

# Instability of Hydromagnetic Double diffusive Convective Flow of Viscoelastic Maxwell Fluid Past Thorough Porous Medium

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## Abstract-

The Present manuscript reports the instability mechanism of Hydromagnetic Double diffusive convective flow in a horizontal layer of viscoelastic Maxwell fluid through porous medium with internal linear heating is presented in this manuscript. The flow is also effected with temperature and concentration gradient in their medium. The Darcy model is adopted in the momentum equation. The onset of stationary and oscillatory instabilities of the viscolastic Maxwell fluid layer is determine between free-free boundaries. The main emphasis is given to the internal heating which is linear in nature. The entire result section is presented in terms of magnetic effect and critical heat source of intensity with respect to other governing physical parameter.

**Keywords** – Viscoelastic Maxwell fluid, Porous media, Soret effect, Dufour effect, Rayleigh number, Internal heat source, magnetic effect.

## 1. INTRODUCTION

Transport of heat through a porous medium is being extensively studied, as understanding the associated transport processes becomes increasingly important. This interest stems from the variety of cases which can be modeled or approximated as transport through porous media, such as packed sphere beds, high performance insulation for buildings, chemical catalytic reactors, grain storage, and many others. The study of convective flow with heat and mass transfer under the influence of chemical reaction with heat source has practical applications in many areas of science and engineering. This phenomenon plays an important role in chemical industry, petroleum industry, cooling of nuclear reactors, and packed-bed catalytic reactors. Natural convection flows occur frequently in nature due to temperature differences, concentration differences, and also due to combined effects. The concentration difference may sometimes produce qualitative changes to the rate of heat transfer. The study of heat generation in many fluids due to exothermic and endothermic chemical reactions and natural convection with heat generation can be added to combustion

modeling. Heat source and chemical reaction effects are crucial in controlling the heat and mass transfer. Recently, the equally problem of hydromagnetic convective flow of a conducting fluid through a porous medium has been investigated.

The study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. Possible applications of this type of flow can be found in many industries. Ajay Kumar Singh [1] has studied unsteady free convection flow of an incompressible micro-polar fluid past in infinite vertical plate with temperature gradient dependent heat source. Ajay Kumar Singh [2] who studied the effects of thermal diffusion on MHD free convection flow through a vertical channel. Acharya et al.[3] studied heat and mass transfer over an accelerating surface with heat source in presence of suction and injection. Atul Kumar Singh [4] investigated the effects of mass transfer on free convection in MHD flow of a viscous fluid. Cortell [5] studied flow and heat transfer of an electrically conducting fluid of second grade over a stretching sheet subject to suction and to a transverse magnetic field. Anjalidevi and Kandasamy [6] have examined the effect of a chemical reaction on the flow in the presence of heat transfer and magnetic field.

Mansour et al.[7] analyzed the effect of chemical reaction and viscous in porous media with suction or injection. However, in engineering and technology, there are occasions where a heat source is needed to maintain the desired heat transfer. At the same time, the suction velocity has also to be normal to the porous plate. Recently S.Shivaiah et al [8] analyze the effect of chemical reaction on unsteady magneto hydrodynamic free convective fluid flow past a vertical porous plate in the presence of suction or injection. More recently B.R.Rout et al [9] investigate the influence of chemical reaction

and the combined effects of internal heat generation and a convective boundary condition on the laminar boundary layer MHD heat and mass transfer flow over a moving vertical flat plate. The lower surface of the plate is in contact with a hot fluid while the stream of cold fluid flows over the upper surface with heat source and chemical reaction.

The flow through porous media is a subject of most common interest and has emerged as a separate intensive research area because heat and mass transfer in porous medium is very much prevalent in nature and can also be encountered in many technological processes. In this context the effect of temperature-dependent heat sources has been studied by Moalem [10] taking into account the steady state heat transfer within porous medium. Rahman and Sattar [11] have investigated the effect of heat generation or absorption on convective flow of a micropolar fluid past a continuously moving vertical porous plate in presence of a magnetic field. The effect of chemical reaction on different geometry of the problem has been investigated by many authors. Das et al. [12] have studied the effect of mass transfer flow past an impulsively started infinite vertical plate with heat flux and chemical reaction.

