Unreliable Batch Arrival Retrial G-queue with Fluctuating Modes of Service, Preemptive Priority and Orbital Search

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Abstract — *In this paper, single server retrial queueing system with two types of customers is analyzed. The server provides M fluctuating modes of service. If the server is free upon the arrival of a batch, one of the customers in the batch receives service immediately and the rest join the orbit. If the server is busy, one of the customers in the arriving batch interrupts the customer in service to commence his own service. The breakdown of the server due to the arrival of a negative customer and an unpredictable breakdown are two different types of system failure. The repair of the failed server due to negative arrival starts after a random amount of time whereas repair due to active breakdown starts instantaneously. If the server is idle in non-empty system, then the server may search for customers in the orbit. Retrial time, service time, delay time and repair time are assumed to be arbitrarily distributed. Using generating functions technique, expressions of the system performance measures are obtained. Stochastic decomposition property and special cases are studied. Finally the effects of several parameters on system measures are shown numerically.*

Keywords— *Retrial queue, positive customers, negative customers, M modes of service, priority, server breakdown, delayed repair and orbital search.*

I. INTRODUCTION

Retrial queueing systems are characterized by the feature that a customer who cannot receive service leaves the service area but after some random delay returns to the system again to request service.

In recent years, interest is growing in queues with negative customers due to their applications in the telecommunication system, neutral networks, multiprocessor computer systems and manufacturing systems. The named G-queue has been adopted for the queue with negative customers in acknowledgement of Gelenbe [7] who first introduced this type of queue. A detailed survey on queueing systems with negative arrivals can be found in Gelenbe [8-10].

Liu et al. [11] obtained the steady state solution for both queueing measures and reliability quantities of an unreliable M/G/1 retrial G-queue with preemptive resume and feedback under N-policy. Aissani [1] obtained the generating function of the number of primary customers in the stationary regime of an M/G/1 retrial queue with negative arrivals and server breakdown. Peng et al. [14] considered M/G/1 retrial G-queue with preemptive resume priority and collisions subject to server breakdowns and repairs and obtained the performance measures. Gao and Wang [6] analysed an M/G/1 – G queue with orbital search and non-persistent customers. Rajadurai et al. [15] studied an $M^X/G/1$ unreliable retrial G-queue with orbital search, feedback and Bernoulli vacation.

In the retrial setup, each service is preceded and followed by the server's idle time because of the ignorance of the status of the server and orbital customers by each other. Server's idle time is reduced by the introduction of search of orbital customers. Search for orbital customers was introduced by Neuts et al. [13] where the authors examined classical queue with search for customers immediately on termination of a service. Artalejo et al. [3] considered a retrial queue with orbital search. Dudin et al. [5] extended the model to a batch arrival retrial queue and performed the steady state analysis of the queueing system. Sumitha and Udaya Chandrika [17] discussed a single server batch arrival retrial queueing system with additional optional service and orbital search. Sumitha and Udaya Chandrika [18] investigated a repairable M/G/1 retrial queue with Bernoulli vacation and orbital search. Deepak et al. [4] considered a retrial model in which at each service completion epoch, two different search mechanisms are switched on to bring the orbital customers to service.

Most of the single server queueing models assume that the server provides service to all customers with the same mean service rate. This is not possible in the real life situation. Baruah et al. [3] studied the behaviour of a batch arrival queueing system equipped with a single server providing arbitrary service in two fluctuating modes. Madan [12] analyzed a single server queue with batch arrivals and general service in three fluctuating modes of different mean service rates. Rajadurai et al. [16] analyzed the repairable batch arrival feedback retrial G-queue with two types of services and J vacations.

In this article batch arrival retrial queueing system with negative customers, fluctuating modes of service, priority, random breakdown, delayed repair and orbital search is considered.

II. Model Description and Definitions

Consider a single server queueing system in which positive customers arrive in batches according to Poisson process with rate λ^+ and negative customers arrive in single according to Poisson process with rate λ^-.

The server serves the customers in M modes of service and the probability of providing i^{th} mode service is p_i ($1 \le i \le M$). If the server is free, the service commences for any one of the positive customers of an arriving batch and the remaining customers join the orbit. If the server is busy, one of the customers in the arriving batch moves the customer in service to the orbit in order to commence his own service with certain probability. The remaining customers along with the interrupted customer join the orbit. The service of the interrupted customer resumes from the beginning. The batch size Y is a random variable with $P(Y = k) = C_k$, the generating function $C(z)$ and first two moments m_1 and m_2. Successive inter-retrial times of any customer are governed by an arbitrary probability distribution function $A(x)$ with corresponding density function $a(x)$ and Laplace Stieltje's transform $A^*(\bullet)$.

The service times of customers in mode i service (i = 1, 2, ..., M) are generally distributed with distribution function $B_i(x)$, density function $b_i(x)$ and Laplace Stieltje's transform $B_i^*(\bullet)$.

The busy server is subject to two different types of breakdown say, type 1 and type 2. Type 1 breakdown is due to negative arrival and type 2 is random breakdown.

Negative arrival removes the customer in service from the system and makes the server down. The repair of the failed service starts after a random amount of time. This delay time follows general distribution with distribution function $S_i(x)$, density function $s_i(x)$ and Laplace Stieltje's transform $S_i^*(\bullet)$.

Repair of random breakdown starts instantaneously and the interrupted customer either remains in service position with probability τ_i until the server is up or leaves the service area with probability $1 - \tau_i$ and keeps returning at times exponentially distributed with rate ω_i. It is assumed that the lifetime of the server in i^{th} mode is exponentially distributed with rate α_i. As soon as the repair is completed, the server continues the service of the interrupted customer or waits for the same customer.

The repair times of type ℓ (= 1, 2) breakdown are generally distributed with distribution function $R_i^{(\ell)}(x)$, density function $r_i^{(\ell)}(x)$ and Laplace Stieltje's transform $R_i^{(\ell)*}(\bullet)$. The server searches for customers in the orbit with probability θ during his idle time.

Let the functions $\eta(x), \mu_i(x), \phi_i(x), \beta_i^{(1)}(x)$ and $\beta_i^{(2)}(x)$ be the hazard rate functions corresponding to repeated attempts, mode i service, delay time, repair time of type 1 and repair time of type 2 respectively.

