Navigational Error Control by Model based Filtering and Smoothing for GPS/INS Integration

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Abstract

Nowadays Global Positioning System (GPS) is the widely used system for navigational aids and tracking of vehicles. However, GPS will not offer uninterrupted and consistent position values i.e. Latitude, Longitude and Altitude all the times as it likely to be blocked by buildings, mountains, etc. Inertial Navigation Systems (INS) provides continues information of position, velocity and attitude all the time. However, the performance of INS deteriorates with time due to the perfromance degradation of inertial sensors. GPS/INS integration provides reliable navigation solution. Existing GPS/INS integration using Kalman Filter (KF) can give correct results only when the system dynamic models are completely known. To estimate the state of vehicle, Extended Kalman Filter (EKF) is used. Since EKF provides the inaccurate navigation during the non linear motion of the vehicle, an Unscented Kalman Filter (UKF) has been employed. Interacting Multiple Model (IMM) filter is more efficient than the conventional single model filter in determining the adequate values of process noise co-variance. In this paper, the application of Interacting Multiple Model Unscented Kalman Filter Two Filter Smoothing (IMM-UKFTFS) approach to GPS/INS integration for the maneuvering vehicle is proposed. The resulting IMM-UKFTFS strategy effectively deals with the non-linear motion and noise covariance problem of navigation. The performance of the proposed IMM-UKFTFS method is examined for a non-linear trajectory which consist of Constant Acceleration (CA) and Coordinated Turn (CT) models. The simulation results show that the proposed IMM-UKFTFS gives better estimate than the existing conventional estimators such as UKF and IMM-UKF.

Keywords: GPS; INS; EKF; UKF; IMM-EKF; IMM-UKF; IMM-UKFTFS; TFS

I. INTRODUCTION

Navigation is the estimation of the state of the moving vehicle. An estimator is an algorithm that uses available measurements to refine the state variables. The GPS measurements are obtained by placing the GPS sensors on the moving vehicle [8]. Estimation techniques, especially the Kalman filtering have found extensive application in navigation systems because they can significantly improve their accuracy. But the KF is a linear estimator [1], [3], [5], it does not give better estimation in non linear motion of the vehicle. So the EKF is introduced to give accurate results for non linear motion model. The EKF linearizes the dynamic state model using Taylor series [8]. An Interacting Multiple Model (IMM) algorithm uses the collection of filters, and each filter is assigned with particular motion model of the moving vehicle [7]. The estimation accuracy of IMM is better than the single model filter like EKF. There are some flaws in EKF, we can use EKF in non linear case; however, it requires linearization of the state model and the measurement model. This leads to poor performance of EKF. In order to overcome the problems in EKF we are going for transformation based statistical approach of Unscented Kalman Filter (UKF). The UKF can be used in IMM algorithm. The estimate of IMM-UKF is better than that of the single model UKF [12] and IMM-EKF.

Smoothing is the post processing technique which resolves the estimation problem. Smoothing is better than filtering by using the additional measurements made after the time of the estimated state vector [14]. Smoothing can be incorporated to the IMM algorithm to achieve better navigation accuracy. There are three types of smoothing, namely Fixed Point Smoothing, Fixed Lag Smoothing and Fixed Interval Smoothing. Due to less complexity, the Fixed Interval Smoothing is popular. The Fixed Interval Smoothing has two types. They are Rauch Tung Striebel Smoother (RTSS) and Two Filter Smoother (TFS) [10]. RTSS is confirmed to work well only for the linear system. TFS gives better results than RTSS during highly non linear behavior of vehicle motion and also the computational time of both the methods are proved to be the same. Hence TFS is chosen which will give reliable result in non linear system and incorporating the TFS into the IMM algorithm for GPS/INS navigation. The paper is organized as follows. Section 2 deals with different model based approach, section 3 deals with the proposed IMM-UKFTFS method, section 4 gives the idea of modeling parameters of GPS/INS navigation, section 5 deals with the concept of GPS/INS integration, section 6 deals with the analysis of various estimation methods.

II. MODEL BASED APPROACH

A. Estimation Methods

A vital problem for GPS/INS navigation is selecting a suitable estimation method. Several approaches have been recognized. They are, Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF), IMM based EKF, and IMM based UKF. However, they suffer from divergence problem i.e., estimation does not converge to the real trajectory.

B. Unscented Kalman Filter (UKF)

In the EKF, the state distribution is propagated analytically through the first-order linearization of the nonlinear system due to which, the posterior mean and covariance could be corrupted. The UKF, which is a derivative-free alternative to the EKF, overcomes this problem by using a deterministic sampling approach. The state distribution is represented using a minimal set of carefully chosen sample points, called sigma points. Like EKF, UKF consists of the same two steps: model forecast and data assimilation, except they are preceded now by another step for the selection of sigma points. The UKF is founded on the intuition that it is easier to approximate a probability distribution than it is to approximate an arbitrary nonlinear function or transformation. The sigma points are chosen so that their mean and covariance to be exactly x(k-1) and P(k-1) respectively. Each sigma point is then propagated through the nonlinear functions yielding a cloud of transformed points at the end. The new estimated mean and covariance are then computed based on their statistics. This process is called unscented transformation [2].

