Finite Difference Analysis of Induction Furnace Wall for Magnesia Ramming Mass

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Abstract

Furnaces are useful for melting different materials for casting process. In this research paper, we had done advanced heat transfer analysis of induction furnace wall made of magnesia ramming mass using explicit finite difference method. We have divided actual geometry of furnace refractory wall into 14 elements and 24 nodes. We have derived explicit finite difference equations for all 24 nodes. We have calculated temperature distribution and thermal stress distribution for all different nodes with respect to time. We have plotted graphs for maximum temperature v/s time and maximum stress v/s time. We found that results indicate the effect of thermal fatigue in the induction furnace wall for magnesia ramming mass. The analysis is very helpful in understanding how thermal fatigue failure of refractory wall happens.

Keywords: *Advanced heat transfer analysis, Temperature distribution, Stress distribution, explicit finite difference method, Magnesia ramming mass*

I. INRODUCTION

Furnace is a term used to identify a closed space here heat is applied to a body in order to raise its temperature. The source of heat may be fuel or electricity. Commonly, metals and alloys and sometimes non-metals are heated in furnaces. The purpose of heating defines the temperature of heating and heating rate. Increase in temperature softens the metals. They become amenable to deformation. This softening occurs with or without a change in the metallic structure. Heating to lower temperatures (below the critical temperature) of the metal softens it by relieving the internal stresses. On the other hand, metals heated to temperatures above the critical temperatures leads to changes in crystal structures and recrystallization like annealing. Some metals and alloys are melted, ceramic products vitrified, coals coked, metals like zinc are vaporized and many other processes are performed in Furnaces. [17]

Induction furnaces are widely used in the iron industry for the casting of the different grades of cast iron products. Refractory wall of induction melting furnace is a key component which is used as insulation layer. It is made of ramming mass like magnesia, silica, alumina etc. The refractory wall is directly influenced by the thermal cycling of the high temperature molten iron in the furnace. Thermal fatigue failure is easy to happen for it because of the larger phase transformation thermal stresses and it has a shorter life. This can cause serious production accidents. Therefore, the service life problem of the refractory wall has always been a focus of attention in the application of this to the industry. [19]

The research on the distribution rule of temperature and thermal stress field and on the fatigue life assessment method for the refractory wall will not only lay foundation for the study on the thermal fatigue of this kind of parts under thermal shock condition of low cycle and high phase transition stresses but also offers effective control for thermal fatigue failure.

A comprehensive literature review on computational investigation on different kinds of furnaces is done to study research trends. [1] A review is done on applications of different numerical methods in heat transfer with its applications. [2] Thermal fatigue analysis of induction melting furnace wall is done for silica ramming mass. [3] A review is done for research on induction heating. [4] A review is done for metal forming analysis using different numerical methods. [5] Transient heat transfer analysis of induction furnace is done by using finite element analysis. [6] Thermal fatigue analysis of induction furnace wall is done for alumina ramming mass. [7]

Thermal analysis of hot wall condenser is done for domestic refrigerator using numerical method for temperature distribution. [8] Optimization of wall thickness is done for minimum heat loss for induction furnace by finite element analysis. [9] A review is done on numerical analysis of furnace. [10] Thermal fatigue analysis of induction furnace wall is done for zirconia. [11] Comparison of finite difference method and finite element method is done for 2 D transient heat transfer problem. [12] Thermal fatigue analysis of induction furnace wall is done for magnesia ramming mass. [13] Advanced mathematical modeling of heat transfer is done for induction furnace wall of zirconia. [14] Advanced heat transfer analysis is done for alumina based refractory wall of induction furnace. [15] Prediction of Life Cycle is done for Induction Furnace Wall for Silica Ramming Mass. [16]

Here, Explicit Finite Difference Method is used to find out temperature and thermal stress variation with respect to time.

II. DEVELOPMENT OF ADVANCED HEAT TRANSFER MODEL

We have divided Induction Furnace Wall into a Nodal Network as shown in Fig. 1. It is divided into 24 nodes. We have derived Explicit Finite Difference Equations for all nodes as per the boundary conditions applied to it. The furnace wall is having thermal conduction heat transfer between different nodes. It is having atmospheric heat convection ha applied from top side of the furnace wall which is open to atmosphere. It is having heat convection from molten metal from inside which is hi. It is having heat convection ho from cooling water which is circulating outside the furnace wall. [18]

To solve this advanced heat transfer problem of induction melting furnace wall which is made from Magnesia Ramming Mass, the following initial and boundary conditions, material properties and basic assumptions are made:

- Refractory Materials for induction furnace wall meets the basic assumptions in the science of mechanics.
- Environmental Temperature is homogeneous at 27° C.
- Ignore the influence of heat radiation.
- Ignore the effect of gravity field.
- The surface of induction melting furnace wall is clean.
- The initial temperature of the induction melting furnace is set 27° C and it is agreement with the ambient temperature during solving the problem.
- Heat convections are considered constant for this analysis.
- Scarp material input inside furnace is considered uniform for our analysis.