The stochastic behavior of the retrial queueing system can be described by the Markov process $\{X(t), t \ge 0\} = \{C(t), N(t), J^*(t), \xi_0(t), \xi_1(t), \xi_2(t), \xi_3(t), \xi_4(t), t \ge 0\}$ where C(t) denotes the server state 0, i, M+i, 2M+i, 3M+i or 4M+i ($1 \le i \le M$) according as the server being idle, busy in providing mode i service, in delay time, under type 1 repair, under type 2 repair or under reserved time. N(t) corresponds to the number of customers in the orbit. If $C(t) = 0$ and $N(t) > 0$, then $\xi_0(t)$ represents the elapsed retrial time. For $N(t) \ge 0$, if $C(t) = i$, $\xi_1(t)$ represents the elapsed service time, if $C(t) = M+i$, $\xi_2(t)$ represents the elapsed delay time, if $C(t) = 2M+i$, $\xi_3(t)$ represents the elapsed repair time of type 1 breakdown, if $C(t) = 3M+i$, $\xi_3(t)$ represents the elapsed repair time of type 2 breakdown and if $C(t) = 4M+i$, $\xi_4(t)$ represents elapsed reserved time.

The state of the interrupted customer $J^*(t)$ is defined as $J^*(t) =$

$$\begin{cases} 0, & \text{if the interrupte d customer remains in service position} \\ 1, & \text{if the interrupte d customer is not in service position} \end{cases}$$

Define the probabilities for the process $\{X(t), t \ge 0\}$

$I_0(t) = P\{C(t)=0, N(t)=0\}$

$I_n(x, t)\,dx = P\{C(t)=0, N(t)=n, x<\xi_0(t)\le x+dx\}, n \ge 1$

For $t \ge 0, x \ge 0, y \ge 0, n \ge 0, i = 1, 2, \ldots, M, j = 0, 1$

$P_{i,n}(x,t)dx = P\{C(t)=i, N(t)=n, x<\xi_1(t)\le x+dx\}$,

$\Pi_{i,n}(x, t)\,dx = P\{C(t) = M+i, N(t = n, x < \xi_2(t) \le x + dx\}$,

$W_{i,n}^{(1)}(x, t)\,dx = P\{C(t) = 2M+i, N(t)=n, x<\xi_3(t)\le x+dx\}$,

$W_{i,j,n}^{(2)}(x, y, t)\,dx\,dy = P\{C(t) = 3M+i, J^*(t) = j, N(t) = n, x < \xi_1(t) \le x + dx, y < \xi_3(t) \le y + dy\}$,

$Q_{i,n}(x, y, t)\,dx\,dy = P\{C(t) = 4M+i, N(t) = n, x < \xi_1(t) \le x + dx, y < \xi_4(t) \le y + dy\}$

III. Steady State Equations

Using the method of supplementary variable technique, we obtain the following system of equations that governs the dynamics of the system behavior.

$$\lambda^{+}I_0=\sum_{i=1}^{M}\left(\int_0^{\infty} P_{i,0}(x)\,\mu_i(x)\,dx+\int_0^{\infty} W_{i,0}^{(1)}(x)\,\beta_i^{(1)}(x)\,dx\right) \quad (1)$$

$$\frac{d}{dx}I_n(x) = -(\lambda^{+}+\eta(x))\,I_n(x), n\geq 1 \quad (2)$$

$$\frac{d}{dx}P_{i,n}(x) = -(\lambda^{+}+\lambda^{-}+\alpha_i+\mu_i(x))\,P_{i,n}(x)+\lambda^{+}(1-\nu)\sum_{k=1}^{n} C_k\,P_{i,n-k}(x)+\int_0^{\infty} W_{i,0,n}^{(2)}(x,y)\,\beta_i^{(2)}(y)\,dy+\omega_i\int_0^{\infty} Q_{i,n}(x,y)\,dy,\quad n\geq 0, i = 1, 2, \ldots, M \quad (3)$$

$$\frac{d}{dx}\Pi_{i,n}(x) = -(\lambda^{+}+\phi_i(x))\,\Pi_{i,n}(x)+\lambda^{+}\sum_{k=1}^{n} C_k\,\Pi_{i,n-k}(x),\quad n\geq 0, i = 1, 2, \ldots, M \quad (4)$$

$$\frac{d}{dx}W_{i,n}^{(1)}(x) = -(\lambda^{+}+\beta_i^{(1)}(x))\,W_{i,n}^{(1)}(x)+\lambda^{+}\sum_{k=1}^{n} C_k\,W_{i,n-k}^{(1)}(x), n\geq 0, i = 1, 2, \ldots, M \quad (5)$$

$$\frac{d}{dy}W_{i,j,n}^{(2)}(x,y) = -(\lambda^{+}+\beta_i^{(2)}(y))\,W_{i,j,n}^{(2)}(x,y)+\lambda^{+}\sum_{k=1}^{n} C_k\,W_{i,j,n-k}^{(2)}(x,y), \geq 0, i = 1, 2, \ldots, M,\ j = 0, 1 \quad (6)$$

$$\frac{d}{dy}Q_{i,n}(x,y) = -(\lambda^{+}+\omega_i)\,Q_{i,n}(x,y)+\lambda^{+}\sum_{k=1}^{n} C_k\,Q_{i,n-k}(x,y), n\geq 0, i = 1, 2, \ldots, M \quad (7)$$

with boundary conditions

$$I_n(0) = (1-\theta)\sum_{i=1}^{M}\left[\int_0^{\infty} P_{i,n}(x)\,\mu_i(x)\,dx+\int_0^{\infty} W_{i,n}^{(1)}(x)\,\beta_i^{(1)}(x)\,dx\right], n\geq 1 \quad (8)$$

$$P_{i,0}(0) = p_i\left[\lambda^{+}C_1 I_0+\int_0^{\infty} I_1(x)\,\eta(x)\,dx+\theta\sum_{j=1}^{M}\left[\int_0^{\infty} P_{j,1}(x)\,\mu_j(x)\,dx+\int_0^{\infty} W_{j,1}^{(1)}(x)\,\beta_j^{(1)}(x)\,dx\right]\right],\quad i = 1, 2, \ldots, M \quad (9)$$

$$P_{i,n}(0) = p_i\left[\lambda^{+}C_{n+1}I_0+\int_0^{\infty} I_{n+1}(x)\,\eta(x)\,dx+\lambda^{+}\sum_{k=1}^{n} C_k\int_0^{\infty} I_{n-k+1}(x)\,dx+\lambda^{+}\nu\sum_{j=1}^{M}\sum_{k=1}^{n} C_k\int_0^{\infty} P_{j,n-k}(x)\,dx+\theta\sum_{j=1}^{M}\left(\int_0^{\infty} P_{j,n+1}(x)\,\mu_j(x)\,dx+\int_0^{\infty} W_{j,n+1}^{(1)}(x)\,\beta_j^{(1)}(x)\,dx\right)\right],\quad n\geq 1, i = 1, 2, \ldots, M \quad (10)$$

