ON (K_1, K_2) - G*Bω - CLOSED SETS IN BiČEch Closure Spaces

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Abstract : *In this paper, we introduce the concepts of closed sets in biČech closure space called (k_1, k_2) - generalized star b ω - closed sets, (k_1, k_2) - generalized star bω - open sets and study their basic properties and its characterizations.*

Keywords : *Bi Č ech closure spaces, (k_1, k_2) - generalized star b ω - closed sets, (k_1, k_2) - generalized star bω - open sets.*

I INTRODUCTION

Čech closure spaces were introduced by Čech [2]. In Čech's approach the operator satisfies idempotent condition among Kuratowski axioms. This condition need not hold for every set *A* of *X*. When this condition is also true, the operator becomes topological closure operator. Thus the concept of closure space is the generalisation of a topological space. Closure functions that are more general than the topological ones have been studied already by Day [6]. A thorough discussion on closure functions is due to Hammer, see eg. [9, 10] and more recently Gnilka [8, 9]. The notion of bitopological space were introduced by J.C. Kelly [7]. Such spaces are equipped with two arbitrary topologies. Furthermore, Kelly extended some of the standard results of separation axioms in a topological space to a bitopological space. In this paper we introduce the **g*bω -** closed sets in biČech closure spaces.

II PRELIMINARIES

Definition 2.1 [4] Two functions k_1 and k_2 from power set X to itself are called bi Č ech closure operators (simply biclosure operator) for X if they satisfies the following properties:

i. $k_1(\varphi) = \varphi$ and $k_2(\varphi) = \varphi$

ii. $A \subset k_1(A)$ and $A \subset k_2(A)$, for any set $A \subset X$

iii. $k_1(A \cup B) = k_1(A) \cup k_1(B)$ and $k_2(A \cup B) = k_2(A) \cup k_2(B)$ for any $A, B \subset X$

(X, k_1, k_2) is called biČech closure space.

Example 2.2 Let X = {a, b, c} and let k_1 and k_2 be defined as:

k1({a})	= {a}	k2({a})	= {a}
k1({b})	= k1({c})	k2({b})	= {b, c}
	= k1({b, c})	k2({c})	= k2({a, c})
	= {b, c}		= {a, c}
k1({a, b})	= k1({a, c})	k2({a, b})	= k2({b, c})
	= k1({X})		=k2({X})
	= X		= X
k1(φ)	= φ	k2(φ)	= φ

Now (X, k_1, k_2) is biČech closure space.

Definition 2.3 [1] A subset A of a biČech closure space (X, k_1, k_2) is called ***biclosed*** if $k_1A = A = k_2A$

Definition 2.4 [3] A subset A in a biČech closure space (X, k_1, k_2) is said to be

i. ***k_i - semi open*** if $A \subseteq k_i\,[int_{ki}(A)]$, i = 1, 2.

ii. ***k_i - semi closed*** if $int_{ki}\,[k_i(A)] \subseteq A$, i = 1, 2.

The intersection of all k_i - semi - closed subsets of X containing A is called the *k_i - semi - closure of A* and is denoted by k_{is} (A).

III (k_1, k_2) - g*bω - CLOSED SETS

In this section, the concept of (k_1, k_2) - g*bω - closed sets in bi$\check{C}$ech closure spaces is defined and some of their characterizations and properties are studied.

Definition 3.1 A subset A in a bi$\check{C}$ech closure space (X, k_1, k_2) is said to be

i. ***k_i - b - closed*** if $(int_{ki}\,(k_i\,(A))) \cup (k_i\,(int_{ki}\,(A))) \subseteq A$.

ii. ***k_i - b - open*** if $A \subseteq (k_i\,(int_{ki}\,(A))) \cap (int_{ki}\,(k_i\,(A)))$.

The intersection of all k_i - b - closed subsets of X containing A is called the *k_i - b - closure of A* and is denoted by k_{ib} (A). The union of all k_i - b - open subsets of X contained in A is called the *k_i - b - interior of A* and is denoted by $int_{k_{ib}}$ (A).