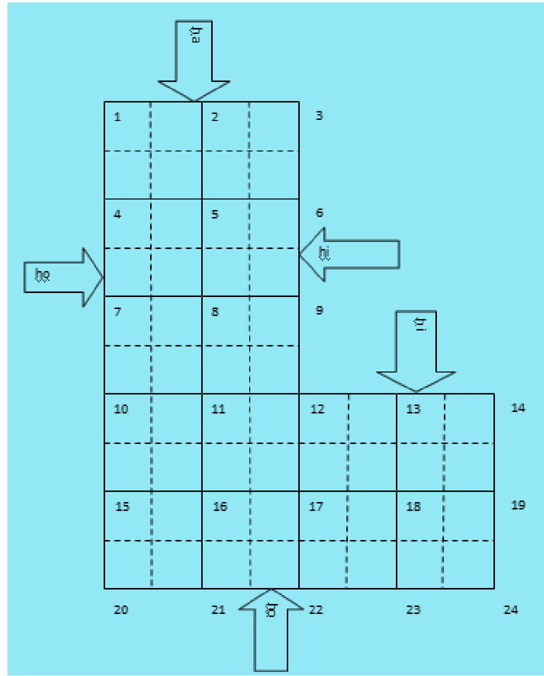

Fig. 1. Nodal Network for Finite Difference Method

Node 1:

$$ha\frac{\Delta x}{2}(T_\infty - T_1^i)+ ho\frac{\Delta y}{2}(T_\infty - T_1^i) + \mathrm{k}\frac{\Delta y}{2}\frac{T_2^i - T_1^i}{\Delta x}+ \mathrm{k}\frac{\Delta x}{2}\frac{T_4^i - T_1^i}{\Delta y} = \rho\frac{\Delta x}{2}\frac{\Delta y}{2}\mathrm{C}\frac{T_1^{i+1} - T_1^i}{\Delta t}$$

$$T_1^{i+1} = ((ha\frac{\Delta x}{2}(T_\infty - T_1^i)+ ho\frac{\Delta y}{2}(T_\infty - T_1^i) + \mathrm{k}\frac{\Delta y}{2}\frac{T_2^i - T_1^i}{\Delta x}+ \mathrm{k}\frac{\Delta x}{2}\frac{T_4^i - T_1^i}{\Delta y})\ \frac{4\Delta t}{\rho \mathrm{C}\Delta x\Delta y})+ T_1^i$$

T[1][i+1] = (((0.5*ha*x*(To-T[1][i])) +(ho*y*(Ta-T[1][i])/2)+(0.5*k*y*(T[2][i]-T[1][i])/x)+(0.5*k*x*(T[4][i]-T[1][i])/y))*((4*t)/(r*c*x*y))) +T[1][i];

Node 2:

$$ha\Delta x(T_\infty - T_2^i)+ \mathrm{k}\frac{\Delta y}{2}\frac{T_1^i - T_2^i}{\Delta x} + \mathrm{k}\frac{\Delta y}{2}\frac{T_3^i - T_2^i}{\Delta x} + \mathrm{k}\Delta x\frac{T_5^i - T_2^i}{\Delta y} = \rho\Delta x\frac{\Delta y}{2}\mathrm{C}\frac{T_2^{i+1} - T_2^i}{\Delta t}$$

$$T_2^{i+1} = ((ha\Delta x(T_\infty - T_2^i)+ \mathrm{k}\frac{\Delta y}{2}\frac{T_1^i - T_2^i}{\Delta x} + \mathrm{k}\frac{\Delta y}{2}\frac{T_3^i - T_2^i}{\Delta x} + \mathrm{k}\Delta x\frac{T_5^i - T_2^i}{\Delta y})\frac{2\Delta t}{\rho \mathrm{C}\Delta x\Delta y}) + T_2^i$$

T[2][i+1] = (((ha*x*(To-T[2][i]))+(0.5*k*y*(T[1][i]-T[2][i])/x)+(0.5*k*y*(T[3][i]-T[2][i])/x) +(0.5*k*x*(T[5][i]-T[2][i])/y))*((2*t)/(r*c*x*y))) +T[2][i];

Node 3:

$$ha\frac{\Delta x}{2}(T_\infty - T_3^i) + hi\frac{\Delta y}{2}(T_h - T_3^i) + \mathrm{k}\frac{\Delta y}{2}\frac{T_2^i - T_3^i}{\Delta x} + \mathrm{k}\frac{\Delta x}{2}\frac{T_6^i - T_3^i}{\Delta y} = \rho\frac{\Delta x}{2}\frac{\Delta y}{2}\mathrm{C}\frac{T_3^{i+1} - T_3^i}{\Delta t}$$

$$T_3^{i+1} = ((ha\frac{\Delta x}{2}(T_\infty - T_3^i) + hi\frac{\Delta y}{2}(T_h - T_3^i) + \mathrm{k}\frac{\Delta y}{2}\frac{T_2^i - T_3^i}{\Delta x} + \mathrm{k}\frac{\Delta x}{2}\frac{T_6^i - T_3^i}{\Delta y})\ \frac{4\Delta t}{\rho\mathrm{C}\Delta x\Delta y}) + T_3^i$$

T[3][i+1] = (((ha*x*(To-T[3][i])*0.5) +(hi*y*(Th-T[3][i])*0.5) +(0.5*k*y*(T[2][i]-T[3][i])/x)+(0.5*k*x*(T[6][i]-T[3][i])/y)) *((4*t)/(r*c*x*y))) +T[3][i];

Node 4:

$$ho\Delta y(T_\infty - T_4^i) + \mathrm{k}\Delta y\frac{T_5^i - T_4^i}{\Delta x} + \mathrm{k}\frac{\Delta x}{2}\frac{T_1^i - T_4^i}{\Delta y} + \mathrm{k}\frac{\Delta x}{2}\frac{T_7^i - T_4^i}{\Delta y} = \rho\frac{\Delta x}{2}\Delta y\mathrm{C}\frac{T_4^{i+4} - T_4^i}{\Delta t}$$

$$T_4^{i+4} = ((ho\Delta y(T_\infty - T_4^i) + \mathrm{k}\Delta y\frac{T_5^i - T_4^i}{\Delta x} + \mathrm{k}\frac{\Delta x}{2}\frac{T_1^i - T_4^i}{\Delta y} + \mathrm{k}\frac{\Delta x}{2}\frac{T_7^i - T_4^i}{\Delta y})\ \frac{2\Delta t}{\rho\mathrm{C}\Delta x\Delta y}) + T_4^i$$