The n-dimensional random variable with mean $\bar{x}$ and covariance P_{xx} is approximated by 2n+1 weighted point by

$$
\left.
\begin{aligned}
&x_0 = \bar{x} \\
&x_i = \bar{x} + \left(\sqrt{(n+k)p_{xx}}\right)_i \quad , i = 1, \dots, n \\
&x_i = \bar{x} - \left(\sqrt{(n+k)p_{xx}}\right)_{i-n}, i = n+1, \dots, 2n \\
&\text{Weights for state } w_s^0 = \frac{k}{n+k} \text{ and weights for covariance } w_c^0 = \frac{k}{n+k} + (1 - \alpha^2 + \beta) \\
&w_s^i = w_c^i = \frac{1}{2(n+k)}
\end{aligned}
\right\}
\tag{1}
$$

In equation (1) the term x denotes sigma point, term w denotes weight, term k is a constant, the term α denotes the spread of sigma points and β is related to distribution of x.

The sigma points are passed through the non-linear function to yield the set of transformed sigma points.

$$\varsigma_i = f(x_i)$$

The mean and covariance are given by the weighted average and the weighted outer product of the transformed points

$$\bar{y} = \sum_{i=0}^{2n} w_s^i \varsigma_i \tag{2}$$

$$P_{yy} = \sum_{i=0}^{2n} w_c^i (\varsigma_i - \bar{y})(\varsigma_i - \bar{y})^T \tag{3}$$

The time prediction and the measurement update is done using this mean and covariance.

C. IMM-UKF

Since the motion model of the vehicle has been changed frequently, more than one estimator is to be considered to meet the changing environment, so IMM has been introduced. As the name implies here, multiple models can be utilized simultaneously [6], [7], [11]. To reduce the noise uncertainty and the system non linearity problem concurrently, the IMM-UKF is introduced [12]. In the IMM-UKF, two UKF filters are used [7]. By using the model probability, the IMM algorithm weights the output of the individual filters. The final combined estimate of IMM-UKF is better than the estimate of UKF which uses the single model. The accuracy of the IMM-UKF is higher than that of the IMM-EKF.

D. Two Filter Smoother (TFS)

Two Filter Smoother (TFS) consists of two filters. They are the forward filter running forward in time and a backward filter running backward in time. At each instant of time, the estimate from the forward filter is based on all the measurements made up to that time and the estimate from the backward filter is based on the measurements made after that time. At each instant of time, the associated estimation uncertainty covariance characterizes the estimation uncertainty based on all these measurements. When the time is reversed, the sign on the dynamic coefficient matrix A is changed, which can make performance of the backward filter model different from that of forward filter model. At each time t, the forward filter generates the covariance matrix $P_{[f]}(t)$ representing the mean-squared uncertainty in the estimate $\hat{x}_{[f]}(t)$ using all measurements z(s) for s≤t . Similarly the backward filter generates the covariance matrix $P_{[b]}(t)$ matrix representing the mean-squared uncertainty in the estimate $\hat{x}_{[b]}(t)$ using all measurements z(s) for s≥t . The optimal smoother combines $\hat{x}_{[f]}(t)$ and $\hat{x}_{[b]}(t)$, using $P_{[f]}(t)$ and $P_{[b]}(t)$ to minimize the resulting covariance matrix $P_{[s]}(t)$ as shown in Figure 1. [9], [10], [4].

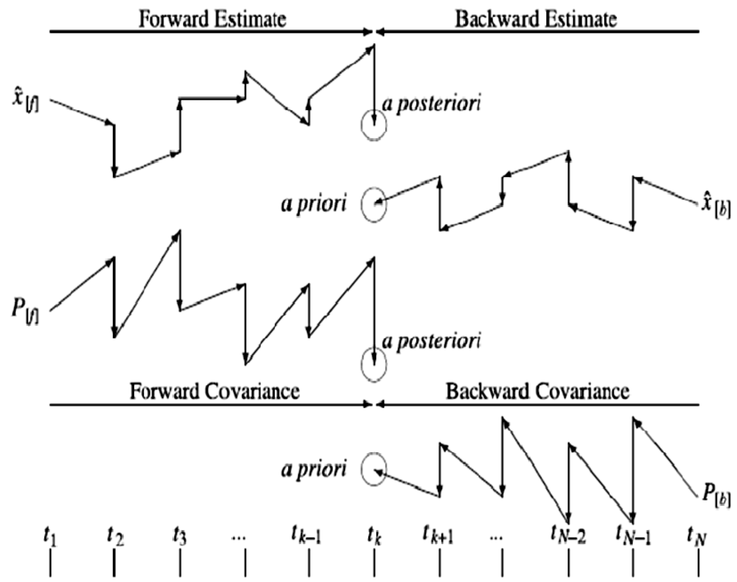

Figure 1. TFS based filter estimates

III. PROPOSED METHOD (IMM-UKFTFS)

To improve the positioning accuracy, IMM based fixed interval Two Filter Smoothing is introduced in this paper. The proposed method incorporates smoothing algorithm into the Interacting Multiple Model approach as shown in Figure 2. Two IMM filters are running parallel and they are assigned with particular model. For two filter smoothing [14], two estimates at time t, one based on forward filtering and other based on backward filtering are considered. The idea is to obtain a smoothed improved estimate by fusion of these two estimates $\hat{x}_{[f]}(t)$ and $\hat{x}_{[b]}(t)$, and its associated co-variances $P_{[f]}(t)$ and $P_{[b]}(t)$ [9]. The IMM-UKFTFS uses the estimation and model probabilities of the forward Unscented Kalman Filter and the backward Unscented Kalman Filter. The filters use its own time update and measurement update equations for filtering as given below.