Definition 3.2 A set A of a bi$\check{C}$ech closure space (X, k_1, k_2) is said to be ***(k_1, k_2) - generalized semi closed*** (briefly, (k_1, k_2) - gs - closed) if $k_{2s}(A) \subseteq U$ whenever $A \subseteq U$ and U is k_1 - open in X.

Definition 3.3 A set A of a bi$\check{C}$ech closure space (X, k_1, k_2) is said to be ***(k_1, k_2) - generalized star b omega closed*** (briefly, (k_1, k_2) - g*bω - closed) if $k_{2b}(A) \subseteq U$ whenever $A \subseteq U$ and U is k_1 - gs - open in X.

Example 3.4 In example 2.2, $\{a\} \subseteq \{a, b\}$ is a (k_1, k_2) - g*bω - closed.

Remark 3.5 By setting $k_1 = k_2$ in definition 3.3, an (k_1, k_2) - g*bω - closed set is a čech g*bω - closed set .

Theorem 3.6 Every k_2 - closed set in X is (k_1, k_2) - g*bω - closed.

Proof. Let A be k_2 - closed in X such that $A \subseteq U$, where U is k_2 - gs - open. Since A is k_2 - closed, $k_2(A) = A \subseteq U$. But $k_{2b}(A) \subseteq k_2(A)$. Therefore $k_{2b}(A) \subseteq U$. Hence A is a (k_1, k_2) - g*bω - closed set in X.

The converse of the above theorem is not true in general as can be seen from the following example.

Example 3.7 In example 2.2, {c} is a (k_1, k_2) - g*bω - closed but not k_2 - closed.

Theorem 3.8 Every k_2 - semi closed set in X is (k_1, k_2) - g*bω - closed.

Proof. Let A be k_2 - semi closed in X such that $A \subseteq U$, where U is k_2 - gs - open. Since A is k_2 - semi closed, $k_{2s}(A) = A \subseteq U$. But $k_{2b}(A) \subseteq k_{2s}(A)$. Therefore $k_{2b}(A) \subseteq U$. Hence A is a (k_1, k_2) - g*bω - closed set in X.

The converse of the above theorem is not true in general as can be seen from the following example.

Example 3.9 In example 2.2, {c} is a (k_1, k_2) - g*bω - closed set but not k_2 - semi closed.

Theorem 3.10 If A and B are two (k_1, k_2) - g*bω - closed sets and so is $A \cap B$.

Proof. Let A and B be two (k_1, k_2) - g*bω - closed sets. Let U be k_1- gs - closed in X. Let $(A \cap B) \subseteq U$. Since $(A \cap B) \subseteq U$, $A \subseteq U$ and $B \subseteq U$. Then $k_{2b}(A) \subseteq U$ and $k_{2b}(B) \subseteq U$ implies $k_{2b}(A) \cap k_{2b}(B) \subseteq U$. Hence $k_{2b}(A \cap B) \subseteq U$. Thus $A \cap B$ is a (k_1, k_2) - g*bω - closed set.

Theorem 3.11 If a subset A of a bi$\check{C}$ech closure space X is (k_1, k_2) - g*bω - closed then $k_{2b}(A) \setminus A$ contains no nonempty k_1- gs - closed set.

Proof. Let A be a (k_1, k_2) - g*bω - closed set and F be a k_1 - gs - closed set such that $F \subseteq k_{2b}(A) \setminus A$. Therefore $A \subseteq F^c$ and $F \subseteq k_{2b}(A)$. Since F is k_1 - gs - closed, F^c is k_1 - gs - open and A is (k_1, k_2) - g*bω - closed, $k_{2b}(A) \subseteq F^c$. Thus $F \subseteq [k_{2b}(A)]^c = X \setminus [k_{2b}(A)]$. Hence $F \subseteq [k_{2b}(A)] \cap [X \setminus [k_{2b}(A)]] =$

φ. Therefore F = φ. Hence $k_{2b}(A) \setminus A$ contains no nonempty k_1- gs - closed set.

Theorem 3.12 Let A be an (k_1, k_2) - g*bω - closed set in X. Then A is k_2 - b - closed if and only if $k_{2b}(A) \setminus A$ is k_1 - gs - closed in X.