T[4][i+1] = (((ho*y*(Ta-T[4][i])) +(k*y*(T[5][i]-T[4][i])/x)+(0.5*k*x*(T[1][i]-T[4][i])/y) + (0.5*k*x*(T[7][i]-T[4][i])/y)) *((2*t)/(r*c*x*y))) +T[4][i];

Node 5:

$$\mathrm{k}\Delta y\frac{T_4^i - T_5^i}{\Delta x} + \mathrm{k}\Delta y\frac{T_6^i - T_5^i}{\Delta x} + \mathrm{k}\Delta x\frac{T_2^i - T_5^i}{\Delta y} + \mathrm{k}\Delta x\frac{T_8^i - T_5^i}{\Delta y} = \rho\Delta x\Delta y\mathrm{C}\frac{T_5^{i+1} - T_5^i}{\Delta t}$$

$$T_5^{i+1} = ((\mathrm{k}\Delta y\frac{T_4^i - T_5^i}{\Delta x} + \mathrm{k}\Delta y\frac{T_6^i - T_5^i}{\Delta x} + \mathrm{k}\Delta x\frac{T_2^i - T_5^i}{\Delta y} + \mathrm{k}\Delta x\frac{T_8^i - T_5^i}{\Delta y})\ \frac{\Delta t}{\rho\mathrm{C}\Delta x\Delta y}) + T_5^i$$

T[5][i+1] = (((k*y*(T[4][i]-T[5][i])/x)+(k*y*(T[6][i]-T[5][i])/x) + (k*x*(T[8][i]-T[5][i])/y)) *((t)/(r*c*x*y))) +T[5][i];

Node 6:

$$hi\Delta y(T_h - T_6^i) + \mathrm{k}\Delta y\frac{T_2^i - T_6^i}{\Delta x} + \mathrm{k}\frac{\Delta x}{2}\frac{T_3^i - T_6^i}{\Delta y} + \mathrm{k}\frac{\Delta x}{2}\frac{T_9^i - T_6^i}{\Delta y} = \rho\frac{\Delta x}{2}\Delta y\mathrm{C}\frac{T_6^{i+1} - T_6^i}{\Delta t}$$

$$T_6^{i+1} = ((hi\Delta y(T_h - T_6^i) + \mathrm{k}\Delta y\frac{T_2^i - T_6^i}{\Delta x} + \mathrm{k}\frac{\Delta x}{2}\frac{T_3^i - T_6^i}{\Delta y} + \mathrm{k}\frac{\Delta x}{2}\frac{T_9^i - T_6^i}{\Delta y})\ \frac{2\Delta t}{\rho\mathrm{C}\Delta x\Delta y}) + T_6^i$$

T[6][i+1] = (((hi*y*(Th-T[6][i])) + (k*y*(T[2][i]-T[6][i])/x)+(0.5*k*x*(T[3][i]-T[6][i])/y) + (0.5*k*x*(T[9][i]-T[6][i])/y)) *((2*t)/(r*c*x*y))) +T[6][i];

Node 7:

$$ho\Delta y(T_\infty - T_7^i) + \mathrm{k}\Delta y\frac{T_8^i - T_7^i}{\Delta x} + \mathrm{k}\frac{\Delta x}{2}\frac{T_4^i - T_7^i}{\Delta y} + \mathrm{k}\frac{\Delta x}{2}\frac{T_{10}^i - T_7^i}{\Delta y} = \rho\frac{\Delta x}{2}\Delta y\mathrm{C}\frac{T_7^{i+7} - T_7^i}{\Delta t}$$

$$T_7^{i+7} = ((ho\Delta y(T_\infty - T_7^i) + \mathrm{k}\Delta y\frac{T_8^i - T_7^i}{\Delta x} + \mathrm{k}\frac{\Delta x}{2}\frac{T_4^i - T_7^i}{\Delta y} + \mathrm{k}\frac{\Delta x}{2}\frac{T_{10}^i - T_7^i}{\Delta y})\ \frac{2\Delta t}{\rho\mathrm{C}\Delta x\Delta y}) + T_7^i$$

T[7][i+1] = (((ho*y*(Ta-T[7][i])) + (k*y*(T[8][i]-T[7][i])/x)+(0.5*k*x*(T[4][i]-T[7][i])/y) + (0.5*k*x*(T[10][i]-T[7][i])/y)) *((2*t)/(r*c*x*y))) +T[7][i];

Node 8:

$$\mathrm{k}\Delta y\frac{T_7^i - T_8^i}{\Delta x} + \mathrm{k}\Delta y\frac{T_9^i - T_8^i}{\Delta x} + \mathrm{k}\Delta x\frac{T_5^i - T_8^i}{\Delta y} + \mathrm{k}\Delta x\frac{T_{11}^i - T_8^i}{\Delta y} = \rho\Delta x\Delta y\mathrm{C}\frac{T_8^{i+1} - T_8^i}{\Delta t}$$

$$T_8^{i+1} = ((\mathrm{k}\Delta y\frac{T_7^i - T_8^i}{\Delta x} + \mathrm{k}\Delta y\frac{T_9^i - T_8^i}{\Delta x} + \mathrm{k}\Delta x\frac{T_5^i - T_8^i}{\Delta y} + \mathrm{k}\Delta x\frac{T_{11}^i - T_8^i}{\Delta y})\ \frac{\Delta t}{\rho\mathrm{C}\Delta x\Delta y}) + T_8^i$$