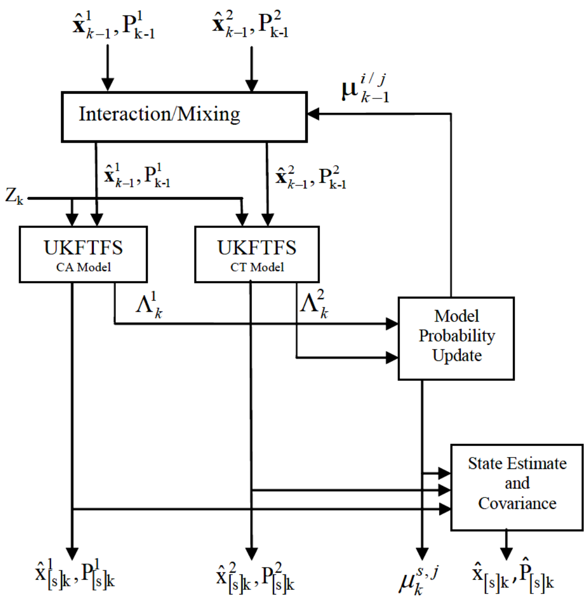

Figure 2. Block Diagram of IMM-UKFTFS

The IMM smoothed estimate $\hat{x}^i_{k,s}$ of x^i_k is estimated as a linear combination of two IMM filters $\hat{x}^i_{k,f}$ and $\hat{x}^i_{k,b}$. Let $\tilde{x}^i_{k,s}$ be the IMM smoothed estimate error given by

$$\tilde{x}^i_{k,s} = \hat{x}^i_{k,s} - x^i_k \qquad (4)$$

The IMM smoothed estimate is given by

$$\hat{x}^i_{k,s} = k_1 \hat{x}^i_{k,f} + k_2 . \hat{x}^i_{k,b} \qquad (5)$$

From the equation (4) we can write

$$x^i_k + \tilde{x}^i_{k,s} = k_1(x^i_k + \tilde{x}^i_{k,f}) + k_2(x^i_k + \tilde{x}^i_{k,b}) \qquad (6)$$

where $\tilde{x}^i_{k,f}, \tilde{x}^i_{k,b}$ are the estimated errors of the IMM forward and backward filters.

$$\tilde{x}^i_{k,s} = (k_1 + k_2 - I)x^i_k + k_1 \tilde{x}^i_{k,f} + k_2 \tilde{x}^i_{k,b} \qquad (7)$$

To make our estimate unbiased, $E(\tilde{x}^i_{k,s}) = 0$

$$k_1 + k_2 - I = 0 \qquad (8)$$

$$k_2 = I - k_1 \qquad (9)$$

Substituting k_2 in the equation (5), we obtain the smoothed estimate.

$$\hat{x}^i_{k,s} = k_1 . \hat{x}^i_{k,f} + (I - k_1)\hat{x}^i_{k,b} \qquad (10)$$

Also it can be written as

$$\hat{x}^i_{k,s} = \hat{x}^i_{k,b} + k_1(\hat{x}^i_{k,f} - \hat{x}^i_{k,b}) \qquad (11)$$

The error covariance matrix of the smoother estimate is obtained by

$$\tilde{x}^i_{k,s} = k_1 \tilde{x}^i_{k,f} + k_2 \tilde{x}^i_{k,b} = k_1 \tilde{x}^i_{k,f} + (I - k_1)\tilde{x}^i_{k,b} \qquad (12)$$

$$P^i_{k,s} = k_1 P^i_{k,f} . k_1^T + (I - k_1) P^i_{k,b} (I - k_1)^T \qquad (13)$$

By minimizing the equation (13) for gain K_1

$$2k_1 P^i_{k,f} - 2(I - k_1) P^i_{k,b} = 0 \qquad (14)$$

$$k_1 = P^i_{k,b} (P^i_{k,f} + P^i_{k,b})^{-1} \qquad (15)$$

$$I - k_1 = I - P^i_{k,b} (P^i_{k,f} + P^i_{k,b})^{-1} \qquad (16)$$

$$I - k_1 = P^i_{k,f} (P^i_{k,f} + P^i_{k,b})^{-1} \qquad (17)$$

Substituting the equations (15) and (17) in the equation (13), we will get

$$P^{i\ -1}_{k,s} = P^{i\ -1}_{k,f} + P^{i\ -1}_{k,b} \qquad (18)$$

The equation for smoothed covariance of forward and backward IMM filter can be written as

$$P^i_{k,s} = [P^{i\ -1}_{k,f} + P^{i\ -1}_{k,b}]^{-1} \qquad (19)$$

The equation (19) proves that the smoothed uncertainty covariance is less than the uncertainty covariance of both forward and backward IMM-UKF. The equation for the IMM smoothed estimate is obtained by substituting (15) and (17) in equation (10)

$$\hat{x}^i_{k,s} = P^i_{k,b} (P^i_{k,f} + P^i_{k,b})^{-1} . \hat{x}^i_{k,f} + P^i_{k,f} . (P^i_{k,f} + P^i_{k,b})^{-1} . \hat{x}^i_{k,b}$$

(20)

$$\hat{x}^i_{k,s} = \frac{P^i_{k,b} . P^i_{k,f}}{(P^i_{k,f} + P^i_{k,b})} [(P^i_{k,f})^{-1} . \hat{x}^i_{k,f} + (P^i_{k,b})^{-1} . \hat{x}^i_{k,b}] \qquad (21)$$

The IMM smoothed estimate is obtained by using the forward and backward uncertainty covariance which is given by,

$$\hat{x}^i_{k,s} = P^i_{k,s} [(P^i_{k,f})^{-1} . \hat{x}^i_{k,f} + (P^i_{k,b})^{-1} . \hat{x}^i_{k,b}] \qquad (22)$$