$$\Pi_{i,n}(0)=\lambda^{-}\int_0^{\infty} P_{i,n}(x)\,dx,\quad n\geq 0, i = 1, 2, \ldots, M \quad (11)$$

$$W_{i,n}^{(1)}(0) = \int_0^{\infty} \Pi_{i,n}(x)\,\phi_i(x)\,dx,\quad n\geq 0, i = 1, 2, \ldots, M \quad (12)$$

$$W_{i,0,n}^{(2)}(x,0) = \tau_i\,\alpha_i\,P_{i,n}(x),\quad n\geq 0, i = 1, 2, \ldots, M \quad (13)$$

$$W_{i,1,n}^{(2)}(x,0) = (1-\tau_i)\,\alpha_i\,P_{i,n}(x),\quad n\geq 0, i = 1, 2, \ldots, M \quad (14)$$

$$Q_{i,n}(x, 0)=\int_0^{\infty} W_{i,1,n}^{(2)}(x,y)\,\beta_i^{(2)}(y)\,dy,\quad n\geq 0, i = 1, 2, \ldots, M \quad (15)$$

IV STEADY STATE SOLUTIONS

Define the generating functions for $|z|\leq 1$ for $i = 1, 2, \ldots, M$ as follows :

$$I(x, z)=\sum_{n=1}^{\infty} I_n(x)\,z^n\ ;\quad P_i(x,z)=\sum_{n=0}^{\infty} P_{i,n}(x)\,z^n\ ;$$

$$\Pi_i(x,z)=\sum_{n=0}^{\infty} \Pi_{i,n}(x)\,z^n\ ;\ W_i^{(1)}(x,z)=\sum_{n=0}^{\infty} W_{i,n}^{(1)}(x)\,z^n\ ;$$

$$W_{i,j}^{(2)}(x,y,z)=\sum_{n=0}^{\infty} W_{i,j,n}^{(2)}(x,y)\,z^n \text{ and}$$

$$Q_i(x, y, z) = \sum_{n=0}^{\infty} Q_{i,n}(x,y)\,z^n, i = 1, 2, \ldots, M ; j = 0, 1$$

Normalizing condition I_0 is

$$I_0+\lim_{z\to 1}\left[\int_0^{\infty} I(x,z)\,dx+\sum_{i=1}^{M}\left(\int_0^{\infty} P_i(x,z)\,dx+\right.\right.$$

$$\int_0^\infty \Pi_i(x,z)\,dx + \int_0^\infty W_i^{(1)}(x,z)\,dx + \sum_{j=0}^{1}\int_0^\infty\int_0^\infty W_{i,j}^{(2)}(x,y,z)\,dx\,dy + \int_0^\infty\int_0^\infty Q_i(x,y,z)\,dx\,dy)] = 1 \quad (16)$$

Multiplying the equations (2) to (15) by z^n and summing over n, we have

$$\left(\frac{d}{dx} + \lambda^+ + \eta(x)\right) I(x,z) = 0 \quad (17)$$

$$\left(\frac{d}{dx} + \lambda^+ + \lambda^- - \lambda^+(1-\nu)C(z) + \alpha_i + \mu_i(x)\right)P_i(x,z) = \int_0^\infty W_{i,0}^{(2)}(x,y,z)\,\beta_i^{(2)}(y)\,dy + \omega_i\int_0^\infty Q_i(x,y,z)\,dy, \quad i=1,2,\ldots,M \quad (18)$$

$$\left(\frac{d}{dx} + \lambda^+ - \lambda^+C(z) + \phi_i(x)\right)\Pi_i(x,z) = 0, i=1,2,\ldots,M \quad (19)$$

$$\left(\frac{d}{dx} + \lambda^+ - \lambda^+ C(z) + \beta_i^{(1)}(x)\right) W_i^{(1)}(x,z) = 0, \quad i=1,2,\ldots,M \quad (20)$$

$$\left(\frac{d}{dy} + \lambda^+ - \lambda^+C(z) + W_{i,j}^{(2)}\right)(x,y,z) = 0, \quad i=1,2,\ldots,M, j=0,1 \quad (21)$$

$$\left(\frac{d}{dy} + \lambda^+ - \lambda^+C(z) + \omega_i\right)Q_i(x,y,z) = 0, \quad i=1,2,\ldots,M \quad (22)$$

$$I(0,z) = (1-\theta)\sum_{i=1}^{M}\left[\int_0^\infty P_i(x,z)\,\mu_i(x)\,dx + \int_0^\infty W_i^{(1)}(x,z)\,\beta_i^{(1)}(x)\,dx\right] - \lambda^+ I_0 \quad (23)$$

$$P_i(0,z) = \frac{p_i}{z}\Big[\lambda^+ C(z) I_0 + I(0,z)\,(A^*(\lambda^+) + C(z)(1 A^*(\lambda^+))) + \lambda^+\nu z C(z)\sum_{j=1}^{M}\int_0^\infty P_j(x,z)\,dx + \theta\sum_{j=1}^{M}\Big[\int_0^\infty P_j(x,z)\,\mu_j(x)\,dx + \int_0^\infty W_j^{(1)}(x,z)\,\beta_j^{(1)}(x)\,dx\Big]\Big], \quad i=1,2,\ldots,M \quad (24)$$

$$\Pi_i(0,z) = \lambda^-\int_0^\infty P_i(x,z)\,dx, \quad i=1,2,\ldots,M \quad (25)$$

$$W_i^{(1)}(0,z) = \int_0^\infty \Pi_i(x,z)\,\phi_i(x)\,dx, \quad i=1,2,\ldots,M \quad (26)$$

$$W_{i,0}^{(2)}(x,0,z) = \tau_i\,\alpha_i\,P_i(x,z), \quad i=1,2,\ldots,M \quad (27)$$

$$W_{i,1}^{(2)}(x,0,z) = (1-\tau_i)\,\alpha_i\,P_i(x,z), \quad i=1,2,\ldots,M \quad (28)$$

$$Q_i(x,0,z) = \int_0^\infty W_{i,1}^{(2)}(x,y,z)\,\beta_i^{(2)}(y)\,dy, \quad i=1,2,\ldots,M \quad (29)$$

Solving the partial differential equations (17) and (19) to (22), we get

$$I(x,z) = I(0,z)\,e^{-\lambda^+ x}(1-A(x)) \quad (30)$$

$$\Pi_i(x,z) = \Pi_i(0,z)\,e^{-\lambda^+(1-C(z))x}(1-S_i(x)), \quad i=1,2,\ldots,M \quad (31)$$

$$W_i^{(1)}(x,z) = W_i^{(1)}(0,z)\,e^{-\lambda^+(1-C(z))x}(1-R_i^{(1)}(x)), \quad i=1,2,\ldots,M \quad (32)$$

$$W_{i,j}^{(2)}(x,y,z) = W_{i,j}^{(2)}(x,0,z)\,e^{-\lambda^+(1-C(z))y}(1-R_i^{(2)}(y)), \quad i=1,2,\ldots,M, j=0,1 \quad (33)$$

$$Q_i(x,y,z) = Q_i(x,0,z)\,e^{-(\lambda^+(1-C(z)) + \omega_i)y}, \quad i=1,2,\ldots,M \quad (34)$$

