

Original Article

Performance of DSSS-BPSK System for Even/Odd Parity Maximal Sequences

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Abstract - The maximal Sequence generated from linear feedback shift registers is used as an input sequence to the Hamming Code generator, where redundant binary bits are added to the original M sequence to generate new sequences. Two sequences can be obtained from each Maximal Sequence: one by even Parity and another by odd Parity. M sequence has an Auto correlation of 2^P-1 where the number of shift registers and cross-correlation value should be as low as possible; the ideal value is zero [1]. Complete analysis of maximal sequences, i.e., Number of valid taps, tap combinations and M-sequences, are presented [2]. Each maximal Sequence produces two codes (even parity sequence and odd Parity sequence). This paper aims to prove that the proposed codes have improved autocorrelation and cross-correlation and increased the number of codes... Further, the improvement in the Probability of error vs. noise ratio is shown through graphs generated in matlab®

Keywords - Autocorrelation, Cross correlation BPSK, Odd parity, Even parity, BER.

1. Introduction

Major problems in Communications and networking require collections of signals that are easily discriminated from a time-shifted replica of itself and distinguishable from every other signal [3]. Earlier property is useful in ranging systems, radar systems, and spread spectrum communications system applications.

Later, the property is employed in a concurrent range of various aims and Code Division Multiple Access systems [4]. Bound on system capacity depends on selected code autocorrelation, cross-correlation and its length [5]. Spreading sequences may be of orthogonal and non-orthogonal type. Walsh codes are orthogonal codes, whereas maximal codes, gold and kasami sequences are non-orthogonal sequences [6]. Codes with impulse autocorrelation and a cross-correlation value of zero are said to be ideal [7].

Spreading code will have half of its elements the same as the original Sequence and half of its elements different from the original Sequence provided when there is any shift in Sequence [8]. Lowering the cross-correlation among the codes than discernment in spread spectrum signals is conceivable at the receiver; this property is helpful in code division multiple access [9].

If the cross-correlation is zero, then there is no correlation between two sequences, i.e., orthogonal sequences [10]. The Sequence produced by the shift register is a function of a

number of stages, feedback connections and initial data stored by flip-flops. The output sequence can be classified as either maximal Sequence or non-maximal. sequence [11] Maximal length sequences have the property that for a 'm' stage linear feedback shift register, the code repetition period is $P = 2^m - 1$ [10-12]. Sequences having lengths smaller than $2^m - 1$ are called non-maximal length sequences [13]. Where C_O = Maximal code with Odd Parity and C_E = Maximal code with Even Parity. Autocorrelation is a similarity between the code and its time-delayed replica.

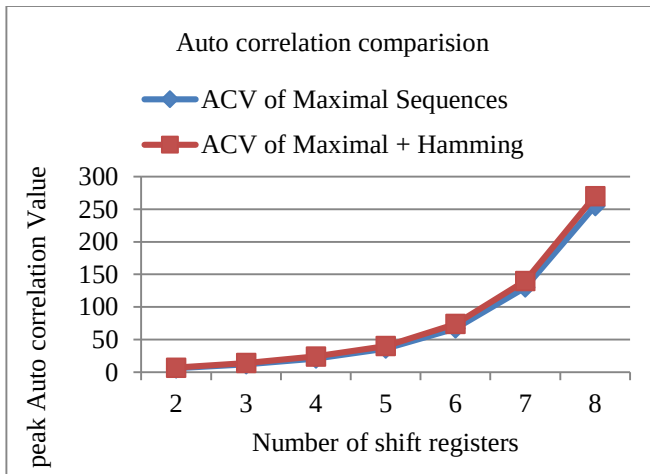
Using autocorrelation, the receiver discriminates signals, i.e. on a yes/no rule. The Sequence becomes more similar to a random binary sequence by increasing its sequence length [14]. The significance of Cryptography and spread-spectrum communications leads to the study of correlation for pseudorandom sequences with good cross-correlation properties. Pseudo-noise spreading sequences determine the communication link efficiency, and properties of spreading codes determine its performance [15].

Autocorrelation of M sequences of each fixed length shift registers for different combinations is compared with the proposed non-maximal Sequence, i.e., generated by introducing redundant bits to the m sequence provided in Table 1. A comparison of autocorrelation values of the Maximal sequence and Maximal-hamming (Even / odd Parity) sequence is shown in Figure 1. From the above figure, it has been observed that maximal Sequence with even and odd Parity exhibits better autocorrelation than normal m-sequence.