T[8][i+1] = (((k*y*(T[7][i]-T[8][i])/x)+(k*y*(T[9][i]-T[8][i])/x) +(k*x*(T[5][i]-T[8][i])/y) + (k*x*(T[11][i]-T[8][i])/y)) *((t)/(r*c*x*y))) +T[8][i];

Node 9:

$$hi\Delta y(T_h - T_9^i) + \mathrm{k}\Delta y\frac{T_8^i - T_9^i}{\Delta x} + \mathrm{k}\frac{\Delta x}{2}\frac{T_6^i - T_9^i}{\Delta y} + \mathrm{k}\frac{\Delta x}{2}\frac{T_{12}^i - T_9^i}{\Delta y} = \rho\frac{\Delta x}{2}\Delta y\mathrm{C}\frac{T_9^{i+1} - T_9^i}{\Delta t}$$

$$T_9^{i+1} = ((hi\Delta y(T_h - T_9^i) + \mathrm{k}\Delta y\frac{T_8^i - T_9^i}{\Delta x} + \mathrm{k}\frac{\Delta x}{2}\frac{T_6^i - T_9^i}{\Delta y} + \mathrm{k}\frac{\Delta x}{2}\frac{T_{12}^i - T_9^i}{\Delta y})\ \frac{2\Delta t}{\rho\mathrm{C}\Delta x\Delta y}) + T_9^i$$

T[9][i+1] = (((hi*y*(Th-T[9][i]))+(k*y*(T[8][i]-T[9][i])/x)+ (0.5*k*x*(T[6][i]-T[9][i])/y) + (0.5*k*x*(T[12][i]-T[9][i])/y)) *((2*t)/(r*c*x*y))) +T[9][i];

Node 10:

$$ho\Delta y(T_\infty - T_{10}^i) + \mathrm{k}\Delta y\frac{T_{11}^i - T_{10}^i}{\Delta x} + \mathrm{k}\frac{\Delta x}{2}\frac{T_7^i - T_{10}^i}{\Delta y} + \mathrm{k}\frac{\Delta x}{2}\frac{T_{15}^i - T_{10}^i}{\Delta y} = \rho\frac{\Delta x}{2}\Delta y\mathrm{C}\frac{T_{10}^{i+10} - T_{10}^i}{\Delta t}$$

$$T_{10}^{i+10} = ((ho\Delta y(T_\infty - T_{10}^i) + \mathrm{k}\Delta y\frac{T_{11}^i - T_{10}^i}{\Delta x} + \mathrm{k}\frac{\Delta x}{2}\frac{T_7^i - T_{10}^i}{\Delta y} + \mathrm{k}\frac{\Delta x}{2}\frac{T_{15}^i - T_{10}^i}{\Delta y})\ \frac{2\Delta t}{\rho\mathrm{C}\Delta x\Delta y}) + T_{10}^i$$

T[10][i+1] = (((ho*y*(Ta-T[10][i]))+(k*y*(T[11][i]-T[10][i])/x)+(0.5*k*x*(T[7][i]-T[10][i])/y) + (0.5*k*x*(T[15][i]-T[10][i])/y)) *((2*t)/(r*c*x*y))) +T[10][i];

Node 11:

$k\Delta y \frac{T_{10}^i - T_{11}^i}{\Delta x} + k\Delta y \frac{T_{12}^i - T_{11}^i}{\Delta x} + k\Delta x \frac{T_8^i - T_{11}^i}{\Delta y} + k\Delta x \frac{T_{16}^i - T_{11}^i}{\Delta y} = \rho \Delta x \Delta y C \frac{T_{11}^{i+1} - T_{11}^i}{\Delta t}$

$T_{11}^{i+1} = ((k\Delta y \frac{T_{10}^i - T_{11}^i}{\Delta x} + k\Delta y \frac{T_{12}^i - T_{11}^i}{\Delta x} + k\Delta x \frac{T_8^i - T_{11}^i}{\Delta y} + k\Delta x \frac{T_{16}^i - T_{11}^i}{\Delta y}) \frac{\Delta t}{\rho C \Delta x \Delta y}) + T_{11}^i$

T[11][i+1] = (((k*y*(T[10][i]-T[11][i])/x)+ (k*y*(T[12][i]-T[11][i])/x)+ (k*x*(T[8][i]-T[11][i])/y) + (k*x*(T[16][i]-T[11][i])/y)) *((t)/(r*c*x*y))) +T[11][i];

Node 12:

$hi \frac{\Delta x}{2}(T_h - T_{12}^i) + hi \frac{\Delta y}{2}(T_h - T_{12}^i) + k\Delta y \frac{T_{11}^i - T_{12}^i}{\Delta x} + k\frac{\Delta y}{2} \frac{T_{13}^i - T_{12}^i}{\Delta x} + k\frac{\Delta x}{2} \frac{T_9^i - T_{12}^i}{\Delta y} + k\Delta x \frac{T_{17}^i - T_{12}^i}{\Delta y} = \rho \frac{3\Delta x \Delta y}{4} C \frac{T_{12}^{i+1} - T_{12}^i}{\Delta t}$