Using equations (28) and (33), equation (29) becomes

$$Q_i(x,0,z) = (1-\tau_i)\,\alpha_i\,P_i(x,z)\,R_i^{(2)*}(h(z)), \quad i=1,2,\ldots,M \quad (35)$$

where $h(z) = \lambda^+ - \lambda^+ C(z)$

Inserting the expressions of $W_{i,0}^{(2)}(x,y,z)$ and $Q_i(x,y,z)$ in equation (18) and solving we get

$$P_i(x,z) = P_i(0,z)\,e^{-g_i(z)x}(1-B_i(x)), \quad i=1,2,\ldots,M \quad (36)$$

where $g_i(z) = h(z) + \lambda^- - \lambda^+\nu C(z) + \alpha_i - \alpha_i R_i^{(2)*}(h(z))\left(\frac{h(z)\tau_i + \omega_i}{h(z) + \omega_i}\right)$

Substituting the expression of $P_i(x,z)$ in equation (25), we get

$$\Pi_i(0,z) = \lambda^- P_i(0,z)\,\bar{B}_i^*(g_i(z)), i=1,2,\ldots,M \quad (37)$$

Inserting equation (37) in equation (31) and substituting the resultant expression of $\Pi_i(x,z)$ in equation (26) we obtain

$$W_i^{(1)}(0,z) = \lambda^- P_i(0,z)\, \bar{B}_i^*(g_i(z))\, S_i^*(h(z)),\qquad i=1,2,\ldots,M \qquad (38)$$

Using equations (32), (36) and (38) in equation (24) and simplifying we get

$$P_i(0,z)=\frac{p_i[\lambda^+ C(z)\, I_0 + I(0,z)\,(A^*(\lambda^+) + C(z)\,(1-A^*(\lambda^+)))]}{[z-\lambda^+\nu z C(z)\sum_{i=1}^{M} p_i \bar{B}_i^*(g_i(z)) - \theta\sum_{i=1}^{M} p_i[B_i^*(g_i(z)) + \lambda^- \bar{B}_i^*(g_i(z))\, S_i^*(h(z))\, R_i^{(1)*}(h(z))]]}$$

$$i=1,2,\ldots,M \qquad (39)$$

Substituting (39) in equation (36) we get the expression of $P_i(x, z)$ in terms of $I(0, z)$. Similarly, substituting equation (39) in equation (38) and inserting the resultant expression in equation (32), we get the expression of $W_i^{(1)}(x,z)$ in terms of $I(0, z)$.

Using the expressions of $P_i(x, z)$ and $W_i^{(1)}(x,z)$ in equation (23) and on simplifying, we obtain

$$I(0,z) = \lambda^+ I_0[(\theta+\bar{\theta}\, C(z))\sum_{i=1}^{M} p_i\,[B_i^*(g_i(z))+\lambda^-\bar{B}_i^*(g_i(z))\, S_i^*(h(z))\, R_i^{(1)*}(h(z))]+\lambda^+\nu z C(z)\sum_{i=1}^{M} p_i\, \bar{B}_i^*(g_i(z)) - z]\,/D(z) \qquad (40)$$

where

$$D(z)= z-\lambda^+\nu z C(z)\sum_{i=1}^{M} p_i\, \bar{B}_i^*(g_i(z)) - (\theta+\bar{\theta}\,(A^*(\lambda^+)+C(z)\,(1-A^*(\lambda^+))))\sum_{i=1}^{M} p_i\,[B_i^*(g_i(z)) + \lambda^-\bar{B}_i^*(g_i(z))\, S_i^*(h(z))\, R_i^{(1)*}(h(z))]$$

Using equation (40), equation (39) becomes

$$P_i(0,z)=I_0\, A^*(\lambda^+)\,\lambda^+\,(C(z)-1)\, p_i\,/\,D(z),\qquad i=1,2,\ldots,M \qquad (41)$$

Substituting the expression $P_i(x, z)$, the equations (27), (28), (35), (37) and (38) yield

$$\Pi_i(0,z)=I_0 A^*(\lambda^+)\,\lambda^+\lambda^-(C(z)-1)p_i\bar{B}_i^*(g_i(z))/D(z),\qquad i=1,2,\ldots,M \qquad (42)$$

$$W_i^{(1)}(0,z)=I_0 A^*(\lambda^+)\,\lambda^+\lambda^-(C(z)-1)\, p_i\,\bar{B}_i^*(g_i(z))\, S_i^*(h(z))\,/D(z),\qquad i=1,2,\ldots,M \qquad (43)$$

$$W_{i,0}^{(2)}(x,0,z) = I_0\, A^*(\lambda^+)\,\lambda^+\,(C(z)-1)\, p_i\,\tau_i\,\alpha_i\, e^{-g_i(z)x}\,(1-B_i(x))\,/\,D(z),\qquad i=1,2,\ldots,M \qquad (44)$$

$$W_{i,1}^{(2)}(x,0,z) = I_0\, A^*(\lambda^+)\,\lambda^+\,(C(z)-1)\, p_i\,(1-\tau_i)\,\alpha_i\, e^{-g_i(z)x}\,(1-B_i(x))\,/\,D(z),\qquad i=1,2,\ldots,M \qquad (45)$$

$$Q_i(x,0,z) = I_0\, A^*(\lambda^+)\,\lambda^+\,(C(z)-1)\, p_i\,(1-\tau_i)\alpha_i\, e^{-g_i(z)x}\,(1-B_i(x))\,R_i^{(2)*}(h(z))\,/D(z),\qquad i=1,2,\ldots,M \qquad (46)$$