The chemical reaction effect on heat and mass transfer flow along a semiinfinite horizontal plate has been studied by Anjalidevi and Kandaswamy [13] and later it was extended for Hiemenz flow by Seddeek et al. [14] and for polar fluid by Patil and Kulkarni [15]. Salem and Abd El-Aziz [16] have reported the effect of hall currents and chemical reaction on hydromagnetic flow of a stretching vertical surface with internal heat generation or absorption. J. Anand Rao et al [17], studied the the chemical reaction effects on an unsteady magneto hydrodynamics free convection fluid flow past a semiinfinite vertical plate embedded in a porous medium with heat absorption.

Magnetoconvection (convection in the presence of a magnetic field) on such processes has been intensively studied by many authors. In recent years, progress has been considerably made in the study of heat and mass transfer in magnetohydrodynamic (MHD) flows due to its application in many devices, like the MHD power generators and Hall accelerators.

Recently Philip Oladapo et al [18] evaluated numerically the effects of heat and mass transfer in the hydromagnetic boundary layer flow of the moving plate, and chemical reaction with Dufour and Soret in the presence of suction/injection. Makinde [19-22] have investigated magneto-

hydrodynamics (MHD) convection in porous medium. A similar problem, involving natural convection about a vertical impermeable flat plate, was tackled by Sparrow and Cess [23]. Kumar Jha and Prasad [24] studied the heat source characteristics on the free – convection and mass transfer flow past an impulsively started infinite vertical plate bounded a saturated porous medium under the action of magnetic field.

Yih [25] studied the free convection effect on MHD-coupled heat and mass transfer from a moving permeable vertical surface. Alan and Rahman [26] examined Dufour and Soret effects on mixed hydrogen-air convective flow past a vertical porous flat plate embedded in a porous medium. The onset of double diffusive convection in a two component coupled stress fluid layer with Soret and Dufour effects was investigated by Gaikwad et al. [27], via linear and non-linear stability analysis. Anwar et al. [28] examined the combined diffusion and impedance effects on heat and mass transfer in an electrically-conducting fluid from a vertical stretching surface in a porous medium in the presence of a uniform transverse magnetic field. Numerical simulations of double-diffusive natural convection of water in a partially heated enclosure with Soret and Dufour coefficients around the density maximum were conducted by Nithyadevi and Yang [29].

Yih [30] numerically analyzed the effect of the transpiration velocity on the heat and mass transfer characteristics of the mixed convection about a permeable vertical plate embedded in a saturated porous medium under the coupled effects of thermal and mass diffusion. Elbashareshy [31] studied the effect of the surface mass flux on the mixed convection along a vertical plate embedded in a porous medium. Pal and Talukdar [32] analyzed the combined effect of the mixed convection with the thermal radiation and chemical reaction on the MHD flow of viscous and electrically conducting fluid past a vertical permeable surface embedded in a porous medium. Mukhopadhyay [33] performed an analysis to investigate the effects of the thermal radiation on the unsteady mixed convection flow and heat transfer over a porous stretching surface in a porous medium. Hayat et al. [34] analyzed a mathematical model in order to study the heat and mass transfer characteristics in the mixed convection boundary layer flow about a linearly stretching vertical surface in a porous medium filled with a viscoelastic fluid, by taking into account the diffusion thermo (Dufour) and thermal diffusion (Soret) effects.

Gaikwad et al.[35] investigated the onset of the double diffusive convection in a two-component couple of the stress fluid layer with the Soret and Dufour effects using both linear and nonlinear stability analyses. Ambethkar[36] studied numerical solutions of heat and mass transfer effects of an unsteady MHD free convective flow past an infinite vertical plate with constant suction. Alam et al.[37] studied the Dufour and Soret effects on a steady MHD combined free-forced convective and mass transfer flow past a semi-infinite vertical plate. Alam and Rahman[38] investigated the Dufour and Soret effects on the mixed convection flow past a vertical porous flat plate with variable suction. Postelnicu[39] discussed influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in a porous media considering the Soret and Dufour effects.

