ambient temperature (Text) and the temperature of the house (Tint) considering different materials in houses with the same dimensions**

Figure 2 shows the simulated temperature behaviours for each of the materials considered, taking into account the exact behaviour of the ambient temperature.

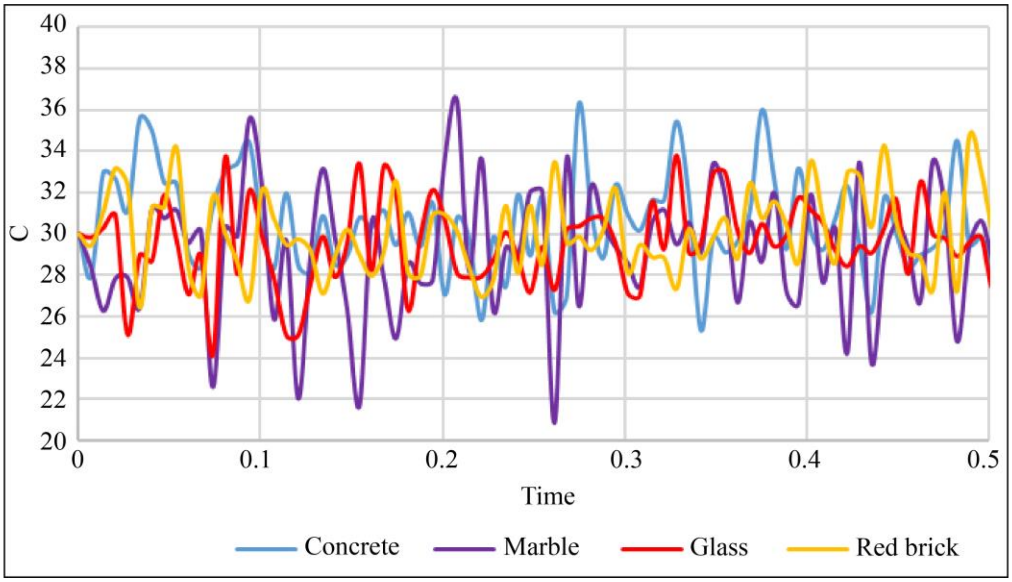


**Fig. 2 Stochastic behaviour of the temperature of the house considering different materials for houses of the same dimensions and the same behaviour of the outside temperature**

In a general way, in Figure 1, we present a visually insightful simulation that captures the stochastic nature of ambient temperature (Text) alongside the resultant temperature of the house (Tint). This simulation is conducted with meticulous consideration of various materials within houses of identical dimensions. The aim is to unravel the diverse thermal responses engendered by different materials in the face of fluctuating ambient conditions.

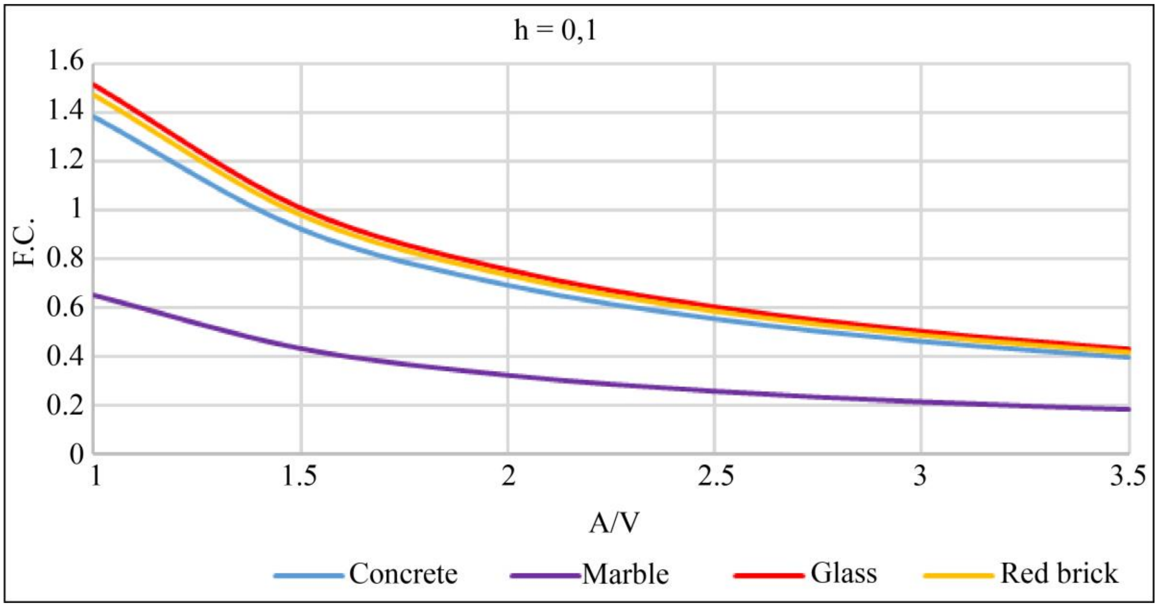
The juxtaposition of Text and Tint in Figure 1 provides a dynamic representation of how various materials influence the internal temperature of a house. This simulation is pivotal in elucidating the nuanced interplay between external and internal temperature dynamics, offering a visual narrative of the impact that material selection can have on the thermal performance of a residence.