Proof. Suppose that A is (k_1, k_2) - g*bω - closed. Let A be k_2 - b - closed. Then $k_{2b}(A) = A$. Therefore $k_{2b}(A) \setminus A = \varphi$ is k_1 - gs - closed in X.

Conversely, suppose that A is (k_1, k_2) - g*bω - closed and $k_{2b}(A) \setminus A$ is k_1 - gs - closed. Since A is (k_1, k_2) - g*bω - closed, $k_{2b}(A) \setminus A$ contains no nonempty k_1 - gs - closed set (by Theorem 3.11). Since $k_{2b}(A) \setminus A$ is k_1 - gs - closed, $k_{2b}(A) \setminus A = \varphi$. Then $k_{2b}(A) = A$. Hence A is k_2 - b - closed.

Theorem 3.13 Let A and B be subsets of X such that $A \subseteq B \subseteq k_{2b}(A)$. If A is (k_1, k_2) - g*bω - closed then B is (k_1, k_2) - g*bω - closed.

Proof. Let A and B be subsets such that $A \subseteq B \subseteq k_{2b}(A)$. Suppose that A is (k_1, k_2) - g*bω - closed. Let $B \subseteq U$ and U be k_1 - gs - open in X. Then $A \subseteq U$. Since A is (k_1, k_2) - g*bω - closed, $k_{2b}(A) \subseteq U$. Since $B \subseteq k_{2b}(A)$, $k_{2b}(B) \subseteq k_{2b}[k_{2b}(A)] = k_{2b}(A) \subseteq U$. Therefore B is (k_1, k_2) - g*bω - closed.

Theorem 3.14 Let X be a bi$\check{C}$ech closure space. If x ∈ X then singleton {x} is either k_1 - gs - closed or $\{x\}^c$ is (k_1, k_2) - g*bω - closed set.

Proof. Let X be a bi$\check{C}$ech closure space. Let x ∈ X and suppose that {x} is not k_1 - gs - closed. Then X \ {x} is not k_1 - gs -open. Consequently, X is the only k_1 - gs - open set containing the set X \ {x}. Therefore X \ {x} is (k_1, k_2) - g*bω - closed.

Theorem 3.15 Let $B \subseteq A \subseteq X$ and suppose that B is (k_1, k_2) - g*bω - closed in X, then B is (k_1, k_2) - g*bω - closed relative to A. The converse is true if A is k_1 - open and (k_1, k_2) - g*bω - closed in X.

Proof. Let B be (k_1, k_2) - g*bω - closed in X. Let $B \subseteq U$ and U be k_1 - gs - open in A. Since U is τ_i - gs - open in A, $U = V \cap A$, where V is k_1 - gs - open in X. Hence $B \subseteq U \subseteq V$. Since B is (k_1, k_2) - g*bω - closed in X, $k_{2b}(B) \subseteq V$. Hence $k_{2b}(B) \cap A \subseteq V \cap A$, which in turn implies that $A \cap k_{2b}(B) \subseteq V \cap A = U$. Therefore B is (k_1, k_2) - g*bω - closed relative to A.

Now to prove the converse, assume the given condition. Let $B \subseteq U$ and U be k_1 - gs - open in X. Then $A \cap U$ is k_1 - gs - open in A. Since $B \subseteq A$ and $B \subseteq U$, $B \subseteq A \cap U$. Since B is (k_1, k_2) - g*bω - closed relative to A, $A \cap k_{2b}(B) \subseteq A \cap U$. Since A is k_1 - open, it is k_1 - gs - open in X. Since $A \subseteq A$ and A is (k_1, k_2) - g*bω - closed in X, $k_{2b}(A) \subseteq A$. Since $B \subseteq A$, $k_{2b}(B) \subseteq k_{2b}(A)$. Hence $k_{2b}(B) \subseteq A$. Therefore, $k_{2b}(B) \cap A = k_{2b}(B) \Rightarrow A \cap k_{2b}(B) = k_{2b}(B)$. Hence $k_{2b}(B) \subseteq A \cap U \subseteq U$. Thus B is (k_1, k_2) - g*bω - closed in X.