By inserting the expressions of $I(0, z)$, $P_i(0, z)$, $\Pi_i(0, z)$ and $W_i^{(1)}(0,z)$ for i = 1, 2, ..., M, into the equations (30), (36), (31) and (32) and integrating with respect to x from 0 to ∞, we get respectively

$$I(z)= I_0(1-A^*(\lambda^+))\,[(\theta+\bar{\theta}\, C(z))\sum_{i=1}^{M} p_i\,[B_i^*(g_i(z)) + \lambda^-\bar{B}_i^*(g_i(z))\, S_i^*(h(z))\, R_i^{(1)*}(h(z))] - z + \lambda^+\nu z C(z)\sum_{i=1}^{M} p_i\,\bar{B}_i^*(g_i(z))]\,/\,D(z) \qquad (47)$$

$$P_i(z)=I_0 A^*(\lambda^+)\,\lambda^+(C(z)-1)p_i\bar{B}_i^*(g_i(z))\,/D(z),\qquad i=1,2,\ldots,M \qquad (48)$$

$$\Pi_i(z)= I_0 A^*(\lambda^+)\,\lambda^+\lambda^-(C(z)-1)p_i\bar{B}_i^*(g_i(z))\,\bar{S}_i^*(h(z))\,/\,D(z),\qquad i=1,2,\ldots,M \qquad (49)$$

$$W_i^{(1)}(z)= I_0 A^*(\lambda^+)\,\lambda^+\lambda^-(C(z)-1)p_i\,\bar{B}_i^*(g_i(z))\, S_i^*(h(z))\,\bar{R}_i^{(1)*}(h(z))/D(z),\qquad i=1,2,\ldots,M \qquad (50)$$

Using equations (44), (45) and (46) in equations (33) and (34) and integrating with respect to x and y from 0 to ∞, we get

$$W_{i,0}^{(2)}(z) = I_0\, A^*(\lambda^+)\,\lambda^+\,(C(z)-1)\, p_i\,\tau_i\,\alpha_i\,\bar{B}_i^*(g_i(z))\,\bar{R}_i^{(2)*}(h(z))\,/\,D(z),\qquad i=1,2,\ldots,M \qquad (51)$$

$$W_{i,1}^{(2)}(z) = I_0 A^*(\lambda^+)\,\lambda^+\,(C(z)-1)\, p_i\,(1-\tau_i)\,\alpha_i\,\bar{B}_i^*(g_i(z))\,\bar{R}_i^{(2)*}(h(z))\,/D(z),\qquad i=1,2,\ldots,M \qquad (52)$$

$$Q_i(z) = I_0 A^*(\lambda^+)\,\lambda^+\,(C(z)-1)\, p_i\,(1-\tau_i)\,\alpha_i\,\bar{B}_i^*(g_i(z))\, R_i^{(2)*}(h(z))\,/\,[(h(z)+\omega_i)\, D(z)],\qquad i=1,2,\ldots,M \qquad (53)$$

I_0 can be determined by using normalizing condition a

$$I_0=[\lambda^+\nu+\lambda^- - \lambda^+\nu\,(m_1+1)\sum_{i=1}^{M} p_i\,(1-B_i^*(\lambda^+\nu+\lambda^-))$$

$$- \lambda^+ m_1 \sum_{i=1}^{M} p_i T_i (1 - B_i^*(\lambda^+ \nu + \lambda^-))$$

$$- (1-\theta) m_1 (1 - A^*(\lambda^+)) \sum_{i=1}^{M} p_i (\lambda^- + \lambda^+ \nu B_i^*(\lambda^+ \nu + \lambda^-))$$

$$- \lambda^+ \lambda^- m_1 \sum_{i=1}^{M} p_i (1 - B_i^*(\lambda^+ \nu + \lambda^-)) (\phi_{i,1}^{(1)} + \beta_{i,1}^{(1)})] /$$

$$[A^*(\lambda^+)(\lambda^- + \lambda^+ \nu \sum_{i=1}^{M} p_i B_i^*(\lambda^+ \nu + \lambda^-))] \quad (54)$$

where $T_i = 1 + \alpha_i (\beta_{i,1}^{(2)} + \frac{1-\tau_i}{\omega_i})$

Result 1

The probability generating function for the number of customers in the retrial queue is

$$P_q(z) = I_0 + I(z) + \sum_{i=1}^{M} (P_i(z) + \Pi_i(z) + W_i^{(1)}(z) + W_{i,0}^{(2)}(z) + W_{i,1}^{(2)}(z) + Q_i(z))$$

$$= I_0 A^*(\lambda^+)(z-1)(1 - \lambda^+ \nu C(z) \sum_{i=1}^{M} p_i \bar{B}_i^*(g_i(z))) / D(z) \quad (55)$$

Result 2

The probability generating function for the number of customers in the system is

$$P_S(z) = I_0 + I(z) + \sum_{i=1}^{M} [z [P_i(z) + W_{i,0}^{(2)}(z) + W_{i,1}^{(2)}(z) + Q_i(z)] + \Pi_i(z) + W_i^{(1)}(z)]$$

$$= I_0 A^*(\lambda^+)(z-1) \sum_{i=1}^{M} p_i (B_i^*(g_i(z)) + \lambda^- \bar{B}_i^*(g_i(z))) / D(z) \quad (56)$$

V PERFORMANCE MEASURES

In this section the performance measures for the system under consideration are derived.

- Probability that the server is idle in non-empty system is

$$I = \lim_{z \to 1} I(z) =$$

$$(1 - A^*(\lambda^+)) [-(\lambda^+ \nu + \lambda^-) + \lambda^+ \nu (m_1 + 1) \sum_{i=1}^{M} p_i (1 - B_i^*(\lambda^+ \nu + \lambda^-))$$

$$+ \lambda^+ m_1 \sum_{i=1}^{M} p_i T_i (1 - B_i^*(\lambda^+ \nu + \lambda^-))$$

$$+ (1-\theta) m_1 \sum_{i=1}^{M} p_i (\lambda^- + \lambda^+ \nu B_i^*(\lambda^+ \nu + \lambda^-))$$

$$+ \lambda^+ \lambda^- m_1 \sum_{i=1}^{M} p_i (1 - B_i^*(\lambda^+ \nu + \lambda^-)) (\phi_{i,1} + \beta_{i,1}^{(1)})] /$$

$$[A^*(\lambda^+)(\lambda^- + \lambda^+ \nu \sum_{i=1}^{M} p_i B_i^*(\lambda^+ \nu + \lambda^-))] \quad (57)$$

- Probability that the server is busy is

$$P = \lim_{z \to 1} \sum_{i=1}^{M} P_i(z) = \frac{\lambda^+ m_1 \sum_{i=1}^{M} p_i (1 - B_i^*(\lambda^+ \nu + \lambda^-))}{(\lambda^- + \lambda^+ \nu \sum_{i=1}^{M} p_i B_i^*(\lambda^+ \nu + \lambda^-))} \quad (58)$$