The model conditioned smoothed estimate $x^j_{k,s}$ and its uncertainty covariance $P^{s,j}_k$ is given by the equation

$$x^j_{k,s} = \sum_{i=1}^{n} \mu^{s,i/j}_{k+1} \hat{x}^i_{k,s} \qquad (23)$$

$$P^j_{k,s} = \sum_{i=1}^{n} \mu^{s,i/j}_{k+1} . \left[P^i_{k,s} + \left(\hat{x}^i_{k,s} - x^j_{k,s} \right) \left(\hat{x}^i_{k,s} - x^j_{k,s} \right)^T \right]$$

(24)

The mixed smoothed probability is calculated by,

$$\mu^{s,i/j}_{k+1} = \frac{1}{r_j} p_{ji} \Lambda^{ji}_k \qquad (25)$$

The term r_j is computed by,

$$r_j = \sum_{1}^{n} p_{ji} \Lambda_k^{ji} \qquad (26)$$

The likelihood Λ_k^{ji} is given by,

$$\Lambda_k^{ji} = N(\Delta_k^{ji}, D_k^{ji}) \qquad (27)$$

Where N( ) is the probability function of measurement in innovation distribution,

$\Delta_k^{ji} = \hat{x}_k^{b,i} - x_k^{f,i}$ is equivalent to measurement innovation.

$D_k^{ji} = \hat{P}_k^{b,i} + P_k^{f,i}$ is combined covariance of forward and backward filter,

Where $\hat{x}_k^{b,i}$, $\hat{P}_k^{b,i}$ are model conditioned backward-time one-step predicted mean and co-variances,

$x_k^{f,i}$, $P_k^{f,i}$ are model-conditioned forward-time filtered means and co-variances,

The overall optimal smoothed estimate and its uncertainty covariance is given by,

$$\hat{x}_k^s = \sum_{i=1}^{n} \mu_k^{s,j} x_{k,s}^j \qquad (28)$$

$$\hat{P}_k^s = \sum_{i=1}^{n} \mu_k^{s,j} . \left[P_{k,s}^j + \left(x_{k,s}^j - \hat{x}_k^s \right)\left(x_{k,s}^j - \hat{x}_k^s \right)^T \right]$$

(29)

The smoothed model probabilities are computed as

$$\mu_k^{s,j} = \frac{1}{r} r_j \mu_k^j \qquad (30)$$

where μ_k^j is the forward time filtered model probability and r is the normalization constant given by,

$$r = \sum_{j=1}^{n} r_j \mu_k^j \qquad (31)$$

IV. MODELING PARAMETERS FOR GPS/INS NAVIGATION

To estimate the position of the moving vehicle using IMM-UKFTFS, two motion models are considered. They are Coordinated Turn or Circular Turn (CT) model and Constant Acceleration (CA) model [13]. The system used in this work includes four sensors and one moving vehicle. Here 2 dimensional Cartesian coordinate system is considered in which the positive ‘x’ and positive ‘y’ axes correspond to East and North of navigation axis respectively.

A. Constant Acceleration (CA) Model

The dynamic model for constant acceleration model can be represented as,

$$\delta x(k) = F\delta x(k-1) + Gv(k-1) \quad (32)$$

The constant acceleration model has sixth order state vector and consists of position error, velocity error and acceleration error of the vehicle in x and y directions and turn rate parameter. The error state vector is given by,

$$\delta x(k) = [\ \delta x_e(k) \ \ \delta y_n(k) \ \ \delta\dot{x}_e(k) \ \ \delta\dot{y}_n(k) \ \ \delta\ddot{x}_e(k) \ \ \delta\ddot{y}_n(k)\ ]$$

(33)

In equation (33) $\delta x_e(k)$ and $\delta y_n(k)$ are the position error parameters in east and north direction, $\delta\dot{x}_e(k)$ and $\delta\dot{y}_n(k)$ are the velocity error parameters in east and north direction, and $\delta\ddot{x}_e(k)$ and $\delta\ddot{y}_n(k)$ are the acceleration error parameters in east and north direction. The error space representation of the constant acceleration model is given by,

$$\begin{bmatrix} \delta x_e(k) \\ \delta y_n(k) \\ \delta\dot{x}_e(k) \\ \delta\dot{y}_n(k) \\ \delta\ddot{x}_e(k) \\ \delta\ddot{y}_n(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 & T & 0 & T^2/2 & 0 \\ 0 & 1 & 0 & T & 0 & T^2/2 \\ 0 & 0 & 1 & 0 & T & 0 \\ 0 & 0 & 0 & 1 & 0 & T \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta x_e(k-1) \\ \delta y_n(k-1) \\ \delta\dot{x}_e(k-1) \\ \delta\dot{y}_n(k-1) \\ \delta\ddot{x}_e(k-1) \\ \delta\ddot{y}_n(k-1) \end{bmatrix} + \begin{bmatrix} T^3/6 & 0 \\ T^2/2 & 0 \\ T & 0 \\ 0 & T^3/6 \\ 0 & T^2/2 \\ 0 & T \end{bmatrix} w(k-1) \qquad (34)$$

The observation matrix H can be represented by,

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(35)

The model transition probability matrix is set to,

$$P_{ji} = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix} \qquad (36)$$

The initial model probabilities are set to,

$$\mu_0 = [0.95 \ \ 0.05] \qquad (37)$$

B. Coordinated Turn (CT) Model

A common way of modelling a turning vehicle is to use the coordinated turn model. In this model, a turn rate parameter Ω is included in the state vector. The dynamic model for coordinated turn model is as follows,

$$\delta x(k) = F\delta x(k-1) + Gv(k-1) \qquad (38)$$

The coordinated turn model has fifth order state vector and consists of position error and velocity error of the vehicle in x and y directions and turn rate parameter. The error state vector is given by,

$$\delta x(k) = [\ \delta x_e(k) \ \ \delta y_n(k) \ \ \delta\dot{x}_e(k) \ \ \delta\dot{y}_n(k) \ \ \delta\Omega(k)\ ]$$