Table 1. Parity maximal hamming codes ACF

No of shift Register	Valid Taps	Non Maximal hamming	Peak value ACF
2	(2,1)	$C_{O1} \& C_{E1}$	6
3	(3,1)	$C_{O1} \& C_{E1}$	11
	(3,2)	$C_{O2} \& C_{E2}$	11
4	(4,1)	$C_{O1} \& C_{E1}$	20
	(4,3)	$C_{O2} \& C_{E2}$	20
5	(5,2)	$C_{O1} \& C_{E1}$	37
	(5,3)	$C_{O2} \& C_{E2}$	37
	(5,4,3,2)	$C_{O3} \& C_{E3}$	37
	(5,4,3,1)	$C_{O4} \& C_{E4}$	37
	(5,4,2,1)	$C_{O5} \& C_{E5}$	37
6	(6,1)	$C_{O1} \& C_{E1}$	70
	(6,5)	$C_{O2} \& C_{E2}$	70
	(6,5,4,1)	$C_{O3} \& C_{E3}$	70
	(6,5,3,2)	$C_{O4} \& C_{E4}$	70
	(6,4,3,1)	$C_{O5} \& C_{E5}$	70
	(6,5,2,1)	$C_{O6} \& C_{E6}$	70

**Fig. 1 Auto correlation comparison**

The Cross-correlation of Maximal sequences of each fixed length shift register for different tap combinations is presented in Table 2.

From the above, 15 cross-correlation tap combinations are possible for five length shift registrars. Of these, nine combinations have a normalised cross-correlation value of 0.2258, five have 0.3548, and one has 0.4858. Similarly, for a length shift registrar, 28 combinations of cross-correlation are possible. Out of which, six combinations have a normalised cross-correlation value of 0.206, nine combinations have 0.238, sis has 0.333, sis has 0.365 and has 0.4285. The Cross-correlation of the Proposed Sequence, i.e., Even /odd Parity maximal Sequence of each fixed length shift registers for different tap combinations, is given in Table 3.

Table 2. M sequence cross-correlation

SR	Possible combinations	CCF	Normalised CCF
3	(3,1) (3,2)	3,-1,5	0.428
4	(4,1) (4,3)	7,3,-1,-5	0.466
5	(5,2) (5,3)	11,7,3,-1,-5,-9	0.3548
	(5,2) (5,4,3,2)	7,-1,-9	0.2258
	(5,2) (5,4,3,1)	7,-1,-9	0.2258
	(5,2) (5,4,2,1)	7,-1,-9	0.2258
	(5,3) (5,4,3,2)	7,-1,-9	0.2258
	(5,3) (5,4,3,1)	7,-1,-9	0.2258
	(5,3) (5,4,2,1)	7,-1,-9	0.2258
	(5,4,3,2) (5,4,3,1)	7,-1,-9	0.2258
	(5,4,3,2) (5,4,2,1)	7,-1,-9	0.2258
	(5,4,3,1) (5,4,2,1)	7,-1,-9	0.2258
		Peak CCV	
6	(6,1) (6,5) ;	15: For 9 Combinations	0.238
	(6,1) (6,5,4,1);	23: For 6 Combinations	0.365
	(6,1) (6,4,3,1) ;		
	(6,1) (6,5,3,2)		
	(6,1) (6,5,2,1) ;		
	(6,5) (6,5,4,1)		
	(6,5) (6,5,3,2) ;		
	(6,5) (6,5,2,1)		
	(6,5) (6,4,3,1) ;		
	(6,5,4,1)		
	(6,5,3,2)		
	(6,5,4,1)		
	(6,5,2,1)		
	(6,5,4,1)		
	(6,5,3,1)		
	(6,5,3,2)		
	(6,5,2,1)		
	(6,5,3,2)		
	(6,4,3,1)		
	(6,5,2,1)		
	(6,4,3,1)		