$T_{12}^{i+1} = ((hi \frac{\Delta x}{2}(T_h - T_{12}^i) + hi \frac{\Delta y}{2}(T_h - T_{12}^i) + k\Delta y \frac{T_{11}^i - T_{12}^i}{\Delta x} + k\frac{\Delta y}{2} \frac{T_{13}^i - T_{12}^i}{\Delta x} + k\frac{\Delta x}{2} \frac{T_9^i - T_{12}^i}{\Delta y} + k\Delta x \frac{T_{17}^i - T_{12}^i}{\Delta y}) \frac{4\Delta t}{3\rho C \Delta x \Delta y}) + T_{12}^i$

T[12][i+1] = (((0.5*hi*x*(Th-T[12][i]))+(0.5*hi*y*(Th-T[12][i])) +(k*y*(T[11][i]-T[12][i])/x)+ (0.5*k*y*(T[13][i]-T[12][i])/x) + (0.5*k*x*(T[9][i]-T[12][i])/y) + (k*x*(T[17][i]-T[12][i])/y)) *((4*t)/(3*r*c*x*y))) +T[12][i];

Node 13:

$hi\Delta x(T_h - T_{13}^i) + k\frac{\Delta y}{2} \frac{T_{12}^i - T_{13}^i}{\Delta x} + k\frac{\Delta y}{2} \frac{T_{14}^i - T_{13}^i}{\Delta x} + k\Delta x \frac{T_{18}^i - T_{13}^i}{\Delta y} = \rho \Delta x \frac{\Delta y}{2} C \frac{T_{13}^{i+1} - T_{13}^i}{\Delta t}$

$T_{13}^{i+1} = ((hi\Delta x(T_h - T_{13}^i) + k\frac{\Delta y}{2} \frac{T_{12}^i - T_{13}^i}{\Delta x} + k\frac{\Delta y}{2} \frac{T_{14}^i - T_{13}^i}{\Delta x} + k\Delta x \frac{T_{18}^i - T_{13}^i}{\Delta y}) \frac{2\Delta t}{\rho C \Delta x \Delta y}) + T_{13}^i$

T[13][i+1] = (((hi*x*(Th-T[13][i]))+(0.5*k*y*(T[12][i]-T[13][i])/x)+ (0.5*k*y*(T[14][i]-T[13][i])/x)+ (k*x*(T[18][i]-T[13][i])/y)) *((2*t)/(r*c*x*y))) +T[13][i];

Node 14:

$hi \frac{\Delta x}{2}(T_h - T_{14}^i) + hi \frac{\Delta y}{2}(T_h - T_{14}^i) + k\frac{\Delta y}{2} \frac{T_{13}^i - T_{14}^i}{\Delta x} + k\frac{\Delta x}{2} \frac{T_{19}^i - T_{14}^i}{\Delta y} = \rho \frac{\Delta x}{2} \frac{\Delta y}{2} C \frac{T_{14}^{i+1} - T_{14}^i}{\Delta t}$

$T_{14}^{i+1} = ((hi \frac{\Delta x}{2}(T_h - T_{14}^i) + hi \frac{\Delta y}{2}(T_h - T_{14}^i) + k\frac{\Delta y}{2} \frac{T_{13}^i - T_{14}^i}{\Delta x} + k\frac{\Delta x}{2} \frac{T_{19}^i - T_{14}^i}{\Delta y}) \frac{4\Delta t}{\rho C \Delta x \Delta y}) + T_{14}^i$

T[14][i+1] = (((0.5*hi*x*(Th-T[14][i]))+(0.5*hi*y*(Th-T[14][i])) +(0.5*k*y*(T[13][i]-T[14][i])/x)+ (0.5*k*x*(T[19][i]-T[14][i])/y)) *((4*t)/(r*c*x*y))) +T[14][i];

Node 15:

$ho\Delta y(T_\infty - T_{15}^i) + k\Delta y \frac{T_{16}^i - T_{15}^i}{\Delta x} + k\frac{\Delta x}{2} \frac{T_{10}^i - T_{15}^i}{\Delta y} + k\frac{\Delta x}{2} \frac{T_{20}^i - T_{15}^i}{\Delta y} = \rho \frac{\Delta x}{2} \Delta y C \frac{T_{15}^{i+15} - T_{15}^i}{\Delta t}$

$T_{15}^{i+15} = ((ho\Delta y(T_\infty - T_{15}^i) + k\Delta y \frac{T_{16}^i - T_{15}^i}{\Delta x} + k\frac{\Delta x}{2} \frac{T_{10}^i - T_{15}^i}{\Delta y} + k\frac{\Delta x}{2} \frac{T_{20}^i - T_{15}^i}{\Delta y}) \frac{2\Delta t}{\rho C \Delta x \Delta y}) + T_{15}^i$

T[15][i+1] = (((ho*y*(Ta-T[15][i]))+(k*y*(T[16][i]-T[15][i])/x)+ (0.5*k*x*(T[10][i]-T[15][i])/y)+(0.5*k*x*(T[20][i]-T[15][i])/y)) *((2*t)/(r*c*x*y))) +T[15][i];

Node 16:

$k\Delta y \frac{T_{15}^i - T_{16}^i}{\Delta x} + k\Delta y \frac{T_{17}^i - T_{16}^i}{\Delta x} + k\Delta x \frac{T_{11}^i - T_{16}^i}{\Delta y} + k\Delta x \frac{T_{21}^i - T_{16}^i}{\Delta y} = \rho \Delta x \Delta y C \frac{T_{16}^{i+1} - T_{16}^i}{\Delta t}$