Mansour et al.[40] investigated the effects of chemical reaction, thermal stratification, Soret and Dufour numbers on MHD free convective heat and mass transfer of a viscous, incompressible and electrically conducting fluid on a vertical stretching surface embedded in a saturated porous medium. Srihari et al.[41] studied the Soret effect on an unsteady MHD free convective mass transfer flow past an infinite vertical porous plate with the oscillatory suction velocity and heat sink. More recently S. R. Vempatl et al [ 42 ] investigate the effect of flow parameters on the free convection and mass transfer of an unsteady magnetohydrodynamic flow of an electrically conducting, viscous, and incompressible fluid past an infinite vertical porous plate under oscillatory suction velocity and thermal radiation. The Dufour (diffusion thermo) and Soret (thermal diffusion) effects are taken into account.

## 2. MATHEMATICAL FORMULATIONS OF THE PROBLEM

Let us Consider an infinite horizontal layer of viscoelastic Maxwell fluid with thickness “ $d$ ,” confined between the planes  $z = 0$  and  $z = d$  in a porous medium of porosity  $\epsilon$  and medium permeability  $k_1$  and is acted upon by gravity  $\mathbf{g}(0, 0, -g)$ . This layer of fluid is heated and soluted in such a way that a constant temperature and concentration distribution is prescribed at the boundaries of the fluid layer. The temperature ( $T$ ) and concentration ( $C$ ) are taken to be  $T_0$  and  $C_0$  at  $z = 0$  and  $T_1$  and  $C_1$  be the difference in temperature and concentration across the boundaries.

Let  $\mathbf{q}(u, V, w)$ ,  $p$ ,  $\rho$ ,  $T$ ,  $C$ ,  $\alpha$ ,  $\alpha'$ ,  $\mu$ ,  $\kappa$ , and  $k'$ ,  $\mathbf{Q}_0$  be the Darcy velocity vector, hydrostatic pressure, density, temperature, solute concentration, coefficient of thermal expansion, an analogous

solvent coefficient of expansion, viscosity, thermal diffusivity, solute diffusivity, and linear heat source of fluid, respectively.

**2.1. Assumptions.** The mathematical equations describing the physical model are based upon the following assumptions.

- (i) Thermo physical properties expect for density in the buoyancy force (Boussinesq hypothesis) are constant.
- (ii) Darcy’s model with time derivative is employed for the momentum equation.
- (iii) The porous medium is assumed to be isotropic and homogeneous.
- (iv) No chemical reaction takes place in a layer of fluid.
- (v) The fluid and solid matrix are in thermal equilibrium state.
- (vi) Radiation heat transfer between the sides of the wall is negligible when compared with other modes of the heat transfer.

**2.2. Governing Equations.** The Governing equations for viscoelastic Maxwell fluid through porous medium is governed in form of partial differential equation which may written as

$$\nabla \cdot \mathbf{q} = 0$$

$$\left(1 + \lambda \frac{\partial}{\partial t}\right) (-\nabla p + \rho(1 - \alpha(T - T_0) + \alpha'(C - C_0)g)) - \frac{\mu}{k_1} \mathbf{q} - \sigma \frac{B_0^2}{\rho} \mathbf{u} = 0$$

$$\sigma \frac{\partial T}{\partial t} + \mathbf{q} \cdot \nabla T = k \nabla^2 T + D_{TC} \nabla^2 C + \mathbf{Q}_0(T' - T_0),$$

$$\epsilon \frac{\partial C}{\partial t} + \mathbf{q} \cdot \nabla C = k' \nabla^2 C + D_{CT} \nabla^2 T - C,$$

(1)

Where  $D_{TC}$  and  $D_{CT}$  are the dufour and Soret coefficients ;  $\sigma = (\rho c_p)_m / (\rho c_p)_f$  is the thermal capacity ,  $c_p$  is specific heat, and the subscript m and f refer to porous medium and fluid, respectively.