- Probability that the server is in failure state is

$$F = R_1 + R_2 = \frac{\lambda^+ m_1 \sum_{i=1}^{M} p_i (1 - B_i^*(\lambda^+ \nu + \lambda^-)) (\lambda^- (\phi_{i,1} + \beta_{i,1}^{(1)}) + \alpha_i \beta_{i,1}^{(2)})}{\lambda^- + \lambda^+ \nu \sum_{i=1}^{M} p_i B_i^*(\lambda^+ \nu + \lambda^-)} \quad (59)$$

- Probability that the server is under reserved time is

$$Q = \lim_{z \to 1} \sum_{i=1}^{M} Q_i(z) = \frac{\lambda^+ m_1 \sum_{i=1}^{M} p_i (1 - B_i^*(\lambda^+ \nu + \lambda^-)) ((1-\tau_i) \alpha_i / \omega_i)}{(\lambda^- + \lambda^+ \nu \sum_{i=1}^{M} p_i B_i^*(\lambda^+ \nu + \lambda^-))} \quad (60)$$

- Mean queue length of the retrial orbit L_q is given by

$$L_q = \lim_{z \to 1} \frac{d}{dz} P_q(z) = \frac{D'(1) Nr''(1) - Nr'(1) D''(1)}{2 D'(1)^2} \quad (61)$$

where $Nr(z)$ denotes the numerator of $P_q(z)$

$$Nr'(1) = I_0 A^*(\lambda^+)(1 - \lambda^+ \nu \sum_{i=1}^{M} p_i \bar{B}_i^*(\lambda^+ \nu + \lambda^-))$$

$$Nr''(1) = -2 I_0 A^*(\lambda^+) \lambda^+ \nu \left(\frac{m_1 \sum_{i=1}^{M} p_i (1 - B_i^*(\lambda^+ \nu + \lambda^-))}{(\lambda^+ \nu + \lambda^-)} \right.$$

$$\left. - \frac{\lambda^+ m_1 \sum_{i=1}^{M} p_i T_i}{(\lambda^+ \nu + \lambda^-)^2} (1 - B_i^*(\lambda^+ \nu + \lambda^-) - f_{i,1} (\lambda^+ \nu + \lambda^-)) \right)$$

$$D'(1) = [\lambda^+ \nu + \lambda^- - \sum_{i=1}^{M} p_i (1 - B_i^*(\lambda^+ \nu + \lambda^-))[\lambda^+ \nu (m_1 + 1) + \lambda^+ m_1 T_i + \lambda^+ \lambda^- m_1 (\phi_{i,1} + \beta_{i,1}^{(1)})] - (1-\theta) m_1 (1 - A^*(\lambda^+)) \sum_{i=1}^{M} p_i (\lambda^+ \nu B_i^*(\lambda^+ \nu + \lambda^-) + \lambda^-)] / (\lambda^+ \nu + \lambda^-)$$

$$D''(1) =$$

$$- \lambda^+ \nu [(2 m_1 + m_2) \sum_{i=1}^{M} p_i \bar{B}_i^*(\lambda^+ \nu + \lambda^-) + 2 (m_1 + 1) \sum_{i=1}^{M} p_i K_{i,5}$$

$+ \sum_{i=1}^{M} p_i K_{i,6}] - L_1 - 2(1-\theta) m_1 (1 - A^*(\lambda^+)) L_2$

$-(1-\theta) m_1 (1 - A^*(\lambda^+)) \sum_{i=1}^{M} p_i (\lambda^+ \nu B_i^*(\lambda^+ \nu + \lambda^-) + \lambda^-$

$/ [\lambda^+ \nu + \lambda^-]$

$L_1 = \sum_{i=1}^{M} p_i (K_{i,4} + \lambda^- K_{i,6}$

$+ \dfrac{2\lambda^{+^2} \lambda^- m_1^2 T_i (\phi_{i,1} + \beta_{i,1}^{(1)})(1 - B_i^*(\lambda^+ \nu + \lambda^-) - (\lambda^+ \nu + \lambda^-) f_{i,1})}{(\lambda^+ \nu + \lambda^-)^2}$

$+ \dfrac{\lambda^- (1 - B_i^*(\lambda^+ \nu + \lambda^-))}{(\lambda^+ \nu + \lambda^-)} (\lambda^{+^2} m_1^2 (\phi_{i,2} + \beta_{i,2}^{(1)}) +$

$\lambda^+ m_2 (\phi_{i,1} + \beta_{i,1}^{(1)})))$

$L_2 = \sum_{i=1}^{M} p_i (K_{i,3} + \lambda^- K_{i,5} +$

$\dfrac{\lambda^+ \lambda^- m_1 (1 - B_i^*(\lambda^+ \nu + \lambda^-))(\phi_{i,1} + \beta_{i,1}^{(1)}))}{(\lambda^+ \nu + \lambda^-)}$

$K_{i,1} = -\lambda^+ m_1 T_i$

$K_{i,2} = -(\lambda^+ m_2 (1-\nu) + \alpha_i (\lambda^{+^2} m_1^2 \beta_{i,2}^{(2)} + \lambda^+ m_2 \beta_{i,1}^{(2)} + 2\lambda^{+^2} m_1^2 \beta_{i,1}^{(2)} \dfrac{(1-\tau_i)}{\omega_i} + \dfrac{(1-\tau_i)}{\omega_i^2}$

$(\lambda^+ m_2 \omega_i + 2\lambda^{+^2} m_1^2)))$

$K_{i,3} = -K_{i,1} f_{i,1}$

$K_{i,4} = \lambda^{+^2} m_1^2 T_i^2 f_{i,2} + f_{i,1}(-K_{i,2})$

$K_{i,5} = \dfrac{\lambda^+ m_1 T_i}{(\lambda^+ \nu + \lambda^-)^2} (1 - B_i^*(\lambda^+ \nu + \lambda^-) - f_{i,1}(\lambda^+ \nu + \lambda^-))$

$K_{i,6} = \dfrac{2\lambda^{+^2} m_1^2 T_i^2 (1 - B_i^*(\lambda^+ \nu + \lambda^-) - f_{i,1}(\lambda^+ \nu + \lambda^-))}{(\lambda^+ \nu + \lambda^-)^3}$

$- \dfrac{K_{i,4}}{(\lambda^+ \nu + \lambda^-)} - \dfrac{K_{i,2}(1 - B_i^*(\lambda^+ \nu + \lambda^-)}{(\lambda^+ \nu + \lambda^-)^2}$

$f_{i,1} = \int_0^\infty x\, e^{-(\lambda^+ \nu + \lambda^-)x} b_i(x)\, dx$

$f_{i,2} = \int_0^\infty x^2\, e^{-(\lambda^+ \nu + \lambda^-)x} b_i(x)\, dx$

- Mean queue length in the system L_S is given by

$$L_S = \lim_{z \to 1} \frac{d}{dz} P_S(z)$$

$$= L_q + P + R_2 + Q \qquad (62)$$

VI RELIABILITY MEASURES

Theorem

The steady state availability ($\mathcal{A}$) and failure frequency ($\mathcal{F}$) of the server are