(39)

The state space representation of the coordinated turn model is given by,

$$\begin{bmatrix}\delta x_e(k)\\ \delta y_n(k)\\ \delta\dot{x}_e(k)\\ \delta\dot{y}_n(k)\\ \delta\Omega(k)\end{bmatrix} = \begin{bmatrix}1 & 0 & \frac{\sin\Omega T}{\Omega} & -\frac{1-\cos\Omega T}{\Omega} & 0\\ 0 & 0 & \cos\Omega T & \sin\Omega T & 0\\ 0 & 1 & \frac{1-\cos\Omega T}{\Omega} & \frac{\sin\Omega T}{\Omega} & 0\\ 0 & 0 & \sin\Omega T & \cos\Omega T & 0\\ 0 & 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\delta x_e(k-1)\\ \delta y_n(k-1)\\ \delta\dot{x}_e(k-1)\\ \delta\dot{y}_n(k-1)\\ \delta\Omega(k-1)\end{bmatrix} + \begin{bmatrix}{}^1/_2 T^2 & 0\\ T & 0\\ 0 & {}^1/_2 T^2\\ 0 & T\\ 0 & 0\end{bmatrix} w(k-1) \quad (40)$$

The observation matrix H for CT model can be represented by,

$$H = \begin{bmatrix}1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\end{bmatrix} \quad (41)$$

The model transition probability matrix is set to,

$$P_{ji} = \begin{bmatrix}0.9 & 0.1\\ 0.1 & 0.9\end{bmatrix} \quad (42)$$

The initial model probabilities are set to,

$$\mu_0 = [0.95 \quad 0.05] \quad (43)$$

V. GPS/INS INTEGRATION USING IMM-UKFTFS

The GPS/INS integration can be done by comparing the output of GPS with that of INS as shown in Figure 3. The difference (δZ_k) between the output of INS and the output of GPS (Z_k) is given to the input of the IMM-UKFTFS. The smoothed estimate of the vehicle position is feedback to the INS to get the corrected INS output.

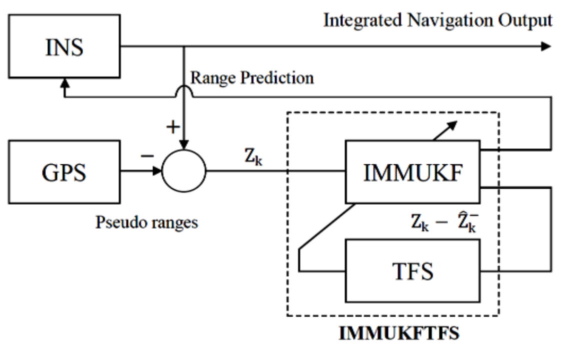

Figure 3. Configuration of Integrated Navigation using IMM-UKFTFS

The INS measurements are created with respect to the GPS measurements by adding error. The GPS measurements are considered as most trusted measurements and are created around the true trajectory as shown in Figure 4. The difference between GPS and INS measurements are taken and are considered as an error in measurement. The error measurement δZ_k is given as an input to CA-CT error modeled IMM-UKFTFS. The error estimated value is subtracted from the INS value and the INS value which resembles the GPS value is obtained. [7, 8]

The trajectory of the error between the GPS and INS values are simulated for 200 time steps with step size T=0.1.The movement of vehicle is given in Table 2. The vehicle starts from origin with acceleration $(\ddot{x}, \ddot{y})$= (1, 0), and at 21s, vehicle starts to turn left with rate Ω = 1, and at 80s, vehicle stops turning and moves for 60 seconds with a constant total acceleration of one, and at 121s, vehicle starts to turn left with rate Ω = 1 and at 180s, vehicle stops turning and moves for 20 seconds with the same acceleration. The error between the GPS position (true trajectory) and the INS measurement is shown in Figure 5. The corrected INS using IMM-UKFTFS is depicted in Figure 6. Figures 7 and 8 show the vehicle error position in north and east direction. Figures 9 and 10 show the model probability of CA and CT models in IMM-UKFTFS.

