Polya - Aeppli Reliability Model of a System

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Abstract

Reliability is the probability of failure - free operation of the system for a specified period of time in a specified environment. Testing reliability of the system is essential to discover the failures and also to remove the fault detections before the system is deployed. In this paper the testing phase of system development is considered and the faults are removed whenever they are detected. The number of fault detection points follows Polya-Aeppli distribution. The test phase time and fault removal time are assumed to follow various distributions. Using these distributions the expected release time of the system and the expected total cost of the system are determined.

keywords: *System reliability, Poisson Process, Geometric Process, Exponential distribution, Polya-Aeppli Distribution, Geometric Distribution, Weibull Distribution , Expected release time of the system , Optimal release time of the system , Expected total cost and Optimal total cost of the system .*

I. INTRODUCTION

System reliability is the probability of failure free operation of the system for a specified period of time in a specified environment. Testing the reliability of any system is essential to fix the fault detections and to remove the detections before it is released in the market. Also this helps to upgrade the system according to the requirements of its users.

The main application of system reliability model is the determination of system release time. Suppose the system is released quickly, the customer experience many faults and suppose the system is released late the system agencies spent more money on delay.

There are many models for system reliability namely Shock models ,Maintenance models and Reliability growth models etc. Esary, Marshall and Proschan[1973] have studied some models for the life distribution of a device that is subject to shocks governed by a Poisson process . A-Hameed and Proschan[1973] have extended the work of Esary,Marshall and Proschan to the case of shocks governed by a non-homogeneous Poission process.

Klefsjo [1981] has considered the pure birth shock model of A-Hameed and Proschan. The survival function has been proved to be IFRA(Increasing Failure Rate Average) and DMRL (Decreasing Mean Residual life) under conditions different from those used by A-Hameed and Proschan . Savits [1988] has considered a system withn components subject to a non-homogeneous Poisson process with continuous mean function. Pellerey[1994] has considered a device subject to shocks governed by a counting process. Ozekici[1995] has used an ageing model for replacement and repair-replacement problems. Lam[1995] has studied an optimal maintenance model for standby systems.

In software reliability model, the optimal release time and the interval estimation of the release time prediction are derived by Xie and Hong in 1999.Again Hoang Pham and Xuemei Zhang in 2003 established a new reliability model addressing the testing coverage time and cost . A software cost model has been developed incorporating the testing cost, fault removal cost and risk cost due to potential problems remaining in the uncovered code.

The Polya-Aeppli distribution is a model that counts the objects that occur in clusters. The number of objects per cluster come from the geometric distribution with probability of successes $1-\rho$, where $0\leq\rho<1$, and the number of clusters follows the Poisson distribution with mean λ (Johnson et al. (2005)). This distribution was first studied by Aeppli in a thesis, followed by Polya five years later. Anscombe (1950) subsequently finalized the derivation of this distribution and gave the name Polya-Aeppli distribution (Minkova (2012)). The probability mass function (PMF) of a Polya-Aeppli random variable N is

$$e^{-\lambda}\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{\left(\lambda(1-\rho)\right)^{i}}{i!}\rho^{k-i}$$

In this paper, the expected system release time is determined using the shock model approach. This paper is organised as follows. In section 2, the expected release time of system for various models are derived. In section 3, the expected total cost of the system are derived. Conclusion is given in Section 4.

Notation

X_i –Test phase time after the i^{th} fault removal and before $(i+1)^{th}$ fault detection .

Y_i – removal time of fault after i^{th} fault detection.

N(t)- Number of fault detections.

W – Release time of the system .

II. EXPECTED RELEASE TIME OF THE SYSTEM:

A. *Model formulation*

Consider the testing phase of system development , the faults are removed whenever they are detected. This removal reduces the total number of faults in the system. The length of time intervals between fault detection should increase and fault removal time should decrease. When the fault detection rate reaches an acceptably low level i.e., it tends to zero, the system can be considered suitable for the customer.