$\mathcal{A} = [I_0 + A^*(\lambda^+)[\lambda^+ \nu + \lambda^- - \lambda^+ \sum_{i=1}^{M} p_i (1 - B_i^*(\lambda^+ \nu + \lambda^-))$

$[m_1 (T_i - 1 + \nu) + \nu + \lambda^- m_1 (\phi_{i,1} + \beta_{i,1}^{(1)})]]] / D'(1)$ (63)

$\mathcal{F} = I_0 A^*(\lambda^+) \lambda^+ m_1 \sum_{i=1}^{M} p_i (1 - B_i^*(\lambda^+ \nu + \lambda^-))(\lambda^- + \alpha_i) /$

$[(\lambda^+ \nu + \lambda^-) D'(1)]$ (64)

Proof

The availability and failure frequency for the system are obtained using the expressions

$\mathcal{A} = I_0 + I + P$

$\mathcal{F} = \lambda^- P + \sum_{i=1}^{M} P_i(1) \alpha_i$

VII STOCHASTIC DECOMPOSITION

Theorem

The number of customers in the system (L_S) can be expressed as the sum of two independent random variables, one of which is the mean number of customers in the batch arrival G-queue with fluctuating modes of service, priority, server breakdown with delayed repair and orbital search (L) and the other is the mean number of customers in the orbit given that the server is idle (L_C).

Proof

The probability generating function $\pi(z)$ of the number of customers in the batch arrival G-queue with fluctuating modes of service, priority, server breakdown with delayed repair and orbital search is

$\pi(z) = I_1 (z-1) \sum_{i=1}^{M} p_i (B_i^*(g_i(z)) + \lambda^- \bar{B}_i^*(g_i(z))) / D_1(z)$ (65)

where

$D_1(z) =$

$z - \lambda^+ \nu z C(z) \sum_{i=1}^{M} p_i \bar{B}_i^*(g_i(z)) - \sum_{i=1}^{M} p_i [B_i^*(g_i(z))$

$+ \lambda^- \bar{B}_i^*(g_i(z)) S_i^*(h(z)) R_i^{(1)*}(h(z))]$

$I_1 = [\lambda^+ \nu + \lambda^- - \lambda^+ \nu (m_1 + 1) \sum_{i=1}^{M} p_i (1 - B_i^*(\lambda^+ \nu + \lambda^-))$

$- \lambda^+ m_1 \sum_{i=1}^{M} p_i T_i (1 - B_i^*(\lambda^+ \nu + \lambda^-))$

$- \lambda^+ \lambda^- m_1 \sum_{i=1}^{M} p_i (1 - B_i^*(\lambda^+ \nu + \lambda^-))(\phi_{i,1} + \beta_{i,1}^{(1)})]$

$/[\lambda^- + \lambda^+ \nu \sum_{i=1}^{M} p_i B_i^*(\lambda^+ \nu + \lambda^-)]$

The probability generating function $\psi(z)$ of the number of customers in the orbit given that the server is idle is

$$\psi(z) = \frac{I_0 + I(z)}{I_0 + I(1)}$$

$$=[[z-\lambda^{+}\nu z C(z)\sum_{i=1}^{M} p_i \bar{B}_i^{*}(g_i(z)) - \sum_{i=1}^{M} p_i (B_i^{*}(g_i(z))$$

$$+\lambda^{-}\bar{B}_i^{*}(g_i(z)) S_i^{*}(h(z)) R_i^{(1)*}(h(z)))] (\lambda^{+}\nu+\lambda^{-})D'(1)]/$$

$$[[\lambda^{+}\nu + \lambda^{-} - \lambda^{+}\sum_{i=1}^{M} p_i (1 - \bar{B}_i^{*}(\lambda^{+}\nu + \lambda^{-}))$$

$$(\nu(m_1+1)+m_1T_i+\lambda^{-}m_1(\phi_{i,1}+\beta_{i,1}^{(1)})] D(z)] \qquad (66)$$

From equations (56), (65) and (66) we observe that

$$P_S(z) = \pi(z)\,\psi(z) \qquad (67)$$

$$L_S = \lim_{z\to 1}\frac{d}{dz} P_S(z) = L + L_C \qquad (68)$$

VIII SPECIAL CASES

Case (i) If $\lambda^{-} = 0$ (no negative customer) then the model under study becomes batch arrival retrial queue with fluctuating modes of service, priority, server breakdown, reserved time and orbital search.

Case (ii) If $M = 1$ and $\alpha_i = 0$ (single type service and no active breakdown) then the results coincide with the results of Sumitha and Udaya Chandrika [19] without collisions and balking .

IX NUMERICAL RESULTS

In this section, numerical results are presented to study the effect of various parameters in the system performance measures. The arbitrary values to the parameters are chosen as follows : $M = 2$, $p_1 = 0.7$, $p_2 = 0.3$, $\lambda^{+} = 0.4$, $\lambda^{-} = 0.2$, $\eta = 8$, $\mu_1 = 25$, $\mu_2 = 24$, $\nu = 0.4$, $\phi_1 = 8$, $\phi_2 = 7$, $\beta_1^{(1)} = 6$, $\beta_2^{(1)} = 5$, $\alpha_1 = 5$, $\alpha_2 = 4$, $\beta_1^{(2)} = 5$, $\beta_2^{(2)} = 4$, $\tau_1 = 0.6$, $\tau_2 = 0.5$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\theta = 0.4$, $m_1 = 1.5$ and $m_2 = 1.0$.

Table 1 to 5 give the computed values of various characteristics of our model like I_0 – the probability that the system is empty, I - the probability that the server is idle in non-empty system, P – the probability that the server is busy, F – the probability that the server is under repair and L_S – the mean system size by varying the rates of η, θ, μ_2, ν and α_1. From the tables it is clear that

- I_0 increases with increase in η, θ and μ_2 but decreases with increase in ν and α_1.
- I increases with increase in α_1 and ν and decreases with increase in η, θ and μ_2.
- P decreases with increase in μ_2 and has no effect with increase in η, θ, ν and α_1.
- F increases with increase in α_1, decreases with increase in μ_2 and is independent of η, θ and ν.
- L_S decreases with increase in η, θ and μ_2 and increases with increase in α_1 and ν.