IV (k_1, k_2) - g*bω - OPEN SETS

In this section, (k_1, k_2) - g*bω - open sets in bi$\check{C}$ech closure space is introduced and their properties are studied.

Definition 4.1 A set A of a bi$\check{C}$echclosure space (X, τ_1, τ_2) is called ***(k_1, k_2) - generalized star b omega open*** (briefly, (k_1, k_2) - g*b ω - open) if its complement is (k_1, k_2) - g*bω - closed.

Theorem 4.2 A subset A of a bi$\check{C}$echclosure space X is (k_1, k_2) - g*bω - open if and only if $F \subseteq int_{k_{2b}}(A)$ whenever $F \subseteq A$ and F is k_1 - gs - closed in X.

Proof. Suppose that A is (k_1, k_2) - g*bω - open. Let $F \subseteq A$ and F be k_1 - gs - closed. Then $A^c \subseteq F^c$ and F^c is k_1 - gs - open. Since A^c is (k_1, k_2) - g*bω - closed, $k_{2b}(A^c) \subseteq F^c$. Since $k_{2b}(A^c) = [\,int_{k_{2b}}(A)]^c$, $[int_{k_{2b}}(A)]^c \subseteq F^c$. Hence $F \subseteq int_{k_{2b}}(A)$.

Conversely, suppose that $F \subseteq int_{k_{2b}}(A)$ whenever $F \subseteq A$ and F is k_1 - gs - closed in X. Let U be k_1 - gs - open in X and $A^c \subseteq U$. Then U^c is k_1 - gs - closed and $U^c \subseteq A$. Hence by assumption $U^c \subseteq int_{k_{2b}}(A)$. That is $k_{2b}(A^c) \subseteq U$. Therefore A^c is (k_1, k_2) - g*bω - closed. Hence A is (k_1, k_2) - g*bω - open.

Theorem 4.3 If a subset A is (k_1, k_2) - g*bω - closed in X then $k_{2b}(A) \setminus A$ is (k_1, k_2) - g*bω - open.

Proof. Suppose that A is (k_1, k_2) - g*bω - closed in X. Let F $\subseteq$ k_{2b}(A) \ A and F be k_1 - gs - closed. Since A is (k_1, k_2) - g*bω - closed, k_{2b}(A) \ A does not contain nonempty k_1 - gs - closed sets (by Theorem 3.11). Hence F = φ. Thus F $\subseteq$ $int_{k_{2b}}$[k_{2b}(A) \ A]. Hence k_{2b}(A) \ A is (k_1, k_2) - g*bω - open.

Theorem 4.4 If a set A is (k_1, k_2) - g*bω - open in X then G = X whenever G is k_1 - gs - open and $int_{k_{2b}}$(A) ∪ A^c $\subseteq$ G.

Proof. Suppose that A is (k_1, k_2) - g*bω - open in X, G is k_1 - gs - open and $int_{k_{2b}}$(A) ∪ $A^c \subseteq$ G. Then $G^c \subseteq \{int_{k_{2b}}(A) \cup A^c\}^c = k_{2b}(A^c) \setminus A^c$. Since A^c is (k_1, k_2) - g*b ω - closed, $k_{2b}(A^c) \setminus A^c$ contains no nonempty k_1 - gs - closed set in X (by Theorem 3.11). Therefore G^c = φ. Hence G = X.

Theorem 4.5 If A and B are two (k_1, k_2) - g*bω - open sets and so is A ∪ B.

Proof. Let A and B be two (k_1, k_2) - g*bω - open sets. Let U be k_1- gs - open in X. Let $(A^c \cap B^c) \subseteq$ U. Since $(A^c \cap B^c) \subseteq$ U, we have $A^c \subseteq$ U and $B^c \subseteq$ U. Then $k_{2b}(A^c) \subseteq$ U and $k_{2b}(B^c) \subseteq$ U implies $k_{2b}(A^c) \cap k_{2b}(B) \subseteq$ U. Hence $k_{2b}(A^c \cap B^c) \subseteq$ U. Thus A ∪ B is (k_1, k_2) - g*bω - open set.

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