B. *Assumptions*

Let X_i be the test phase time after the i^{th} fault removal and before $(i+1)^{th}$ fault detection. Assume $\{X_i, i = 1,2, \dots n\}$ be stochastically increasing geometric process. If $F(t) = \Pr(X_1 \le t)$ is the distribution function of X_1 with mean μ_1 then the distribution function of X_n is $F(a^{n-1}t) = \Pr(X_n \le t), a \le 1$ with mean $\frac{\mu_1}{a^{n-1}}$. Let Y_i be the removal time of fault after i^{th} fault detection. Assume that $\{Y_i, i = 1,2, \dots n\}$ form a monotonically decreasing geometric process. If $G(t) = \Pr(Y_1 \le t)$ then the distribution function of Y_n is $G(b^{n-1}t) = \Pr(Y_n \le t), b \le 1$ with mean $\frac{\mu_2}{b^{n-1}}$. Also assume that X_i and Y_i are independent. Let N(t) be the random variable denoting the number of fault detections.

The release time of the system is

$$W = X_0 + \sum_{i=1}^{N}(X_i + Y_i)$$

where X_0 is test phase time before the first fault detection .

The expected release time of the system is

$$E(W) = E(X_0) + \sum_{i=1}^{N} E(X_i + Y_i)$$

$$= E(X_0)\, P(N(t)=0) + [\sum_{i=1}^{n} \{E(X_i) + E(Y_i)\}]\; P(N(t) = k)$$

$$= \mu_1\, P(N(t)=0) + [(\mu_1+\mu_2) + \{(\frac{\mu_1}{a} + \frac{\mu_2}{b}) + (\frac{\mu_1}{a^2} + \frac{\mu_2}{b^2}) + \ldots(\frac{\mu_1}{a^{k-1}} + \frac{\mu_2}{b^{k-1}})\}\; P(N(t) = k)$$

Model 1

Let the test phase time of the system X_1 follows exponential distribution with parameter A, the fault removal time Y_1 follows exponential distribution with parameter B and the number of faults detections N(t) follows Polya-Aeppli distribution with parameter λ , ρ.

where $E(X_1) = \frac{1}{A}$,A>0 ; $E(Y_1) = \frac{1}{B}, B > 0$; $E(N) = (\frac{\lambda}{(1-\rho)})$

then $E(W) = \frac{1}{A}e^{-\lambda} + \{e^{-\lambda}\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho))^i}{i!}\rho^{k-i}\}\{\sum_{i=1}^{k}(\frac{1}{Aa^{i-1}} + \frac{1}{Bb^{i-1}})\}$

k =0,1,2,…

When $\rho = 0$, it becomes

$$E(W) = \frac{1}{A}e^{-\lambda}$$

Model 2

Let the testing phase of the system X_1 follows exponential distribution with parameter A , the fault removal time Y_1 follows Weibull distribution with parameter B,β and the number of faults detections N follows Polya-Aeppli distribution with parameter λ , ρ.

where $E(X_1) = \frac{1}{A}, A > 0$; ; $E(Y_1) = \frac{1}{B}, B > 0$; ; $E(N) = (\frac{\lambda}{(1-\rho)})$

then $E(W) = \frac{e^{-\lambda}}{A} + [\sum_{i=1}^{k}(\frac{1}{Aa^{i-1}} + \frac{B\Gamma(\frac{1}{\beta}+1)}{b^{i-1}})]$ k=0,1,2,…

When ρ=0 , it becomes $E(W) = \frac{e^{-\lambda}}{A}$

Model 3

Let the testing phase of the system X_1 follows Weibull distribution with parameter A ,α , the fault time removal Y_1 follows Exponential distribution with parameter B,β and the number of faults detections N follows Polya-Aeppli distribution with parameter λ , ρ.

where $E(X_1) = A\,\Gamma(\frac{1}{\alpha}+1)$, $A > 0$; $E(Y_1) = \frac{1}{B}, B > 0$; and $E(N(t) = (\frac{\lambda}{(1-\rho)})$

$$E(W) = A\,\Gamma(\frac{1}{\alpha}+1)e^{-\lambda} + \left[\sum_{i=1}^{k}\left(\frac{A\Gamma\left(\frac{1}{\alpha}+1\right)}{a^{i-1}} + \frac{1}{Bb^{i-1}}\right)\right]\left\{e^{-\lambda}\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho))^{i}}{i!}\rho^{k-i}\right\}$$