Here the wall temperature and concentration assumed to be constant w.r.t the boundaries of the fluid layer. Therefore , the boundary condition are define as follows

$$w = 0, T = T_0, C = C_0 \text{ at } z = 0$$

$$w = 0, T = T_1, C = C_1 \text{ at } z = d$$

(2)

2.3 **Steady state and its solutions.** The steady state solution can be obtained by assuming

$$u = v = w = 0, p = p(z), T = T_s(z), C = C_s(z) \quad (3)$$

The steady state solution is given by

$$T_s = T_0 - \Delta T \left(\frac{z}{d}\right),$$

$$C_s = C_0 - \Delta C \left(\frac{z}{d}\right) \quad (4)$$

$$p_s = p_0 - p_0 g \left(z + \alpha \frac{\Delta T}{2d} z^2 + \alpha' \frac{\Delta C}{2d} z^2\right)$$

Where subscript 0 shows the value of the variable at boundary  $z = 0$

2.4 **Disturbance in flow : In order to investigate the stability of the flow dynamic, it necessary to give imposed infinitesimal perturbations on the basic state** which is well documented in the book of Chandershakra rao (1992), The perturbation on the base flow is defined as .

$$q = 0 + q', T = 0 + T', C = C_s + C', p = p_s + p' \quad (5)$$

where the parameters  $q', T', C', p'$  is known as the perturbed quantities of the mean flow dynamics. Substituting (5) into (1) and neglecting higher order terms of the perturbed quantities, then we get

$$\nabla \cdot q' = 0$$

$$\left(1 + \lambda \frac{\partial}{\partial t}\right) (-\nabla p' + \rho_0 (\alpha T' + \alpha' C') g) = \frac{\mu}{k_1} q' - \sigma \frac{B_0^2}{\rho} u$$

$$\sigma \frac{\partial T'}{\partial t} - \omega' \frac{\Delta T}{d} = \kappa (\nabla^2 + Q) T' + D_{TC} \nabla^2 C'$$

$$\varepsilon \frac{\partial C'}{\partial t} - \omega' \frac{\Delta C}{d} = \kappa' \nabla^2 C' + D_{CT} \nabla^2 T' \quad (6)$$

the dimensionless parameters are defines as follows.

$$\begin{aligned} (x'', y'', z'') &= \frac{1}{d} (x', y', z'), & (u'', v'', w'') &= \frac{d}{\mu} (u', v', w'), \\ t'' &= \frac{k}{\sigma d^2} t', & T'' &= \frac{T'}{\Delta T}, & C'' &= \frac{C'}{\Delta C}, \\ p'' &= \frac{k_1 d^2}{\mu k} p', & Q &= \frac{h^2 Q_0}{\alpha_m (\rho c_p)_f}, & M &= \sigma \frac{B_0^2}{\mu} V \end{aligned} \quad (7)$$

Remove asterisk for the simplicity

$$\nabla \cdot q = 0$$

$$\left(1 + F \frac{\partial}{\partial t}\right) (-\nabla p + RaT + Ra_s C) - q - MU = 0$$

$$\frac{\partial T}{\partial t} - w = (\nabla^2 + Q)T + D_f \nabla^2 C,$$

$$\frac{\varepsilon \partial C}{\sigma \partial t} - w = \frac{1}{Le} \nabla^2 C - C + S_r \nabla^2 T \quad (8)$$

The different non-dimensional parameters are defined as follows.

,  $Ra = \frac{\alpha d k_1 \Delta T g \rho_0}{\mu k}$  is the thermal Rayleigh number

,  $Ra_s = \frac{\alpha' d k_1 \Delta C g \rho_0}{\mu k'}$  is the solutal Rayleigh number,

$Le = \frac{k}{\sigma d^2}$  is the Lewis number,  $F = \left(\frac{k}{\sigma d^2}\right) \lambda$

is the stress relaxation parameter and Q is the rate of heat addition per unit mass by internal sources.

$D_f = \frac{D_{TC}}{k} \frac{\Delta C}{\Delta T}$  is the Dufour parameter, and

$S_r = \frac{D_{CT}}{k} \frac{\Delta T}{\Delta C}$  is the Soret parameter .The non dimensional boundary conditions are

The nondimensional boundary conditions are

$$w = T = C = 0 \text{ at } z = 0, z = 1 \quad (9)$$

### 3. NORMAL MODES AND STABILITY ANALYSIS

The disturbances of the mean flow is taken into account in term of normal modes analysis which is well documented in the book of Drazin (1987).

$$[w, T, C] = [W(z), \Theta(z), \Gamma(z)] \exp(i k_x x + i k_y y + nt) \quad (10)$$