$T_{16}^{i+1} = ((k\Delta y \frac{T_{15}^i - T_{16}^i}{\Delta x} + k\Delta y \frac{T_{17}^i - T_{16}^i}{\Delta x} + k\Delta x \frac{T_{11}^i - T_{16}^i}{\Delta y} + k\Delta x \frac{T_{21}^i - T_{16}^i}{\Delta y}) \frac{\Delta t}{\rho C \Delta x \Delta y}) + T_{16}^i$

T[16][i+1] = (((k*y*(T[15][i]-T[16][i])/x)+ (k*y*(T[17][i]-T[16][i])/x)+(k*x*(T[11][i]-T[16][i])/y)+(k*x*(T[21][i]-T[16][i])/y)) *((t)/(r*c*x*y))) +T[16][i];

Node 17:

$k\Delta y \frac{T_{16}^i - T_{17}^i}{\Delta x} + k\Delta y \frac{T_{18}^i - T_{17}^i}{\Delta x} + k\Delta x \frac{T_{12}^i - T_{17}^i}{\Delta y} + k\Delta x \frac{T_{22}^i - T_{17}^i}{\Delta y} = \rho \Delta x \Delta y C \frac{T_{17}^{i+1} - T_{17}^i}{\Delta t}$

$T_{17}^{i+1} = ((k\Delta y \frac{T_{16}^i - T_{17}^i}{\Delta x} + k\Delta y \frac{T_{18}^i - T_{17}^i}{\Delta x} + k\Delta x \frac{T_{12}^i - T_{17}^i}{\Delta y} + k\Delta x \frac{T_{22}^i - T_{17}^i}{\Delta y}) \frac{\Delta t}{\rho C \Delta x \Delta y}) + T_{17}^i$

T[17][i+1] = (((k*y*(T[16][i]-T[17][i])/x)+ (k*y*(T[18][i]-T[17][i])/x)+(k*x*(T[12][i]-T[17][i])/y)+(k*x*(T[22][i]-T[17][i])/y)) *((t)/(r*c*x*y))) +T[17][i];

Node 18:

$k\Delta y \frac{T_{17}^i - T_{18}^i}{\Delta x} + k\Delta y \frac{T_{19}^i - T_{18}^i}{\Delta x} + k\Delta x \frac{T_{13}^i - T_{18}^i}{\Delta y} + k\Delta x \frac{T_{23}^i - T_{18}^i}{\Delta y} = \rho \Delta x \Delta y C \frac{T_{18}^{i+1} - T_{18}^i}{\Delta t}$

$T_{18}^{i+1} = ((k\Delta y \frac{T_{17}^i - T_{18}^i}{\Delta x} + k\Delta y \frac{T_{19}^i - T_{18}^i}{\Delta x} + k\Delta x \frac{T_{13}^i - T_{18}^i}{\Delta y} + k\Delta x \frac{T_{23}^i - T_{18}^i}{\Delta y}) \frac{\Delta t}{\rho C \Delta x \Delta y}) + T_{18}^i$

T[18][i+1] = (((k*y*(T[17][i]-T[18][i])/x)+ (k*y*(T[19][i]-T[18][i])/x)+(k*x*(T[13][i]-T[18][i])/y)+(k*x*(T[23][i]-T[18][i])/y)) *((t)/(r*c*x*y))) +T[18][i];

Node 19:

$hi\Delta y(T_h - T_{19}^i) + \mathrm{k}\Delta y \frac{T_{18}^i - T_{19}^i}{\Delta x} + \mathrm{k}\frac{\Delta x}{2}\frac{T_{14}^i - T_{19}^i}{\Delta y} + \mathrm{k}\frac{\Delta x}{2}\frac{T_{24}^i - T_{19}^i}{\Delta y} = \rho\,\frac{\Delta x}{2}\Delta y \mathrm{C}\frac{T_{19}^{i+1} - T_{19}^i}{\Delta t}$

$T_{19}^{i+1} = ((hi\Delta y(T_h - T_{19}^i) + \mathrm{k}\Delta y \frac{T_{18}^i - T_{19}^i}{\Delta x} + \mathrm{k}\frac{\Delta x}{2}\frac{T_{14}^i - T_{19}^i}{\Delta y} + \mathrm{k}\frac{\Delta x}{2}\frac{T_{24}^i - T_{19}^i}{\Delta y})\ \frac{2\Delta t}{\rho \mathrm{C}\Delta x\Delta y}) + T_{19}^i$

T[19][i+1] = (((hi*y*(Th-T[19][i]))+(k*y*(T[18][i]-T[19][i])/x)+(0.5*k*x*(T[14][i]-T[19][i])/y)+(0.5*k*x*(T[24][i]-T[19][i])/y)) *((2*t)/(r*c*x*y))) +T[19][i];

Node 20:

$ho\,\frac{\Delta x}{2}(T_\infty - T_{20}^i) + \ ho\,\frac{\Delta y}{2}(T_\infty - T_{20}^i) + \mathrm{k}\frac{\Delta y}{2}\frac{T_{21}^i - T_{20}^i}{\Delta x} + \mathrm{k}\frac{\Delta x}{2}\frac{T_{15}^i - T_{20}^i}{\Delta y} = \rho\,\frac{\Delta x}{2}\frac{\Delta y}{2}\mathrm{C}\frac{T_{20}^{i+20} - T_{20}^i}{\Delta t}$