When $\rho=0$, $E(W) = A\,\Gamma(\frac{1}{\alpha}+1)e^{-\lambda}$

Model 4

Let the test phase of thesystem X_1 follows Weibull distribution with parameter A,α , the fault removal time Y_1 follows Weibull distribution with parameter B, β and the number of faults detections N follows Polya-Aeppli distribution with parameter λ , ρ

where $E(X_1) = A\Gamma\left(\frac{1}{\alpha}+1\right)$, $A > 0$; $E(Y_1) = \frac{1}{B}$, B>0; and $E(N(t) = \left(\frac{\lambda}{(1-\rho)}\right)$

$$E(W) = A\,\Gamma(\frac{1}{\alpha}+1)e^{-\lambda} + \left[\sum_{i=1}^{k}\left(\frac{A\Gamma\left(\frac{1}{\alpha}+1\right)}{a^{i-1}} + \frac{B\Gamma\left(\frac{1}{\beta}+1\right)}{b^{i-1}}\right)\right]\left\{e^{-\lambda}\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho))^{i}}{i!}\rho^{k-i}\right\}$$

When $\rho=0$, $E(W) = A\,\Gamma(\frac{1}{\alpha}+1)e^{-\lambda}$

III. EXPECTED TOTAL COST

It is essential to determine when the testing phase of the system should be stopped so that the expected cost is minimized and the reliability of the software product satisfies customer's requirements as well.

In this section Analytical expression of the total cost of all the models are derived .

A. Assumptions

(i) The cost to perform testing is proportional to the testing time.
(ii) Expected testing cost is $E_1(TC)$.
(iii) The cost of removing faults is proportional to the fault removal time.
(iv) Expected fault removal cost is $E_2(TC)$.
(v) There is a risk cost . In Exploration system,for cost and performance,while ensuring that the risks to the mission and crew were acceptable .
(vi) Expected risk cost $E_3(TC)$.
(vii) The risk cost is of the form

$$E_3(TC)=C_3(1+ct)e^{-ct}M$$

The expected total cost E(TC) is

$$E(TC)=E_1(TC)+E_2(TC)+E_3(TC)$$

B. Expected Total Cost

Cost Model 1

Let the test phase time X_1 follows exponential distribution with parameter A.

Then $E(X_1)=\frac{1}{A}$, A>0; $E_1(TC)=C_1\sum_{i=1}^{k}\frac{1}{Aa^{i-1}}\, e^{-\lambda}\left\{\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho))^i}{i!}\rho^{k-i}\right\}$

Let the fault removal time Y_1 follows exponential distribution with parameter B

then $E(Y_1)=\frac{1}{B}$, B>0; $E_2(TC)=C_2\sum_{i=1}^{k}\frac{1}{Bb^{i-1}}\, e^{-\lambda}\left\{\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho))^i}{i!}\rho^{k-i}\right\}$

Then the expected total cost is ,

$$E(TC)= C_1\sum_{i=1}^{k}\frac{1}{Aa^{i-1}}\, e^{-\lambda}\left\{\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho))^i}{i!}\rho^{k-i}\right\}$$

$$+\ C_2\sum_{i=1}^{k}\frac{1}{Bb^{i-1}}\, e^{-\lambda t}\left\{\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho)t)^i}{i!}\rho^{k-i}\right\}$$

$$+\ C_3(1+ct)e^{-ct}M$$

Cost Model 2

Let the test phase time X_1 follows exponential distribution with parameter A. Then $E(X_1)=\frac{1}{A}$, A>0;

$$E_1(TC)=C_1\sum_{i=1}^{k}\frac{1}{Aa^{i-1}}\left\{e^{-\lambda t}\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho)t)^i}{i!}\rho^{k-i}\right\}$$

Let fault removal time Y_1 follows Weibull distribution with parameter B, β . Then