Where the parameters  $k_x, k_y$  are called as wave numbers along with different coordinate axis  $x$  and  $y$  respectively, and  $n$  is defined as growth rate of disturbances. By using eq (10), (8) becomes

$$(D^2 - a^2 - M)W + (1 + Fn)(a^2 Ra \Theta + a^2 Ra_s \Gamma) = 0$$

$$W + (D^2 - a^2 - n - Q) \Theta + D_f (D^2 - a^2) \Gamma = 0$$

$$W + S_r (D^2 - a^2) \Theta + \left(\frac{1}{Le} (D^2 - a^2) - \frac{\varepsilon}{\sigma} n\right) \Gamma = 0$$

Where  $D = \frac{d}{dz}$  and  $a = \sqrt{k_x^2 + k_y^2}$  is define as dimensionless wave number.

The corresponding free – free boundary conditions are

$$\begin{aligned} W = 0, D^2 W = 0, \Theta = 0, \Gamma = 0, \text{ at } z = 0 \\ W = 0, D^2 W = 0, \Theta = 0, \Gamma = 0, \text{ at } z = 1 \end{aligned} \quad (12)$$

We assume the solution to  $W, \Theta,$  and  $\Gamma$  is of the form

$$W = W_0 \sin \pi z, \Theta = \Theta_0 \sin \pi z, \Gamma = \Gamma_0 \sin \pi z, \quad (13)$$

Those satisfy the boundary conditions (12).

Substituting solution (13) in (11), integrating each equation from  $z = 0$  to  $z = 1$  by parts, we obtain the following matrix equation as

$$\begin{bmatrix} J - M & -a^2(1 + Fn)Ra & -a^2(1 + Fn)Ra_s \\ -1 & (J + n + Q) & D_f J \\ -1 & S_r J & \left(\frac{J}{Le} - \frac{\epsilon n}{\sigma}\right) \end{bmatrix} \begin{bmatrix} W_0 \\ \Theta_0 \\ \Gamma_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (14)$$

Where  $J = \pi^2 + a^2$

The nontrivial solution corresponding to the matrix given in eq (14)

$$Ra = \frac{(J - M)(J + n + Q)\left(\frac{J}{Le} - \frac{\epsilon n}{\sigma}\right) - S_r D_f J^2}{a^2(1 + Fn)J\left(\frac{1}{Le} - D_f\right) + \frac{\epsilon n}{\sigma}} + \frac{(S_r J - (J + n + Q))}{J\left(\frac{1}{Le} - D_f\right) - \frac{\epsilon n}{\sigma}} Ra_s \quad (15)$$

For neutral instability  $n = i\omega$ , (where  $\omega$  is real and dimensionless frequency of oscillation) and equating real and imaginary parts of (15), we have

$$\begin{aligned} (J + M) \left( \left( \frac{J^2}{Le} + \frac{\epsilon \omega^2}{\sigma} + \frac{QJ}{Le} + QKr \right) - S_r D_f J^2 \right) + \\ a^2 Ra \left( D_f - \frac{1}{Le} \right) + \frac{\epsilon \omega^2 F}{\sigma} - a^2 Rs \left( J + Q - S_r J - \omega^2 F \right) = 0 \\ \left( \frac{J^2}{Le} \right) + a^2 Ra \left( FJ \left( D_f - \frac{1}{Le} \right) - F \right) - a^2 Rs \left( JF - S_r JF + FQ \right) = 0 \end{aligned} \quad (16)$$

For stationary convection  $\omega=0$  ( $n=0$ ), we have

$$Ra = \frac{(J + M)(S_r D_f J^2 - J - Q)\left(\frac{J}{Le}\right)}{a^2\left(D_f J - \frac{1}{Le}\right)} + \frac{a^2[J + Q - S_r]}{D_f J - \left(\frac{J}{Le}\right)a^2} Ra_s \quad (17)$$

Her the onset instability is measured in form of stationary convection. The different parameter is defined as , the Rayleigh number Ra is a function of dimensionless wave number  $a$ , Dufour parameter  $D_f$ , Soret parameter  $S_r$ , Lewis number Le and solutal Rayleigh number  $Ra_s$ , internal heat source  $Q$ , and the magnetic number M. Thus for stationary convection of viscoelastic Maxwell fluid is work as an ordinary Newtonian fluid.