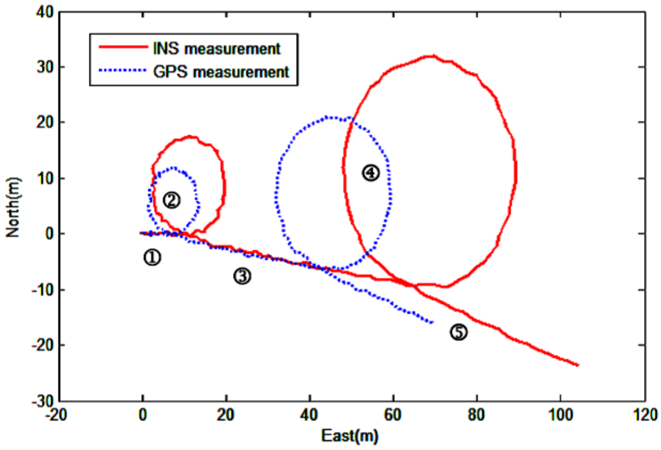

Figure 4. GPS (True trajectory) and INS Measurement of Simulated Vehicle

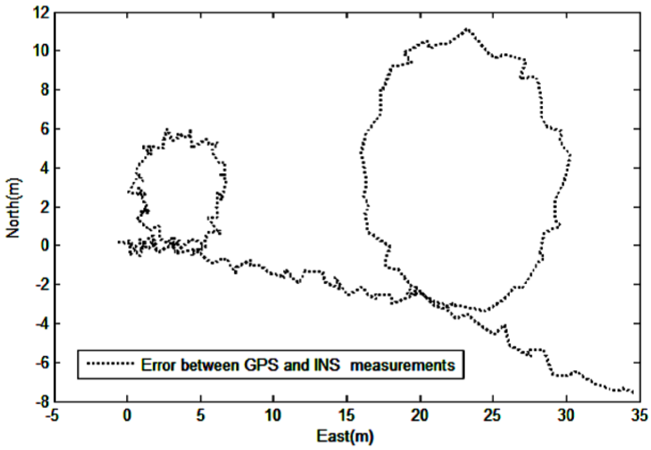

Figure 5. Error Between GPS and INS Measurement

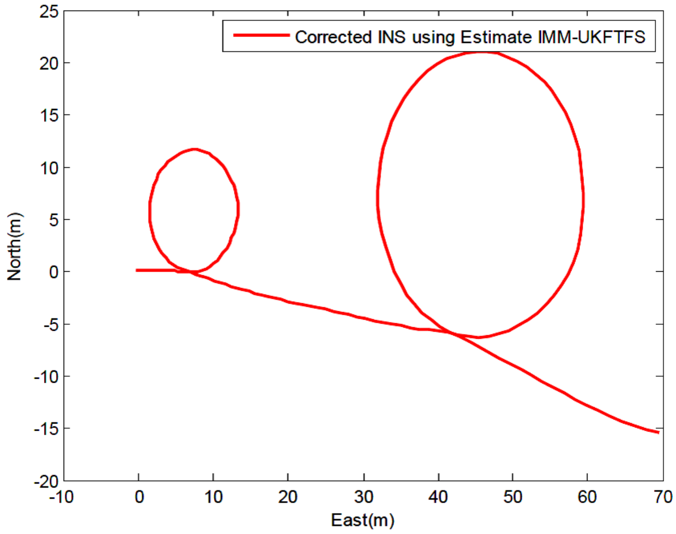

Figure 6. Corrected INS using IMMUKFTFS

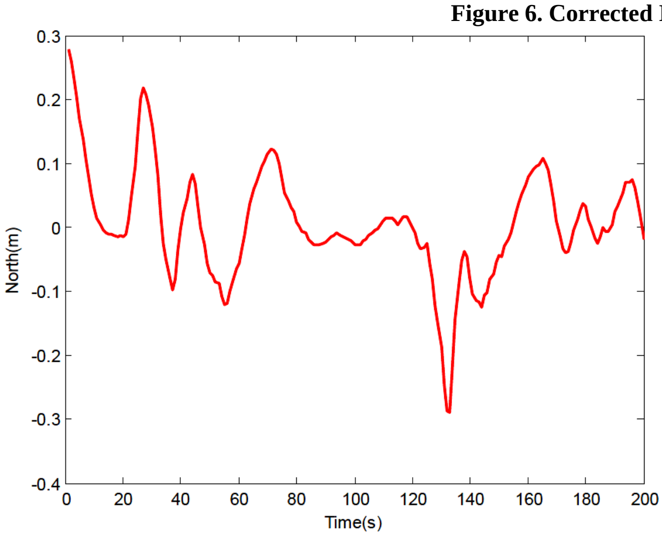

Figure 7. Vehicle Position Error Estimate in North Direction using IMM-UKFTFS

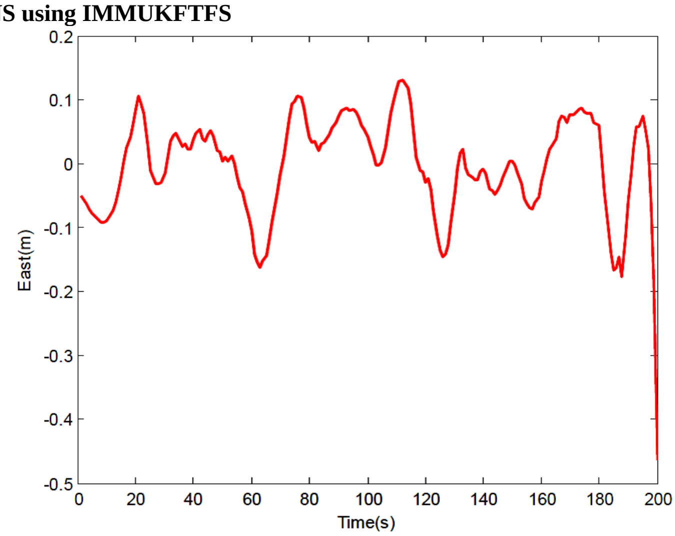

Figure 8. Vehicle Position Error Estimate in East Direction using IMM-UKFTFS

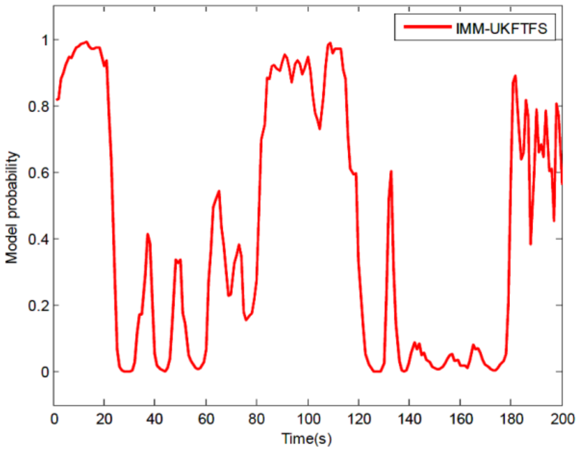

Figure 9. Constant Acceleration Model Probability in IMM-UKFTFS

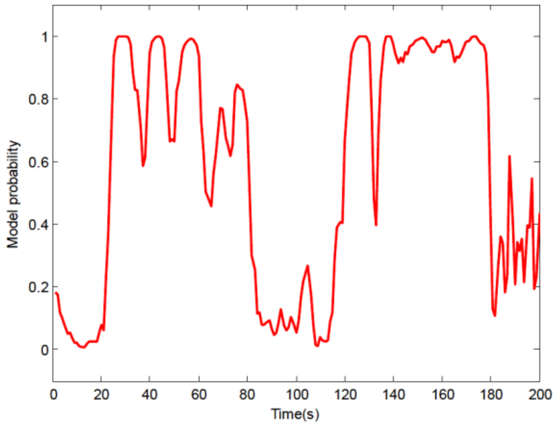

Figure 10. Constant Acceleration Model Probability in IMM-UKFTFS

VI. PERFORMANCE ANALYSIS

The estimates of erroneous INS and corrected INS using UKF, IMM-UKF, and IMM-UKFTFS are depicted in Figures 11 and 12. From these figures it can be revealed that the estimated trajectory is estimated and smoothed as close as to the true trajectory using IMM-UKFTFS. The Figures 13 and 14 give the constant acceleration and coordinated turn model probabilities for IMM-UKF and IMM-UKFTFS. The model probabilities depicted by IMM-UKFTFS is more likely to the true model than that depicted by IMM-UKF. The Figures 13 and 14 give the idea about the changes in model probability between constant acceleration model and coordinated turn model with respect to the actual vehicle dynamics. The Figures 15 and 16 show the vehicle position error estimate of proposed schemes in north and east components. From Figures 15 and 16 it can be proved that the error in north and east direction is very less in IMM-UKFTFS when compared to UKF and IMM-UKF.