$E(Y_1) = B\Gamma\left(\frac{1}{\beta}+1\right), B > 0.$

$$E_2(TC)=C_2\sum_{i=1}^{k}\left(\frac{B\,\Gamma(\frac{1}{\beta}+1)}{b^{\,i-1}}\right)\left\{e^{-\lambda t}\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho)t)^i}{i!}\rho^{k-i}\right\}$$

$$E_3(TC)=C_3(1+ct)e^{-ct}M$$

Then the expected total cost is ,

$$E(TC) = C_1\sum_{i=1}^{k}\frac{1}{Aa^{\,i-1}}\left\{e^{-\lambda t}\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho)t)^i}{i!}\rho^{k-i}\right\}$$

$$+ C_2\sum_{i=1}^{k}\left(\frac{B\,\Gamma(\frac{1}{\beta}+1)}{b^{\,i-1}}\right)\left\{e^{-\lambda t}\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho)t)^i}{i!}\rho^{k-i}\right\} + C_3(1+ct)e^{-ct}M$$

Cost Model 3

Let test phase time X_1 follows Weibull distribution with parameter A, α . Then $E(X_1) = A\Gamma\left(\frac{1}{\alpha}+1\right), A > 0.$

$$E_1(TC) = C_1\sum_{i=1}^{k}\left(\frac{A\,\Gamma(\frac{1}{\alpha}+1)}{a^{\,i-1}}\right)\left\{e^{-\lambda t}\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho)t)^i}{i!}\rho^{k-i}\right\}$$

Let fault removal time Y_1 follows exponential distribution with parameter B.

Then $E(Y_1) = \frac{1}{B}$, B>0 .

$$E_2(TC) = C_2\sum_{i=1}^{k}\left(\frac{1}{Bb^{\,i-1}}\right)\left\{e^{-\lambda t}\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho)t)^i}{i!}\rho^{k-i}\right\}$$

Then the expected total cost is ,

$$E(TC) = C_1\sum_{i=1}^{k}\left(\frac{A\,\Gamma(\frac{1}{\alpha}+1)}{a^{\,i-1}}\right)\left\{e^{-\lambda t}\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho)t)^i}{i!}\rho^{k-i}\right\}$$

$$+ C_2 \sum_{i=1}^{k} \left(\frac{1}{Bb^{i-1}} \right) \left\{ e^{-\lambda t} \sum_{i=1}^{k} \binom{k-1}{i-1} \frac{(\lambda(1-\rho)t)^i}{i!} \rho^{k-i} \right\} + C_3(1+ct)e^{-ct} M$$

Cost Model 4

Let test phase time X_1 follows Weibull distribution with parameter A, α . Then

$$E(X_1) = A\Gamma\left(\frac{1}{\alpha} + 1\right), A > 0.$$

$$E_1(TC) = C_1 \sum_{i=1}^{k} \left(\frac{A\Gamma(\frac{1}{\alpha} + 1)}{a^{i-1}} \right) \left\{ e^{-\lambda t} \sum_{i=1}^{k} \binom{k-1}{i-1} \frac{(\lambda(1-\rho)t)^i}{i!} \rho^{k-i} \right\}$$

Let fault removal time Y_1 follows Weibull distribution with parameter B, β . Then

$$E(Y_1) = B\Gamma\left(\frac{1}{\beta} + 1\right), B > 0.$$

$$E_2(TC) = C_2 \sum_{i=1}^{k} \left(\frac{B\Gamma(\frac{1}{\beta} + 1)}{b^{i-1}} \right) \left\{ e^{-\lambda t} \sum_{i=1}^{k} \binom{k-1}{i-1} \frac{(\lambda(1-\rho)t)^i}{i!} \rho^{k-i} \right\}$$

Then the expected total cost is

$$E(TC) = C_1 \sum_{i=1}^{k} \left(\frac{A\Gamma(\frac{1}{\alpha} + 1)}{a^{i-1}} \right) + C_2 \sum_{i=1}^{k} \left(\frac{B\Gamma(\frac{1}{\beta} + 1)}{b^{i-1}} \right) \left\{ e^{-\lambda t} \sum_{i=1}^{k} \binom{k-1}{i-1} \frac{(\lambda(1-\rho)t)^i}{i!} \rho^{k-i} \right\}$$

$$+ C_3(1+ct)e^{-ct} M$$

IV. APPLICATION OF POLYA-AEPPLI RELIABILITY MODEL IN SOFTWARE DEVELOPMENT

Software reliability is the probability of failure free software operation for a specified period of time in a specified environment. Testing the reliability of any software is essential to fix the errors and to rectify before its launch. Also this helps to upgrade the software according to the requirements of its users.