Table 1 Performance Measures by varying η

η	I_0	I	P	F	L_S
7	0.8258	0.0040	0.0241	0.0256	0.5826
9	0.8267	0.0031	0.0241	0.0256	0.5669
11	0.8272	0.0026	0.0241	0.0256	0.5569
13	0.8276	0.0022	0.0241	0.0256	0.5500
15	0.8279	0.0019	0.0241	0.0256	0.5450

Table 2 Performance Measures by varying θ

θ	I_0	I	P	F	L_S
0.15	0.8075	0.0223	0.0241	0.0256	0.6014
0.20	0.8113	0.0185	0.0241	0.0256	0.5957
0.25	0.8150	0.0148	0.0241	0.0256	0.5902
0.30	0.8188	0.0110	0.0241	0.0256	0.5846
0.35	0.8225	0.0073	0.0241	0.0256	0.5792

Table 3 Performance Measures by varying μ_2

μ_2	I_0	I	P	F	L_S
22	0.8213	0.0037	0.0248	0.0263	0.5930
24	0.8263	0.0035	0.0241	0.0256	0.5737
26	0.8305	0.0033	0.0235	0.0250	0.5577
28	0.8342	0.0031	0.0230	0.0245	0.5442
30	0.8373	0.0030	0.0226	0.0240	0.5325

Table 4 Performance Measures by varying ν

ν	I_0	I	P	F	L_S
0.4	0.8263	0.0035	0.0241	0.0256	0.5659
0.5	0.8257	0.0041	0.0241	0.0256	0.5673
0.6	0.8243	0.0055	0.0241	0.0256	0.5690
0.7	0.8220	0.0078	0.0241	0.0256	0.5711
0.8	0.8010	0.0088	0.0241	0.0256	0.5737

Table 5 Performance Measures by varying α_1

α_1	I_0	I	P	F	L_S
4	0.8473	0.0025	0.0241	0.0223	0.4909
5	0.8263	0.0034	0.0241	0.0256	0.5737
6	0.8053	0.0045	0.0241	0.0289	0.6613
7	0.7843	0.0055	0.0241	0.0323	0.7540
8	0.7633	0.0065	0.0241	0.0356	0.8523

X CONCLUSIONS

This paper deals with steady state analysis for the batch arrival retrial G-queue with M modes of service, priority, active breakdown, delayed repair and orbital search. Probability generating functions of the number of customers in the orbit and in the system are obtained. Some important system characteristics are derived. Finally, an illustrative numerical example is presented.

References

[1] Aissani, A. (2010). An M/G/1 Retrial Queue with Negative Arrivals and Unreliable Server, Proceedings of the World Congress on Engineering, Vol. 1, June 30-July 2, London, U.K.

[2] Artalejo, J.R., Joshua, V.C. and Krishnamoorthy, A. (2002). An M/G/1 Retrial Queue with Orbital Search by the Server, Advances in Stochastic Modelling, 41-54.

[3] Baruah, M., Madan, K.C. and Eldabi, T. (2014). A Batch Arrival Single Server Queue with Server Providing General Service in Two Fluctuating Modes and Reneging During Vacation and Breakdowns, Journal of Probability and Statistics, 2, 1-12.

[4] Deepak, T.G., Dudin, A.N., Joshua, G.C. and Krishnamoorthy, A. (2013). On an $M^{[X]}/G/1$ Retrial System with Two Types of Search of Customers from the Orbit, Stochastic Analysis and Applications, 31 (1), 92-107.

[5] Dudin, A.N., Krishnamoorthy, A., Joshua, V.C. and Tsarenkov, G.V. (2004). Analysis of the BMAP/G/1 Retrial System with Search of Customers from the Orbit, European Journal of Operational Research, 157 (1), 169-169.

[6] Gao, S. and Wang, J. (2014). Performance and Reliability Analysis of an M/G/1-G Retrial Queue with Orbital Search and Non-Persistent Customers, European Journal of Operational Research, 236 (2), 561-572.

[7] Gelenbe, E. (1989). Random Neural Networks with Negative and Positive Signals and Product Form Solution, Neural Computation, 1, 502-510.

[8] Gelenbe, E. (1991). Product-form Queueing Networks with Negative and Positive Customers, Journal of Applied Probability, 28, 656-663.

[9] Gelenbe, E. (1994). G-Networks : A Unifying Model for Neural and Queueing Networks, Annals of Operations Research, 48, 433-461.

[10] Gelenbe, E. (2000). The First Decade of G-Networks, European Journal of Operation Research, 126, 231-232.

[11] Liu, Z., Wu, J. and Yang, G. (2009). An M/G/1 Retrial G-Queue with Preemptive Resume and Feedback under N-Policy Subject to Server Breakdowns and Repairs, Computers and Mathematics with Applications, 58 (9), 1792-1807.

[12] Madan, K.C. (2014). On a Single Server Queue with Arrivals in Batches of Variable Size, General Service in Three Fluctuating Modes, Balking, Random Breakdowns and a Stand-by Server During Breakdown Periods, Revista Investiacion Operacional, 35 (3), 189-200.

[13] Neuts, M.F. and Ramalhoto M.F. (1984). A Service Model in which the Server is Required to Search for Customers, Journal of Applied Probability, 21 (1), 157-166.

[14] Peng, Y., Liu, Z. and Wu, J. (2014). An M/G/1 Retrial G-Queue with Preemptive Resume Priority and Collisions Subject to the Server Breakdowns and Delayed Repairs, Journal of Applied Mathematics and Computing, 44 (1), 187-213.

[15] Rajadurai, P., Chandrasekaran, V.M. and Saravanarajan, M.C. (2015). Analysis of an $M^{[X]}/G/1$ Unreliable G-queue with Orbital Search and Feedback under Bernoulli Vacation Schedule, OPSEARCH, 53(1), 197-223.

[16] Rajadurai, P., Saravanarajan, M.C. and Chandrasekaran, V.M. (2015). Analysis of Repairable $M^{[X]}/(G_1, G_2)/1$-Feedback Retrial G-Queue with Balking and Starting Failures under atmost J Vacations, Journal of Applications and Applied Mathematics,10(1),13-39.

[17] Sumitha, D. and Udaya Chandrika, K. (2011), Numerical Analysis of Two Phase Bulk Arrival Retrial Queue with Orbital Search, International Journal of Advanced Scientific and Technical Research, 2 (1) : 483-493.

[18] Sumitha, D. and Udaya Chandrika, K. (2012). Performance Analysis of Repairable M/G/1 Retrial Queue with Bernoulli Vacation and Orbital Search, International Journal of Mathematical Archive, 3 (2), 412-419.

[19] Sumitha, D. and Udaya Chandrika, K. (2015). Batch Arrival Retrial Queue with Positive and Negative Customers, Priority or Collisions, Delayed Repair and Orbital Search, Elixir International Journal : Applied Mathematics, 88, 36182-36189.