$T_{20}^{i+20} = ((ho\,\frac{\Delta x}{2}(T_\infty - T_{20}^i) + \ ho\,\frac{\Delta y}{2}(T_\infty - T_{20}^i) + \mathrm{k}\frac{\Delta y}{2}\frac{T_{21}^i - T_{20}^i}{\Delta x} + \mathrm{k}\frac{\Delta x}{2}\frac{T_{15}^i - T_{20}^i}{\Delta y})\ \frac{4\Delta t}{\rho \mathrm{C}\Delta x\Delta y}) + T_{20}^i$

T[20][i+1] = (((0.5*ho*x*(Ta-T[20][i]))+(0.5*ho*y*(Ta-T[20][i]))+(0.5*k*y*(T[21][i]-T[20][i])/x)+ (0.5*k*x*(T[15][i]-T[20][i])/y))*((4*t)/(r*c*x*y))) +T[20][i];

Node 21:

$ho\Delta x(T_\infty - T_{21}^i) + \mathrm{k}\frac{\Delta y}{2}\frac{T_{20}^i - T_{21}^i}{\Delta x} + \mathrm{k}\frac{\Delta y}{2}\frac{T_{22}^i - T_{21}^i}{\Delta x} + \mathrm{k}\Delta x\,\frac{T_{16}^i - T_{21}^i}{\Delta y} = \rho\Delta x\,\frac{\Delta y}{2}\mathrm{C}\frac{T_{21}^{i+1} - T_{21}^i}{\Delta t}$

$T_{21}^{i+1} = ((ho\Delta x(T_\infty - T_{21}^i) + \mathrm{k}\frac{\Delta y}{2}\frac{T_{20}^i - T_{21}^i}{\Delta x} + \mathrm{k}\frac{\Delta y}{2}\frac{T_{22}^i - T_{21}^i}{\Delta x} + \mathrm{k}\Delta x\,\frac{T_{16}^i - T_{21}^i}{\Delta y})\ \frac{2\Delta t}{\rho \mathrm{C}\Delta x\Delta y}) + T_{21}^i$

T[21][i+1] = (((0.5*ho*x*(Ta-T[21][i]))+(0.5*k*y*(T[20][i]-T[21][i])/x)+(0.5*k*y*(T[22][i]-T[21][i])/x)+ (k*x*(T[16][i]-T[21][i])/y))*((2*t)/(r*c*x*y))) +T[21][i];

Node 22:

$ho\Delta x(T_\infty - T_{22}^i) + \mathrm{k}\frac{\Delta y}{2}\frac{T_{21}^i - T_{22}^i}{\Delta x} + \mathrm{k}\frac{\Delta y}{2}\frac{T_{23}^i - T_{22}^i}{\Delta x} + \mathrm{k}\Delta x\,\frac{T_{17}^i - T_{22}^i}{\Delta y} = \rho\Delta x\,\frac{\Delta y}{2}\mathrm{C}\frac{T_{22}^{i+1} - T_{22}^i}{\Delta t}$

$T_{22}^{i+1} = ((ho\Delta x(T_\infty - T_{22}^i) + \mathrm{k}\frac{\Delta y}{2}\frac{T_{21}^i - T_{22}^i}{\Delta x} + \mathrm{k}\frac{\Delta y}{2}\frac{T_{23}^i - T_{22}^i}{\Delta x} + \mathrm{k}\Delta x\,\frac{T_{17}^i - T_{22}^i}{\Delta y})\ \frac{2\Delta t}{\rho \mathrm{C}\Delta x\Delta y}) + T_{22}^i$

T[22][i+1] = (((ho*x*(Ta-T[22][i]))+(0.5*k*y*(T[21][i]-T[22][i])/x)+(0.5*k*y*(T[23][i]-T[22][i])/x)+ (k*x*(T[17][i]-T[22][i])/y))*((2*t)/(r*c*x*y))) +T[22][i];

Node 23:

$ho\Delta x(T_\infty - T_{23}^i) + \mathrm{k}\frac{\Delta y}{2}\frac{T_{22}^i - T_{23}^i}{\Delta x} + \mathrm{k}\frac{\Delta y}{2}\frac{T_{24}^i - T_{23}^i}{\Delta x} + \mathrm{k}\Delta x\,\frac{T_{18}^i - T_{23}^i}{\Delta y} = \rho\Delta x\,\frac{\Delta y}{2}\mathrm{C}\frac{T_{23}^{i+1} - T_{23}^i}{\Delta t}$

$T_{23}^{i+1} = ((ho\Delta x(T_\infty - T_{23}^i) + \mathrm{k}\frac{\Delta y}{2}\frac{T_{22}^i - T_{23}^i}{\Delta x} + \mathrm{k}\frac{\Delta y}{2}\frac{T_{24}^i - T_{23}^i}{\Delta x} + \mathrm{k}\Delta x\,\frac{T_{18}^i - T_{23}^i}{\Delta y})\ \frac{2\Delta t}{\rho \mathrm{C}\Delta x\Delta y}) + T_{23}^i$

T[23][i+1] = (((ho*x*(Ta-T[23][i]))+(0.5*k*y*(T[22][i]-T[23][i])/x)+(0.5*k*y*(T[24][i]-T[23][i])/x)+ (k*x*(T[18][i]-T[23][i])/y))*((2*t)/(r*c*x*y))) +T[23][i];

Node 24:

$ho\,\frac{\Delta x}{2}(T_\infty - T_{24}^i) + \ hi\,\frac{\Delta y}{2}(T_h - T_{24}^i) + \mathrm{k}\frac{\Delta y}{2}\frac{T_{23}^i - T_{24}^i}{\Delta x} + \mathrm{k}\frac{\Delta x}{2}\frac{T_{19}^i - T_{24}^i}{\Delta y} = \rho\,\frac{\Delta x}{2}\frac{\Delta y}{2}\mathrm{C}\frac{T_{24}^{i+1} - T_{24}^i}{\Delta t}$