The main application of software reliability model is the determination of software release time. Suppose the software is released quickly, the customer experience many failures and suppose the software is released late the software agencies spent more money on delay.

There are many models for software reliability and are mostly based on the NHPP with a mean value function [2,4,5,7,8]. Hoang Pham and Xuemei Zhang (1997) gave a new reliability model based on total error content and error detection rate and applied to widely used data sets. It is showed that this model fits the failure data very well when compared to other existing NHPP software reliability models.

The optimal release time and the interval estimation of the release time prediction are derived by Xie and Hong in 1999.Again Hoang Pham and Xuemei Zhang in 2003 established a new reliability model addressing the testing coverage time and cost . A software cost model has been developed incorporating the testing cost, fault removal cost and risk cost due to potential problems remaining in the uncovered code.

The expected software release time is determined using the shock model approach. The expected release time of software for various models are derived and the expected total cost of software are derived.

Notation

X_i –Test phase time after the i^{th} fault removal and before $(i+1)^{th}$ fault detection .

Y_i –Wrong code removal time after i^{th} fault detection.

N(t)- Number of fault detections.

W – Software release time.

A. Expected Release Time of Software:

1. Model formulation

Consider the testing phase of software development ,the faults are removed whenever they are detected. This removal reduces the total number of faults in the software. The length of time intervals between fault detection should increase and fault removal time should decrease. When the fault detection rate reaches an acceptably low level i.e., it tends to zero, the software can be considered suitable for the customer .

2. Assumptions

Let X_i be the test phase time after the i^{th} fault removal and before $(i+1)^{th}$ fault detection. Assume $\{X_i, i = 1,2, \dots n\}$ be stochastically increasing geometric process. If $F(t) = \Pr\,(X_1 \le t)$ is the distribution function of X_1 with mean μ_1 then the distribution function of X_n is $F(a^{n-1}t) = \Pr(X_n \le t)\,, a \le 1$ with mean $\frac{\mu_1}{a^{n-1}}$. Let Y_i be the wrong code removal time after i^{th} fault detection. Assume that $\{Y_i, i = 1,2, \dots n\}$ form a monotonically decreasing geometric process. If $G(t) = \Pr\,(Y_1 \le t)$ then the distribution function of Y_n is $G(b^{n-1}t) = \Pr(Y_n \le t)\,, b \le 1$ with mean $\frac{\mu_2}{b^{n-1}}$. Also assume that X_i and Y_i are independent. Let N(t) be the random variable denoting the number of fault detections.

The release time of the software is

$$W = X_0 + \sum_{i=1}^{N} (X_i + Y_i)$$

where X_0 is test phase times before the first fault detection of first fault.

The expected release time of the software is

$$E(W) = E(X_0) + \sum_{i=1}^{N} E(X_i + Y_i)$$

$$= E(X_0)P(N(t)=0) + [\sum_{i=1}^{n} \{E(X_i) + E(Y_i)\}]\ P(N(t) = k$$

$$= \mu_1 P(N(t) = 0) + [(\mu_1 + \mu_2) + (\frac{\mu_1}{a} + \frac{\mu_2}{b}) + (\frac{\mu_1}{a^2} + \frac{\mu_2}{b^2}) + \ldots(\frac{\mu_1}{a^{k-1}} + \frac{\mu_2}{b^{k-1}})]\,P(N(t) = k)$$

$$E(W) = \mu_1 P(N(t) = 0) + \left[\mu_1\left(1 + \frac{1}{a} + \frac{1}{a^2} + \cdots + \frac{1}{a^{k-1}}\right) + \mu_2\left(1 + \frac{1}{b} + \frac{1}{b^2} + \cdots + \frac{1}{b^{k-1}}\right)\right]$$

$$P(N(t) = k)$$

Model 1

Let the test phase time of software X_1 follows exponential distribution with parameter A, the wrong code removal time Y_1 follows exponential distribution with parameter B and the number of faults detections N(t) follows Polya-Aeppli distribution with parameter λ , ρ.