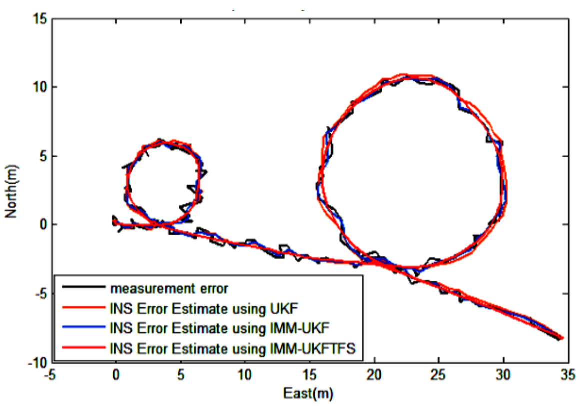

Figure 11. Estimates of Error Produced by UKF, IMMUKF, IMMUKF-TFS

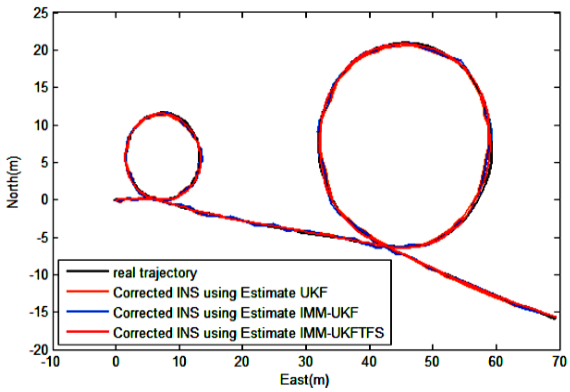

Figure 12. Estimates of Corrected INS Produced by UKF, IMMUKF, IMMUKF-TFS

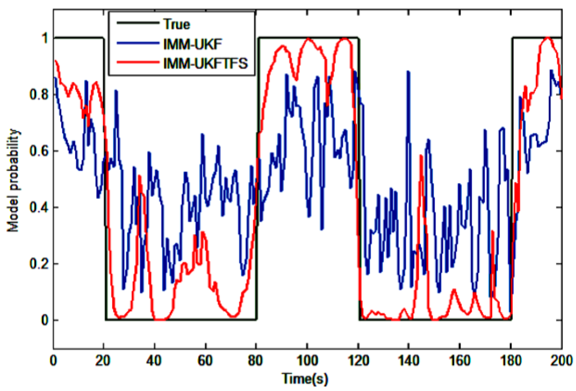

Figure 13. Constant Acceleration Model Probability

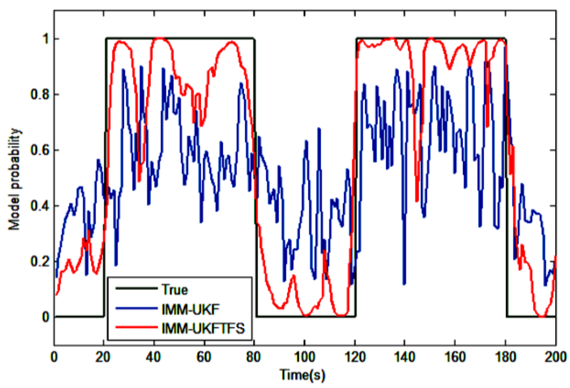

Figure 14. Coordinated Turn Model Probability

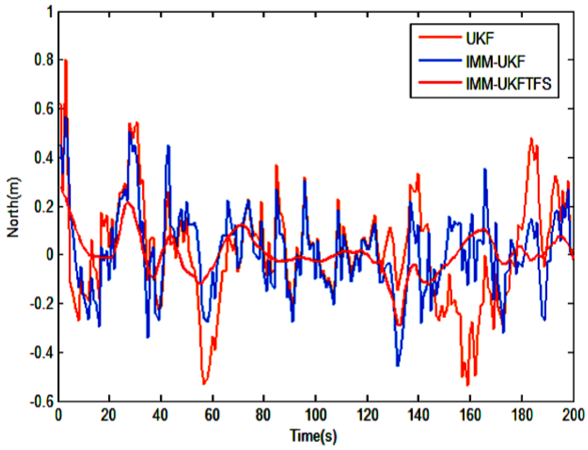

Figure 14. Vehicle Position Estimate Error in North

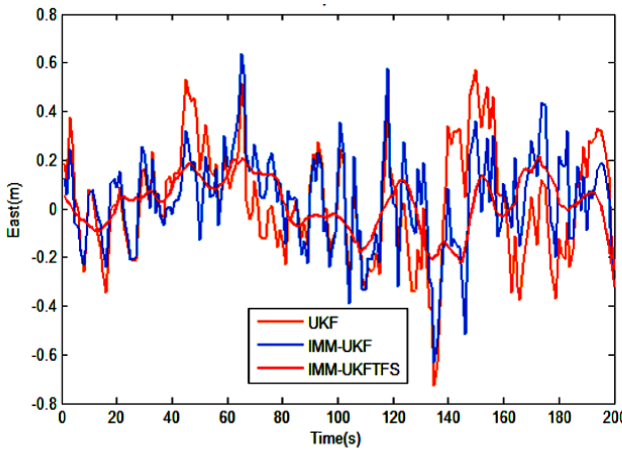

Figure 15. Vehicle Position Estimate Error in East

Table 1. Description of the Vehicle Motion

Segment Number	Time Interval (sec)	Motion
①	0-20	Constant Acceleration straight line
②	21-80	Counter clockwise turn with turn rate Ω=1
③	81-120	Constant Acceleration straight line
④	121-180	Counter clockwise turn with turn rate Ω=1
⑤	181-200	Constant Acceleration straight line

Table 2. Comparison of Mean Square Error of Different Estimation Methods Integrated CA-CT Error Models

Methods	Integrated CA-CT Error Model	
	North	East
UKF	0.0372	0.0417
IMM-UKF	0.0137	0.0167
IMM-UKFTFS	0.005	0.0051

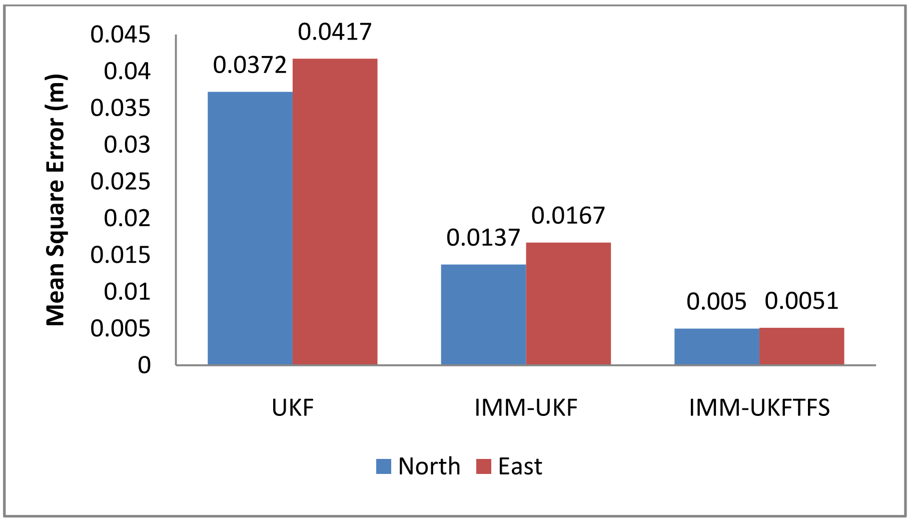

Figure 16. Comparison of Mean Square Error of Different Estimation methods for CA-CT Error Models

VII.CONCLUSION