$T_{24}^{i+1} = ((ho\,\frac{\Delta x}{2}(T_\infty - T_{24}^i) + \ hi\,\frac{\Delta y}{2}(T_h - T_{24}^i) + \mathrm{k}\frac{\Delta y}{2}\frac{T_{23}^i - T_{24}^i}{\Delta x} + \mathrm{k}\frac{\Delta x}{2}\frac{T_{19}^i - T_{24}^i}{\Delta y})\ \frac{4\Delta t}{\rho \mathrm{C}\Delta x\Delta y}) + T_{24}^i$

T[24][i+1] = (((0.5*ho*x*(Ta-T[24][i]))+(0.5*hi*y*(Th-T[24][i])) +(0.5*k*y*(T[23][i]-T[24][i])/x)+ (0.5*k*x*(T[19][i]-T[24][i])/y))*((4*t)/(r*c*x*y))) +T[24][i];

III. PROGRAMMING & SOLUTION

With the help of a computer program we can solve the matrix created by finite difference equations for 24 nodes. We can calculate temperature distribution and stress distribution with respect to time.

Table 1
Material Property And Boundary Conditions

Material Properties and Boundary Conditions for Magnesia Ramming Mass			Unit
1	Internal Film Co-efficient hi	200	W/m^2 K
2	External Film Co-efficient ho	40	W/m^2 K
3	Atmosphere Film Co-efficient ha	10	W/m^2 K
4	Density	3300	Kg/m^3
5	Time Interval Δt	10	Seconds
6	Thermal Conductivity k	4	W/m K
7	Temperature outside Furnace Wall	303	Kelvin
8	Temperature inside Furnace Wall	1773	Kelvin
9	Temperature of Air	303	Kelvin
10	Specific Heat	880	J/kg K
11	Elasticity Constant	180000	N/ m^2
12	Thermal Expansion Co-efficient	0.00000 138	m/ K
13	Ultimate Stress	600	MPa

IV. RESULTS AND DISCUSSION

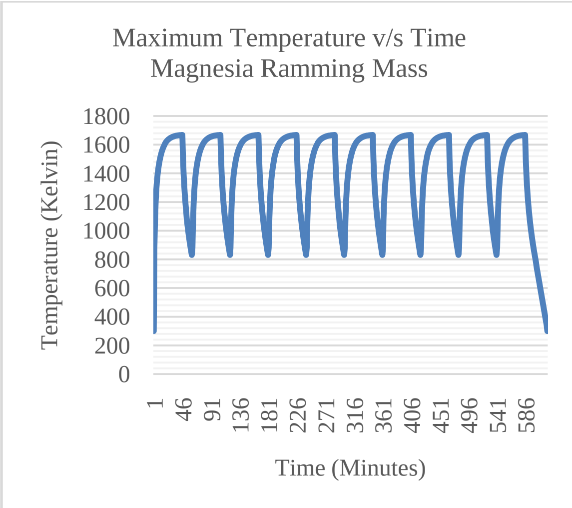

Fig. 2.Maximum Temperature v/s Time Graph for Magnesia Ramming Mass

We can see from the Fig. 2 that maximum temperature is increasing from atmospheric temperature 300 K and reaches to maximum temperature 1667 K in 45 minutes and then starts reducing and reaches to 831 K in next 15 minutes. It again starts increasing and reaches to maximum 1667 K after 105 minutes and again starts reducing. There are 10 similar temperature cycles in one day.

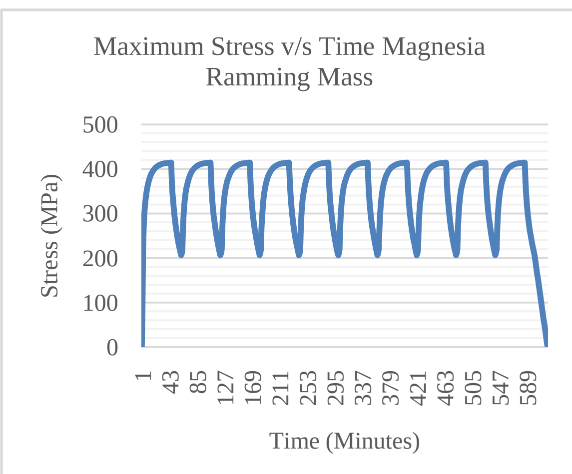

Fig. 3.Maximum Stress v/s Time Graph for Magnesia Ramming Mass

We can see from the Fig. 3 that maximum thermal stress is increasing from initial condition 0 MPa and reaches to maximum stress 414 MPa in 45 minutes and then starts reducing and reaches to 206 MPa in next 15 minutes. It again starts increasing and reaches to maximum stress 414 MPa after 105 minutes and again it starts reducing. There are 10 similar thermal stress cycles in one day.

V. CONCLUSION

Induction melting furnaces are highly used now-a-days for melting of different kinds of materials. We have found variation of maximum temperature and maximum thermal stress with reference to time. From the graph, we can conclude that induction furnace wall which is made from magnesia ramming mass is under the effect of low cycle thermal fatigue. The reason for its low life span is thermal fatigue behavior of its loading conditions.

VI. FUTURE SCOPE

This analysis can be utilized for prediction of life cycle of induction furnace wall which is made up of magnesia based refractory materials. It can also be utilized to improve efficacy of furnace and optimization of wall thickness.

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