where $E(X_1) = \frac{1}{A},\ A > 0$; $E(Y_1) = \frac{1}{B}\ ,\ B > 0$; $E(N)=\left(\frac{\lambda}{(1-\rho)}\right)$

then $E(W) = \frac{1}{A} e^{-\lambda} + \{e^{-\lambda} \sum_{i=1}^{k} \binom{k-1}{i-1} \frac{(\lambda(1-\rho))^i}{i!} \rho^{k-i}\}\{\sum_{i=1}^{k} (\frac{1}{Aa^{i-1}} + \frac{1}{Bb^{i-1}})\}$

k= 0,1,2,…

When $\rho = 0$, it becomes

$$E(W) = \frac{1}{A} e^{-\lambda}$$

Model 2

Let the testing phase of software X_1 follows exponential distribution with parameter A, the wrong code removal time Y_1 follows Weibull distribution with parameter B, β and the number of faults detections N follows Polya-Aeppli distribution with parameter λ , ρ.

where $E(X_1) = \frac{1}{A},\ A > 0$; $E(Y_1) = B\Gamma\left(\frac{1}{\beta} + 1\right)\ ,\ B > 0$; $E(N)=\left(\frac{\lambda}{(1-\rho)}\right)$

then

$$E(W) = \frac{e^{-\lambda}}{A} + \left[\sum_{i=1}^{k} \left(\frac{1}{Aa^{i-1}} + \frac{B\Gamma\left(\frac{1}{\beta}+1\right)}{b^{i-1}} \right) \right] \quad k=0,1,2,\ldots$$

When $\rho = 0$, it becomes $E(W) = \frac{e^{-\lambda}}{A}$

Model 3

Let the testing phase of software X_1 follows weibull distribution with parameter A,α , the wrong code removal time Y_1 follows exponential distribution with parameter B, β and the number of faults detections N follows Polya-Aeppli distribution with parameter λ , ρ.

where $E(X_1) = A\Gamma\left(\frac{1}{\alpha}+1\right)$, $A > 0$; $E(Y_1) = \frac{1}{B}$, $B > 0$; and E(N(t))=$\left(\frac{\lambda}{(1-\rho)}\right)$

$$E(W)=A\Gamma\left(\frac{1}{\alpha}+1\right)e^{-\lambda} + \left[\sum_{i=1}^{k} \left(\frac{A\Gamma\left(\frac{1}{\alpha}+1\right)}{a^{i-1}} + \frac{1}{Bb^{i-1}} \right) \right] \left\{ e^{-\lambda} \sum_{i=1}^{k} \binom{k-1}{i-1} \frac{(\lambda(1-\rho))^{i}}{i!} \rho^{k-i} \right\}$$

When ρ=0 , $E(W) = A\Gamma\left(\frac{1}{\alpha}+1\right)e^{-\lambda}$

Model 4

Let the test phase of software X_1 follows Weibull distribution with parameter A,α , the wrong code removal time Y_1 follows Weibull distribution with parameter B, β and the number of faults detections N follows Polya-Aeppli distribution with parameter λ , ρ

where $E(X_1) = A\Gamma\left(\frac{1}{\alpha}+1\right)$, $A > 0$; $E(Y_1) = \frac{1}{B}$, $B > 0$; and E(N(t))=$\left(\frac{\lambda}{(1-\rho)}\right)$

$$E(W) = A\Gamma\left(\frac{1}{\alpha}+1\right)e^{-\lambda} + \left[\sum_{i=1}^{k} \left(\frac{A\Gamma\left(\frac{1}{\alpha}+1\right)}{a^{i-1}} + \frac{B\Gamma\left(\frac{1}{\beta}+1\right)}{b^{i-1}} \right) \right] \left\{ e^{-\lambda} \sum_{i=1}^{k} \binom{k-1}{i-1} \frac{(\lambda(1-\rho))^{i}}{i!} \rho^{k-i} \right\}$$