Table 3. Cross-correlation of even /odd parity maximal sequence

No of SR	Valid Taps	Code	Non Max. Hamming	Combinations	Cross-Correlation Values	Normalised CCF
2	(2,1)	C ₁	C _{O1} & C _{E1}	(C _{O1}) (C _{E1})	4,0,-4	0.666
3	(3,1)	C ₁	C _{O1} & C _{E1}	(C _{E1}) (C _{E2})	3,1	0.2727
	(3,2)	C ₂	C _{O2} & C _{E2}	(C _{E1}) (C _{O2})	3,-1,-5	0.2727
				(C _{O1}) (C _{E2})	7,3,-1,-5	0.6363
				(C _{O1}) (C _{O2})	3,7,-1,-5	0.2727
4	(4,1)	C ₁	C _{O1} & C _{E1}	(C _{E1}) (C _{E2})	8,4,0,-4,-12	0.4
	(4,3)	C ₂	C _{O2} & C _{E2}	(C _{E1}) (C _{O2})	6,2,-2,-6,-10	0.3
				(C _{O1}) (C _{E2})	10,6,2,-2,-6,-10	0.5
				(C _{O1}) (C _{O2})	8,4,0,-4,-8	0.4
					Peak CCF	
5	(5,2)	C ₁	C _{O1} & C _{E1}	Total 40 Combinations	7: For 2 Combinations	0.1891
	(5,3)	C ₂	C _{O2} & C _{E2}		9: For 4 Combinations	0.2432
	(5,4,3,2)	C ₃	C _{O3} & C _{E3}		11: For 7 Combinations	0.2972
	(5,4,3,1)	C ₄	C _{O4} & C _{E4}		13: For 16 Combinations	0.3513
	(5,4,2,1)	C ₅	C _{O5} & C _{E5}		15: For 6 Combinations	0.4054
					17: For 4 Combinations	0.4594
					19: For 1 Combination	0.5135
6	(6,1)	C ₁	C _{O1} & C _{E1}	Total 66 Combinations	16: For 7 Combinations	0.2285
	(5,1)	C ₂	C _{O2} & C _{E2}		18: For 12 Combinations	0.2571
	(6,5)	C ₃	C _{O3} & C _{E3}		20: For 13 Combinations	0.2857
	(5,4)	C ₄	C _{O4} & C _{E4}		22: For 10 Combinations	0.3142
	(6,5,4,1)	C ₅	C _{O5} & C _{E5}		24: For 8 Combinations	0.3428
	(6,4,3,1)	C ₆	C _{O6} & C _{E6}		26: For 4 Combinations	0.3714
	(6,5,3,2)	C ₇	C _{O7} & C _{E7}		28: For 4 Combinations	0.40
	(6,5,2,1)	C ₈	C _{O8} & C _{E8}		30: For 1 Combination	0.4285
					32: For 1 Combination	0.4571
					52: For 1 Combination	0.7428
					56: For 5 Combinations	0.8

Table 4. Percentage improvement in cross-correlation of proposed sequence

Shift Registrar	No. of Codes with Better CCF	M Sequence CCF	Proposed Sequence CCF	Improvement in Cross-Correlation	Percentage of Improvement
3	3	0.428	0.2727	0.1553	56.9%
4	1	0.466	0.30	0.1666	55.3%
4	2	0.466	0.20	0.266	133.3%
5	2	0.2258	0.1891	0.0367	19.4%
5	4	0.3548	0.243	0.1118	46.0%
5	7	0.3548	0.2972	0.0576	19.38%
5	16	0.3548	0.3513	0.0035	1.0%
5	6	0.4838	0.4054	0.0784	19.33%
5	4	0.4838	0.4594	0.0244	5.31 %

Cross-correlation of M sequences of each fixed length shift registers for different combinations is compared with the proposed non-maximal Sequence, i.e., generated by introducing redundant bits to m sequence presented in Table 3

(Percentage improvement in cross-correlation of proposed Sequence) and Table 4 (Increased Number of codes with better cross-correlation).

2. Matlab Simulation

A Matlab software simulation program was used to calculate the probability error analysis of the AWGN baseband BPSK Modulation scheme with no noise power spectrum. The variables of the modulation simulation environment are mentioned in Table 5 and Table 6 below.