Expanding our investigation, Figure 2 delves deeper into the specific simulated temperature behaviours associated with each material under consideration. Notably, this figure maintains a consistent external temperature profile, allowing for a direct comparison of the materials' thermal responses within the same environmental context. By isolating the impact of materials on internal temperature dynamics, Figure 2 becomes a valuable tool for discerning the thermal characteristics inherent to each material.

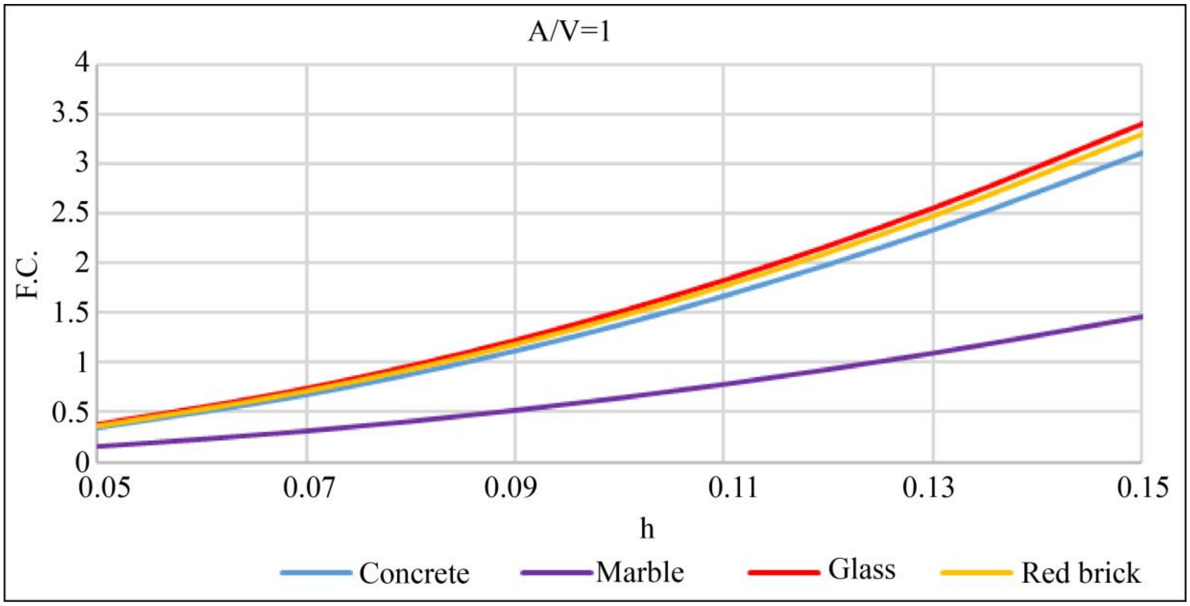
As depicted in Figure 2, the stochastic behaviour of house temperature varies significantly depending on the material employed, even when subjected to an identical external temperature profile. This underscores the critical role that material selection plays in shaping the thermal envelope of a dwelling. From insulating materials that exhibit a more stable internal temperature to those with a more responsive and dynamic behaviour, the figure provides a nuanced portrayal of the thermal outcomes associated with different construction materials.

In summary, the predicted results indicate that, as expected, temperature fluctuations inside homes are mainly dampened, which is a condition that ensures that living spaces protect people against inclement weather, thus fulfilling their primary function [13]. On the other hand, Figure 2 shows how the ability of a house to cushion fluctuations depends on the material selected. In this case, glass is shown to be the best insulator (these are the materials that have the lowest thermal conductivity). At the same time, the most significant fluctuations correspond to marble, which is the material with the highest thermal conductivity. This also corresponds to the modern trend of designing the walls of homes using different materials [14]] to increase the insulation of the exterior. This is because when selecting a material, it is necessary to consider not only its thermal properties but also its ability to withstand stresses in such a way that the house is also protected from natural disasters, such as hurricanes and earthquakes.

Finally, Figure 3 shows how the comfort factor decreases with the ratio between the area of the walls and the volume of the house considering the type of material as a parameter (Figure 3. A) and how the comfort factor increases with the thickness of the wall (Figure 3. B).



**A. Influence of wall area**



B. Influence of wall thickness

Fig. 3 Behavior of the comfort factor with respect to the dimensions of the house

## 4. Conclusion

The focus of our current research lies in the development and application of a stochastic model aimed at predicting the intricate dynamics of temperature within residential structures. This model is meticulously designed to account for the inherent uncertainty associated with the random fluctuations in ambient temperature. By leveraging the insights gained from this model, coupled with the robust Monte Carlo simulation method, we generate comprehensive predictions regarding temperature behaviour in a residential setting.