When ρ=0, $E(W) = A\Gamma\left(\frac{1}{\alpha}+1\right)e^{-\lambda}$

B. Expected Total Cost

It is essential to determine when the software testing should be stopped so that the expected cost is minimized and the reliability of the software product satisfies customer's requirements as well.

In this section Analytical expression of the total cost of all the models are derived .

Assumptions

(i) The cost to perform testing is proportional to the testing time.
(ii) Expected testing cost is $E_1(TC)$.
(iii) The cost of removing faults is proportional to the wrong code removal time.
(iv) Expected fault removal cost is $E_2(TC)$.
(v) There is a risk cost . A software provider has to pay each customer a certain amount of money for potential faults in uncovered code. Assume that there are M customers in this system.
(vi) Expected risk cost due to fault remaining in the uncovered code $E_3(TC)$.
(vii) The risk cost is of the form

$$E_3(TC)=C_3(1+ct)e^{-ct}M$$

The expected total cost E(TC) is

$$E(TC)=E_1(TC)+E_2(TC)+E_3(TC)$$

Expected Total Cost

Cost Model 1

Let the test phase time X_1 follows exponential distribution with parameter A.

Then $E(X_1)=\frac{1}{A}$, $A>0$; $E_1(TC)=C_1\sum_{i=1}^{k}\frac{1}{Aa^{i-1}}e^{-\lambda}\left\{\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho))^{i}}{i!}\rho^{k-i}\right\}$

Let the wrong code removal time Y_1 follows exponential distribution with parameter B

then $E(Y_1)=\frac{1}{B}$, $B>0$; $E_2(TC)=C_2\sum_{i=1}^{k}\frac{1}{Bb^{i-1}}\ e^{-\lambda}\left\{\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho))^{i}}{i!}\rho^{k-i}\right\}$

Then the expected total cost is ,

$$E(TC)= C_1\sum_{i=1}^{k}\frac{1}{Aa^{i-1}}e^{-\lambda}\left\{\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho))^{i}}{i!}\rho^{k-i}\right\}$$

$$+\ C_2\sum_{i=1}^{k}\frac{1}{Bb^{i-1}}\ e^{-\lambda t}\left\{\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho)t)^{i}}{i!}\rho^{k-i}\right\}$$

$$+\ C_3(1+ct)e^{-ct}M$$

Cost Model 2

Let the test phase time X_1 follows exponential distribution with parameter A. Then $E(X_1)=\frac{1}{A}$, A>0;

$$E_1(TC)=C_1\sum_{i=1}^{k}\frac{1}{Aa^{i-1}}\left\{e^{-\lambda t}\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho)t)^i}{i!}\rho^{k-i}\right\}$$

Let Y_1 follows Weibull distribution with parameter B, β. Then $E(Y_1)=B\Gamma\left(\frac{1}{\beta}+1\right), B>0.$

$$E_2(TC)=C_2\sum_{i=1}^{k}\left(\frac{B\Gamma(\frac{1}{\beta}+1)}{b^{i-1}}\right)\left\{e^{-\lambda t}\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho)t)^i}{i!}\rho^{k-i}\right\}$$

$$E_3(TC)=C_3(1+ct)e^{-ct}M$$

Then the expected total cost is ,

$$E(TC) = C_1\sum_{i=1}^{k}\frac{1}{Aa^{i-1}}\left\{e^{-\lambda t}\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho)t)^i}{i!}\rho^{k-i}\right\}$$

$$+ C_2\sum_{i=1}^{k}\left(\frac{B\Gamma(\frac{1}{\beta}+1)}{b^{i-1}}\right)\left\{e^{-\lambda t}\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho)t)^i}{i!}\rho^{k-i}\right\} + C_3(1+ct)e^{-ct}M$$