Table 5. Increased codes with better CCF

S.No	Normalised CCF Range	No. of Maximal Codes	No. of proposed M-Seq.	Increased No. of Seq.
5	0.10 – 0.20	nil	2	2
5	0.20 – 0.30	9	11	2
5	0.30 - 0.40	1	22	21
6	0.20 – 0.30	9	31	22
6	0.30 - 0.35	6	18	12
6	0.35 - 0.40	nil	7	7

Table 6. modulation simulation variables

Parameter	Value
Spreading code	M-Sequence hamming (Even /odd Parity)
SR and Valid taps	3 and (1,3)
Code length	11
SNR(dB)	0 to 10 dB
Channel & Modulation	AWGN - BPSK (Base Band)
Total bits	15,000
Maxi. code & length	[1 1 1 0 1 0 0] & 7
Proposed sequence (maxi. + Even Parity)	[1 1 1 1 0 1 0 1 0 1 0] & 11

The proposed maximal Sequence of length 11 and the original maximal Sequence of length 7 can be used to spread 15000 transmitted bits in the AWGN channel using BPSK modulation. The results are presented in the figure below.

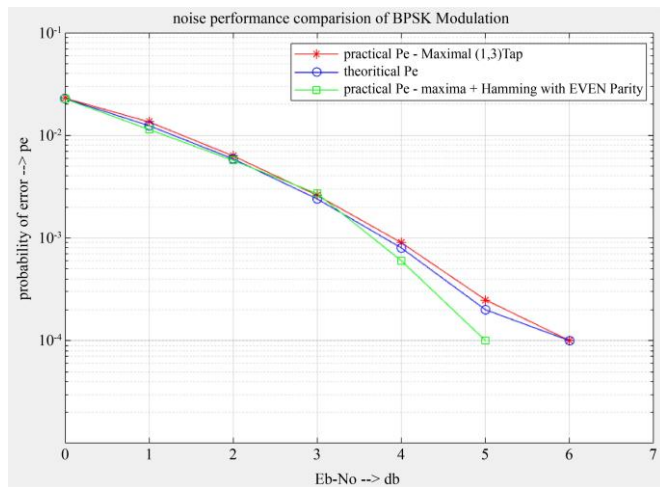


Fig. 2 BPSK performance comparison

The performance of the proposed even Parity maximal sequence (green line) is better than theoretical BPSK error

probability and practical Probability of error for direct Sequence seven chip spreading code from (3,1) with BPSK.

Table 7. Modulation simulation variables

Parameter	Value
Spreading code	Maximal Sequence with hamming (Even /Odd Parity)
Shift registers and tap combinations	4 and (1,4)
Code length	20
SNR(dB)	0 to 19dB
Channel	AWGN
Modulation	BPSK(Base Band)
Total bits	15,000
Maximal Sequence and its length	[1 1 1 1 0 1 0 1 1 0 0 1 0 0 0] & 15
Proposed Sequence (maximal Sequence + Even Parity) and its length	[1 1 1 1 0 0 1 0 1 1 0 0 1 1 0 0 0 0 1 0] & 20

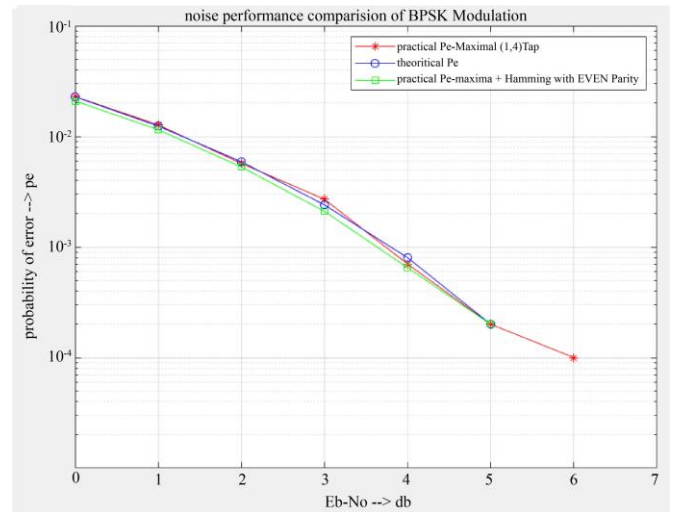


Fig. 3 BPSK performance comparison

The performance of the proposed even Parity maximal sequence (green line) is better than theoretical BPSK error probability and practical Probability of error for direct Sequence fifteen-length chip spreading code generated from (4,1) Tap linear feedback shift register using BPSK.