In this paper, GPS navigation with error modeling using IMM-UKFTFS method is presented. Since TFS provides better results than RTS during high nonlinear characteristics of navigation applications and also the computational times for both TFS and RTS methods were almost the same[10], the IMM based TFS algorithm is considered in this work. The proposed methods have been tested by using two combinations of model trajectories. The navigation accuracy based on the proposed methods has been compared to the single model filter, UKF and multiple model filter, IMM-UKF. While comparing UKF and IMM-UKF, the output of IMM-UKF is better than UKF. Because IMM-UKF uses both CA and CT models for vehicle motion whereas UKF uses single model either CA or CT for vehicle motion. But a smoothed estimate obtained from IMM-UKFTFS is superior to IMM-UKF because they use additional smoothing measurements. Further, the mean square error of IMM-UKFTFS is lesser than other estimators. The comparison of Mean square error estimates of UKF, IMM-UKF and IMM-UKFTFS using CA and CT models are given in Table2. In both the cases it is found that IMM-UKFTFS method gives less mean square error compared to UKF and IMM-UKF methods. Even though the computational complexity is higher for IMM-UKFTFS than other estimation techniques, it has been confirmed that there is significant improvement in both navigation accuracy and tracking capability. From the simulation results, we can conclude that IMM-UKFTFS is better than IMM-UKF and gives accurate positioning.

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