Cost Model 3

Let X_1 follows Weibull distribution with parameter A, α. Then $E(X_1)=A\Gamma\left(\frac{1}{\alpha}+1\right), A>0.$

$$E_1(\,TC) = C_1\sum_{i=1}^{k}\left(\frac{A\Gamma(\frac{1}{\alpha}+1)}{a^{i-1}}\right)\left\{e^{-\lambda t}\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho)t)^i}{i!}\rho^{k-i}\right\}$$

Let Y_1 follows exponential distribution with parameter B. Then $E(Y_1) = \frac{1}{B}$, B>0 .

$$E_2(TC) = C_2\sum_{i=1}^{k}\left(\frac{1}{Bb^{i-1}}\right)\left\{e^{-\lambda t}\sum_{i=1}^{k}\binom{k-1}{i-1}\frac{(\lambda(1-\rho)t)^i}{i!}\rho^{k-i}\right\}$$

Let Y_1 follows exponential distribution with parameter B. Then $E(Y_1) = \frac{1}{B}$, B>0 .

$$E_2(TC) = C_2 \sum_{i=1}^{k} \left(\frac{1}{Bb^{i-1}} \right) \left\{ e^{-\lambda t} \sum_{i=1}^{k} \binom{k-1}{i-1} \frac{(\lambda(1-\rho)t)^i}{i!} \rho^{k-i} \right\}$$

Then the expected total cost is ,

$$E(TC) = C_1 \sum_{i=1}^{k} \left(\frac{A\Gamma(\frac{1}{\alpha}+1)}{a^{i-1}} \right) \left\{ e^{-\lambda t} \sum_{i=1}^{k} \binom{k-1}{i-1} \frac{(\lambda(1-\rho)t)^i}{i!} \rho^{k-i} \right\}$$

$$+ C_2 \sum_{i=1}^{k} \left(\frac{1}{Bb^{i-1}} \right) \left\{ e^{-\lambda t} \sum_{i=1}^{k} \binom{k-1}{i-1} \frac{(\lambda(1-\rho)t)^i}{i!} \rho^{k-i} \right\} + C_3(1+ct)e^{-ct} M$$

Cost Model 4

Let X_1 follows Weibull distribution with parameter A, α. Then $E(X_1) = A\Gamma\left(\frac{1}{\alpha}+1\right)$, $A > 0.$

$$E_1(TC) = C_1 \sum_{i=1}^{k} \left(\frac{A\Gamma(\frac{1}{\alpha}+1)}{a^{i-1}} \right) \left\{ e^{-\lambda t} \sum_{i=1}^{k} \binom{k-1}{i-1} \frac{(\lambda(1-\rho)t)^i}{i!} \rho^{k-i} \right\}$$

Let Y_1 follows Weibull distribution with parameter B, β. Then $E(Y_1) = B\Gamma\left(\frac{1}{\beta}+1\right)$, $B > 0.$

$$E_2(TC)=C_2 \sum_{i=1}^{k} \left(\frac{B\Gamma(\frac{1}{\beta}+1)}{b^{i-1}} \right) \left\{ e^{-\lambda t} \sum_{i=1}^{k} \binom{k-1}{i-1} \frac{(\lambda(1-\rho)t)^i}{i!} \rho^{k-i} \right\}$$

Then the expected total cost is

$$E(TC)=C_1 \sum_{i=1}^{k} \left(\frac{A\Gamma(\frac{1}{\alpha}+1)}{a^{i-1}} \right) +C_2 \sum_{i=1}^{k} \left(\frac{B\Gamma(\frac{1}{\beta}+1)}{b^{i-1}} \right) \left\{ e^{-\lambda t} \sum_{i=1}^{k} \binom{k-1}{i-1} \frac{(\lambda(1-\rho)t)^i}{i!} \rho^{k-i} \right\} + C_3(1+ct)e^{-ct} M$$

V. CONCLUSION

In this paper four Mathematical models are constructed to find the expected release time and the expected total cost of a system and application of Polya-Appeli reliability model in software development are derived .

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