The critical cell size at the onset of instability is calculated from.

$$\left(\frac{\partial Ra}{\partial a}\right)_{a=a_c} = 0 \quad \text{which gives } a_c = \pi$$

The corresponding critical Rayleigh number  $Ra_T$  for the steady onset is

$$Ra_T = 4\pi^2 \left( \frac{S_r D_f Le - 1}{D_f Le - 1} \right) + \frac{(S_r - 1)Le}{1 - D_f Le} Ra_s \quad (18)$$

$$\text{If } S_r = D_f = Ra_s = 0 \quad \text{then } Ra_T = 4\pi^2 \quad (19)$$

#### 4. RESULT AND DISCUSSION

The onset of double diffusive convection in a horizontal layer of Maxwell viscoelastic fluid in the presence of magnetic , heating effect ,Soret and Dufour in a porous medium is investigated analytically. The expressions for both the stationary and oscillatory convection is obtained using normal mode analysis in terms of critical rayleigh numbers, which characterize the stability of the system, are obtained analytically. It has been observed that the stationary critical Rayleigh number is found to be independent from viscoelastic parameter F, In this continuation we found that the Maxwell viscoelastic binary fluid behaves like ordinary Newtonian binary fluid during stationary convection. It is also important that the stationary critical Rayleigh number and critical wave number are independent of viscoelastic parameter because of the absence of base flow in the present case.

The expressions for the stationary and oscillatory Rayleigh numbers for different values of the parameters such as internal Rayleigh number, soret number , dufour number , lewis number , magnetic and heat parameter are computed and the results are depicted in figures.

The neutral stability curves in  $Ra_T - \alpha$  plane is plotted in different physical condition to analysis the linear stability of the system , for different values of parameters. In Figures. 1–6 the stability curve in a different plan is depicted. The wide range of physical parameter is taken into account during the study. Figure 1 shows the stability curve between  $Ra_T - \alpha$  for different value of Dufore parameter  $D_f$  while fixing the other parameter at  $S_r = 0.001, Le = 1.0, Ra_s = 10.0, M = 0.01,$  and  $Q = 10$ . The Figure 2 shows the variation of  $Ra_T$  with respect to  $\alpha$  for the different values of Magnetic number (M) while fixing the values of the other physical parameters at  $S_r = 0.001, D_f = 0.2, Le = 1.0, Ra_s = 10.0, Q =$

10 with variation in one of the parameters. It is clear from these figures that the neutral curves are connected in a topological sense. This connectedness allows the linear stability criteria to be expressed in terms of the critical Rayleigh number,  $Ra_T$ , below which the system is stable and unstable above. From these figures we can easily identify the points where the over stable solutions branch off from the stationary convection. Also we observe that for smaller values of the wave number, each curve is a margin of the oscillatory instability, and at some fixed wave number depending on the other parameters, the overstability disappears and the curve forms the margin of stationary convection. The characteristic curves for different

value of  $D_f$  and  $M$  have been presented in Fig. 1 and 2, respectively. The characteristic curves exhibit one or two minima corresponding to the oscillatory or stationary points, where the oscillating solutions branch off from the stationary solution. At these points both stationary and oscillatory convection occur simultaneously. Although stationary convection does not depend on  $D_f$  and  $M$ , however, oscillatory Rayleigh number decreases on increasing both  $D_f$  and  $M$ , thus advancing the onset of convection. In both curves it can be clearly seen that the slope is changing rapidly w.r.t the different parametric values.