Crucially, our investigations extend beyond theoretical constructs, delving into the practical implications of diverse materials on temperature management. Through a systematic analysis encompassing various materials, we derive nuanced results that provide a more holistic understanding of the thermal characteristics of a house. This multifaceted approach allows us to evaluate the performance of different materials under real-world conditions, offering valuable insights for architects, builders, and homeowners alike.

Integral to our study is the introduction of a comfort index or factor, a novel metric devised to measure the dwelling's capacity to mitigate temperature fluctuations quantitatively. This index serves as a valuable tool in assessing the overall thermal performance of a residence, considering its ability to maintain comfort in the face of varying ambient temperatures. Findings reveal compelling correlations between thermal comfort and specific architectural and material parameters. Notably, we observe that thermal comfort experiences a discernible boost with a decrease in the thermal conductivity of materials. Additionally, increased wall thickness emerges as a contributing factor to enhanced thermal comfort. Furthermore, a noteworthy revelation underscores the significance of the exterior wall area-to-house volume ratio, indicating that a reduction in this ratio positively influences thermal comfort.

In essence, our research contributes significantly to the understanding of factors influencing thermal comfort in residential structures. The implications of our findings extend beyond theoretical discourse, offering practical guidance for designing energy-efficient and comfortable living spaces. Architects, builders, and homeowners stand to benefit from the nuanced insights provided by our study, fostering advancements in sustainable and comfortable housing.

## Funding Statement

This paper was supported by the UAT/SIP/041 2023 project.

## Acknowledgments

The authors thank the staff who attended the materials characterization and the Nanoscience and Nanotechnology Research National Laboratory (LINAN) at IPICyT for providing access to their facilities.

## References

[1] V. Grandson, R. Cubillos, and R. Barrios, "Resilient Design Aspects Applied to the Envelope that Determine Thermal Comfort in Social Housing," *Construction Engineering Magazine*, vol. 36, no. 2, pp. 197-209, 2021. [CrossRef] [Google Scholar] [Publisher Link]

[2] Seyed Morteza Hosseini et al., "A Morphological Approach for Kinetic Façade Design Process to Improve Visual and Thermal Comfort," *Building and Environment*, vol. 153, pp. 186-204, 2019. [CrossRef] [Google Scholar] [Publisher Link]

[3] Lasse Rohde et al., "Framing Holistic Indoor Environment: Definitions of Comfort, Health and Well-Being," *Indoor and Built Environment*, vol. 29, no. 8, pp.1118-1136, 2020. [CrossRef] [Google Scholar] [Publisher Link]

[4] Camila Salazar-Fernández et al., "The Perceived Impact of COVID-19 on Comfort Food Consumption Over Time: The Mediational Role of Emotional Distress," *Nutrients*, vol. 13, no. 6, pp. 1-12, 2021. [CrossRef] [Google Scholar] [Publisher Link]

[5] Qiantao Zhao, Zhiwei Lian, and Dayi Lai, "Thermal Comfort Models and their Developments: A Review," *Energy and Built Environment*, vol. 2, no. 1, pp.21-33, 2021. [CrossRef] [Google Scholar] [Publisher Link]

[6] Kudabayev Ruslan et al., "Modeling the Thermal Regime of a Room in a Building with a Thermal Energy Storage Envelope," *Mathematical Modelling of Engineering Problems*, vol. 9, no. 2, 2022.

[7] Robert Byron Bird, Edwin Niblock Lightfoot, and Warren E. Stewart, *Transport Phenomena*, Wiley, pp. 1-780, 1960. [Google Scholar] [Publisher Link]

[8] Crispin W. Gardiner, *Handbook of Stochastic Methods for Physics, Chemistry, and the Natural Sciences*, Springer-Verlag, pp. 1-442, 1983. [Google Scholar] [Publisher Link]

[9] N.G. Van Kampen, *Stochastic Processes in Physics and Chemistry*, Elsevier Science, pp. 1-480, 1992. [Google Scholar] [Publisher Link]

[10] Jack Philip Holman, *Heat Transfer*, McGraw-Hill, pp. 1-696, 1997. [Google Scholar] [Publisher Link]

[11] Jeffrey D. Camm, and James R. Evans, *Management Science: Modelling, Analysis and Interpretation*, South-Western College Publishing, pp. 1-865, 1996. [Google Scholar] [Publisher Link]

[12] Wayne L. Winston, S. Christian Albright, and Mark Nathan Broadie, *Practical Management Science: Spreadsheet Modeling and Applications*, Duxbury Press, pp. 1-796, 1997. [Google Scholar] [Publisher Link]

[13] G. N. Uzakov et al., "Mathematical Modeling of the Combined Heat Supply System of a Solar House," *Energy. News of Higher Educational Institutions and Energy Associations of the CIS*, vol. 65, no. 5, pp. 412-421, 2022. [CrossRef] [Google Scholar] [Publisher Link]

[14] C. Suresh, Tapano Kumar Hotta, and Sandip K. Saha, "Phase Change Material Incorporation Techniques in Building Envelopes for Enhancing the Building Thermal Comfort-A Review," *Energy and Buildings*, vol. 268, 2022. [CrossRef] [Google Scholar] [Publisher Link]