3. Conclusion

The proposed sequences have better auto-correlation and improved cross-correlation. Increased autocorrelation and decreased cross-correlation reduced the Probability of error, thereby reducing transmitting power and radiation levels. The number of sequences obtained from the proposed method is higher than that obtained by m sequences; therefore, many consumers can be accommodated into the CDMA cellular system with existing resources.

With decreased power level and increased user accommodation in the cellular system, the proposed parity/odd Parity maximal codes can be applied to the rake receiver and can be used in 5G technologies to reduce the effect of multipath fading.

The future scope of the paper includes the proposed method of altering the codes that can be applied to gold sequences and kasami sequences to study their correlation behaviour.

References

- [1] R. Gold, "Maximal Recursive Sequences with 3-Valued Recursive Cross-Correlation Function," *IEEE Transactions on Information Theory*, vol. 14, no. 1, pp. 154-156, 1968. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [2] Zenon Syroka, Tomasz Zając, and Paweł Dubiłowicz, "Generation of Linear Maximal Length Sequences," *2010 International Conference on Computer Information Systems and Industrial Management Applications (CISIM)*, Krakow, Poland, pp. 309-313, 2010. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [3] A.Y.C. Quan, "A Note on High-Speed Generation of Maximal Length Sequences," *IEEE Transactions on Computers*, vol. 23, no. 2, pp. 201-203, 1974. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [4] John J. Komo et al., "Maximal Length Sequences for QPSK," *IEEE Conference on Proceedings of The Twentieth Southeastern Symposium on System Theory*, Charlotte, NC, USA, pp. 332-335, 1998. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [5] R. Gold, "Optimal Binary Sequences for Spread Spectrum Multiplexing (Corresp.)," *IEEE Transactions on Information Theory*, vol. 13, no. 4, pp. 619-621, 1967. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [6] J.J. Komo, "Crosscorrelation of m-sequences Over Nonprime Finite Fields," *Electronics Letters*, vol. 25, no. 4, 1989. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [7] R. Kohno, R. Meidan, and L.B. Milstein, "Spread Spectrum Access Methods for Wireless Communications," *IEEE Communications Magazine*, vol. 33, no. 1, pp. 58-67, 1995. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [8] Esmail H. Dinan, and Bijan Jabbari, "Spreading Codes for Direct Sequence CDMA and Wideband CDMA Cellular Networks," *IEEE Communications Magazine*, vol. 36, no. 9, pp. 48-54, 1998. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [9] Olanrewaju Bola Wojuola, "Cross-Correlation Index and Multiple-Access Performance of Even-Degree Gold Codes," *2019 21st International Conference on Advanced Communication Technology (ICACT)*, PyeongChang, Korea (South), pp. 111-114, 2019. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [10] D.V. Sarwate, and M.B. Pursley, "Cross Correlation Properties of Pseudorandom and Related Sequences," *Proceedings of the IEEE*, vol. 68, no. 5, pp. 593-619, 1980. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [11] Deepak Kedia, "Comparative Analysis of Peak Correlation Characteristics of Non-Orthogonal Spreading Codes for Wireless Systems," *International Journal of Distributed and Parallel Systems*, vol. 3, no. 3, pp. 63-74, 2012. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [12] Sujit Jos et al., "Method of Generating Multiple Sets of Orthogonal Codes with Wide Choice of Spreading Factors," *IEEE Wireless Communications Letters*, vol. 1, no. 5, pp. 492-495, 2012. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [13] Yan Jun, and Zhang Xiang-Li, "A New Design of Gold-Like Binary Sequences," *2017 2nd Asia-Pacific Conference on Intelligent Robot Systems (ACIRS)*, Wuhan, China, pp. 118-124, 2017. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [14] S.W. Golomb, "Shift-Register Sequences and Spread-Spectrum Communications," *Proceedings of IEEE 3rd International Symposium on Spread Spectrum Techniques and Applications (ISSSTA'94)*, Oulu, Finland, vol. 1, pp. 14-15, 1994. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [15] Zhang Xinyu, "Analysis of M-Sequence and Gold-Sequence in CDMA System," *2011 IEEE 3rd International Conference on Communication Software and Networks*, Xi'an, China, pp. 466-468, 2011. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]