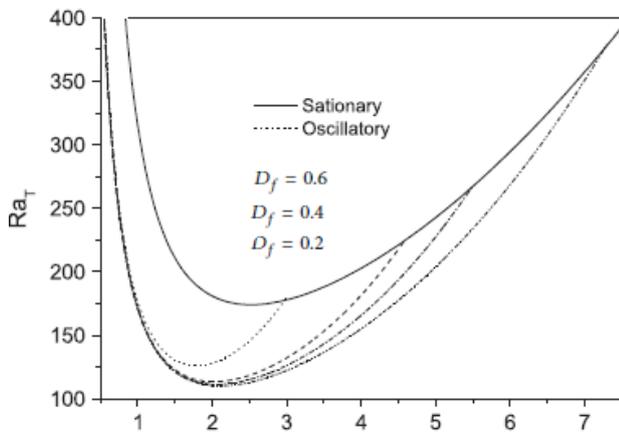


Figure 1 Variation of  $Ra_T$ - $\alpha$  for different values of  $D_f$

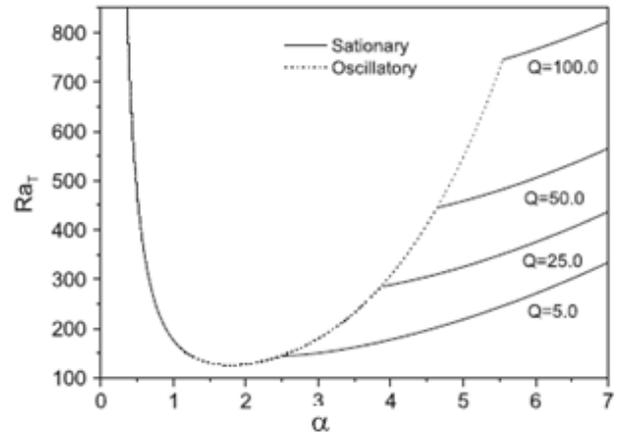


Figure 3 Variation of  $Ra_T$ - $\alpha$  for different values of  $Q$

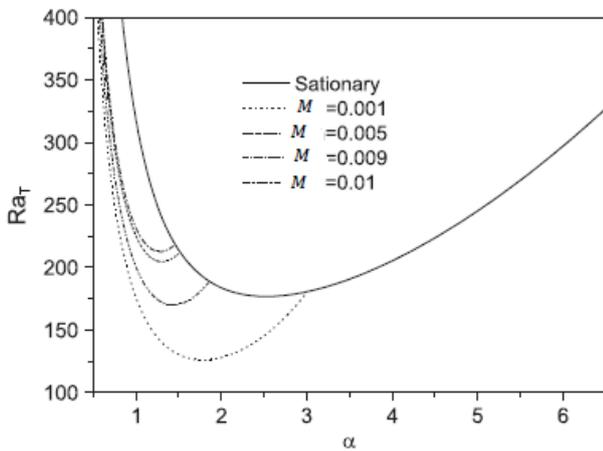


Figure 2 Variation of  $Ra_T$ - $\alpha$  for different values of  $M$

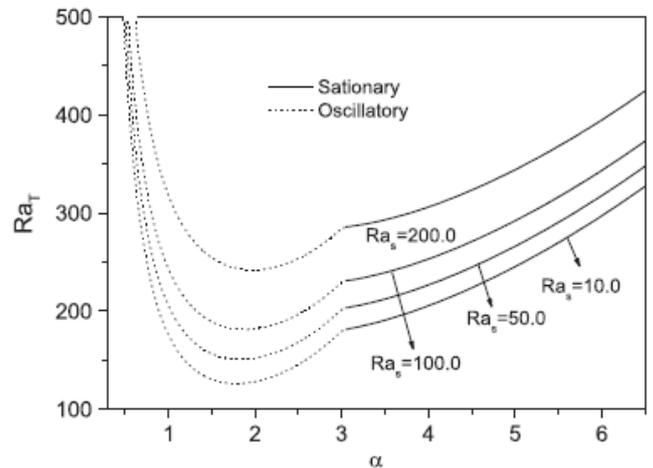


Figure 4 Variation of  $Ra_T$ - $\alpha$  for different values of  $Ra_s$

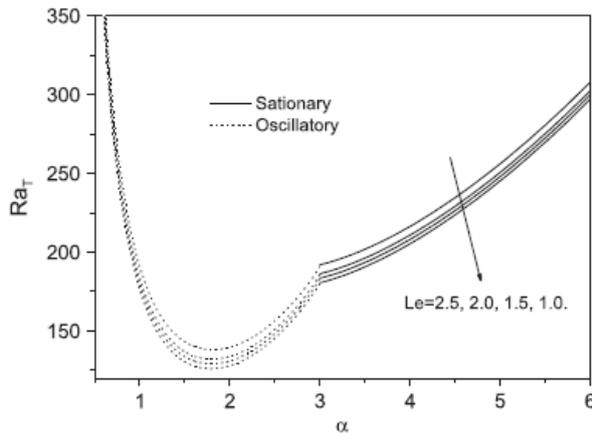


Figure 5 Variation of  $Ra_T - \alpha$  for different values of  $Le$

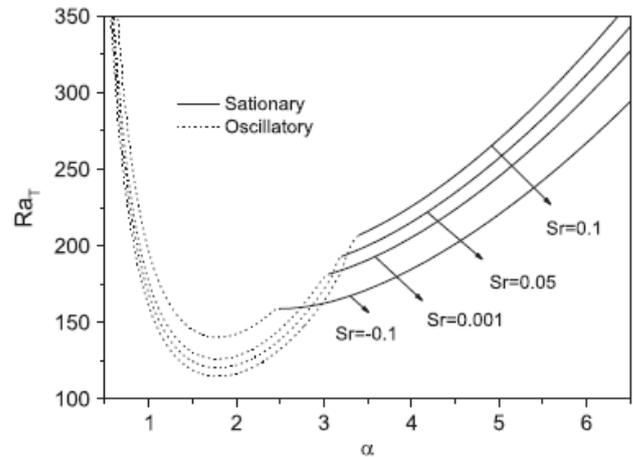


Figure 6 Variation of  $Ra_T - \alpha$  for different values of  $Sr$

Figure.3 depicts the effect of heat source intensity ( $Q$ ) on the neutral stability curve. Here we fixed the other Physical paramete  $S_r = 0.001, D_f = 0.2, Le = 1.0, Ra_s = 10.0, M = 0.01$ . Now we can see that the effect of increasing  $Q$  is to increase the critical value of Rayleigh number and the corresponding wave number, implying that  $Q$  has a stabilizing effect on the double diffusive convection in porous medium. This figure also indicates that for small  $Q$ , the instability manifests as stationary convection, while as  $Q$  is increased, the instability sets in as oscillatory convection. Fig. 4 indicates the effect of scalesolute Rayleigh number on neutral stability curve. We find that the effect to increasing  $Ra_s$  is to increase the critical value of Rayleigh number and the corresponding wave number, indicating that the effect of solute Rayleigh number  $Ra_s$  is to inhibit the onset of convection. Similarly Figure 5 is plotted for the different values of Lewis number ( $Le$ ), respectively, for fixed values of other parameters at  $Sr = 0.001, D_f = 0.2, Ra_s = 10.0, M = 0.001, Q = 10$ . Here we can see that quantitatively that the heat source intensity is increases as we incedared the value of  $Le$  it means that the lewis number play an important role of flow stability to stabilize it. Finally in Fiureg. 6, we investigate the effect of Soret parameter on the neutral stability for fixed values of other parameters at  $Le = 02, = 0.2, Ra_s = 10.0, M = 0.001, Q = 10$ . It can be observed that an increment in  $Sr$  decreases the minimum of  $Ra_T$  Rayleigh number for oscillatory state, however, it increases the minimum value of Rayleigh number for stationary state. Here we can also see that the soret parameter is stabilizing the flow strength.

## 5. CONCLUSIONS

A linear stability analysis of hydromagnetic double diffusive convection in a horizontal layer of Maxwell viscoelastic fluid in the presence of Soret and Dufour in a porous medium is performed analytically. The stationary and oscillatory convection is analyzed in detail during study.

The main conclusions are as follows.

- (i) In stationary convection Maxwell viscoelastic fluid behaves like ordinary Newtonian fluid.
- (ii) Dufour parameter, Soret parameter, and Lewis parameter have both stabilizing and destabilizing effects on the stationary convection.
- (iii) The Heat source parameter ( $Q$ ) is shows the stabilizing character
- (iv) Solutal Rayleigh number destabilizes the stationary convection.
- (v) In limiting case when  $S_r = D_f = Rs = 0$  the critical thermal Rayleigh number obtained is the same as reported by Nield
- (vi) Effect of increasing  $Q, Ra_s, Le$  is found to increase the onset of stationary and oscillatory convection.
- (vii) On increasing the value of  $D_f, M$ , the value of Rayleigh number corresponding to stationary, and oscillatory convection decrease, thus it advances the onset of convection.

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