Vibration Analysis of Cantilever Beams with Inclined Cracks

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***Abstract* — *In this paper the dynamic parameters of an inclined cracked rectangular cross-section cantilever beam in transverse vibration are studied. Vibration analysis is an effective tool used for crack detection under non-destructive testing for machinery. The modelled structures for rectangular cross-section beams of different crack depths and crack locations and crack angles were developed using CATIA. Modal analysis and harmonic analysis were performed to obtain modal parameters and harmonic responses using ANSYS and also Modal Analysis using HyperMesh. It is seen that due to presence of cracks the stiffness, frequency response functions (FRF's) and natural frequency changes. The Natural Frequencies obtained from modal analysis using ANSYS, HyperMesh and MATLAB are verified with the harmonic analysis results.*

Keywords — *Inclined cracks, HyperMesh, Natural Frequency, Theoretical calculations, Cantilever Beams, Modal Analysis, Harmonic Analysis.*

I. Introduction

Fracture is the separation of a body into two or more pieces as a result of an imposed stress. Fast fracture occurs when a pre-existing crack in a material suddenly becomes unstable and grows rapidly through the material. This form of fracture is highly undesirable. It is a catastrophic failure that occurs without warning.

There are two types of fracture found for most engineering materials, either ductile or brittle. Ductile fracture is characterized by large amounts of plastic deformation before failure. This form of fracture involves large amounts of energy absorption (high toughness). Brittle fracture is characterized by low plastic deformation and small amounts of energy absorption at failure (low toughness).It takes a lot of force to deform a glass rod. With the load shown the rod is not broken. But if the rod has a small notch, the force required to cause fracture is greatly reduced, and the rod breaks. If we apply a force to a copper rod that contains a small notch the rod plastically deforms and the notch does not decrease the force required to break the rod. The fracture of a copper rod is an example of ductile fracture. In ductile fracture there is a lot of plastic deformation, and significant energy is absorbed before the fracture.

II. Problem Formulation and Modelling

In this present work it has been analyzed that the crack can be detected in the various structures through visual inspection or by the method of measuring natural frequency, mode shape and structural damping [11]. As the measurement of natural frequency and mode shape is quite easy as compared to other parameters, so in this chapter a logical approach has been adopted to develop the expression to calculate the natural frequency and the mode shape of the cantilever beam with the presence of a transverse crack and the effect of natural frequency in the presence of crack.

A. Specifications of test structure

The design limitations according to B.P. Nandwana and S.K. Maiti [1, 2] for inclined edge crack cantilever beams, $L/b \geq 12$, $a \leq t/2$ and $\theta \leq 45°$.

In our present work we consider cantilever beams having inclined cracks as shown in fig. 3.1 with two materials aluminum and copper. The γ values ranging from 0.25 to 0.70 (relative crack location) and the α values ranging from 0.15 to 0.50 (relative crack depth). Three angles of cracks are considered 15 degree, 30 degree and 45 degrees. To model the transverse vibration the crack is represented by a rotational spring of stiffness K_t. The beam can be conveniently divided into two segments, one on either side of the spring. The governing equation of transverse vibration of each segment is [1, 2]

$$\frac{d^2}{dx^2}[EI\frac{d^2 x}{dx^2}]-\omega^2\rho AU=0 \quad (1)$$

Where '*Q* ' is displacement, ' *A*' is the cross-sectional area, 'ρ ' is the mass density, ' *ω* ' is natural frequency of the vibration of the beam, ' *I* ' is the moment of inertia, '*E* ' is the Young's modulus of elasticity and the *x*-axis is aligned with the axis of the beam. The equations for the each part are given below:

$$\frac{d^4 Q_1}{d\gamma^4} - \lambda^4 Q_1 = 0 \quad (2)$$

$$\frac{d^4 Q_2}{d\gamma^4} - \lambda^4 Q_2 = 0 \quad (3)$$

$\lambda^4 = (\omega^2\rho AL^4)/EI$ and $\gamma = L_1/L$ are non-dimensional parameters. Solving the above two equations, we get displacements on each part of the beam.

$$Q_1(\gamma) = A_1\cos\lambda\gamma + A_2\cos\lambda\gamma + A_3\sin\lambda\gamma + A_4\sin\lambda\gamma \quad (4)$$

$$Q_2(\gamma) = A_5\cos\lambda\gamma + A_6\cos\lambda\gamma + A_7\sin\lambda\gamma + A_8\sin\lambda\gamma \quad (5)$$

Where, A_1 to A_8 are arbitrary constants to be calculated from the boundary conditions. The boundary conditions at the fixed and free ends, respectively, are

$$Q_{1,\gamma=0}=0,\; Q_1'{}_{,\gamma=0}=0\;;\; Q_2''{}_{,\gamma=0}=0,\; Q_2'''{}_{,\gamma=0}=0 \qquad (6)$$

The continuity of displacement, bending moment and shear forces at the crack location (say $\gamma = L_1/L$) and jump condition in the slope can be written in the following form:

$$Q_1(\gamma) = Q_2(\gamma),\; Q_1''(\gamma) = Q_2''(\gamma),\; Q_1'''(\gamma) = Q_2'''(\gamma) \qquad (7)$$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\cos\lambda & \cosh\lambda & -\sin\lambda & \sinh\lambda \\ 0 & 0 & 0 & 0 & \sin\lambda & \sinh\lambda & -\cos\lambda & \cosh\lambda \\ \cos\alpha & \cosh\alpha & \sin\alpha & \sinh\alpha & -\cos\alpha & -\cosh\alpha & -\sin\alpha & -\sinh\alpha \\ -\cos\alpha & \cosh\alpha & -\sin\alpha & \sinh\alpha & \cos\alpha & -\cosh\alpha & \sin\alpha & -\sinh\alpha \\ \sin\alpha & \sinh\alpha & -\cos\alpha & \cosh\alpha & -\sin\alpha & -\sinh\alpha & \cos\alpha & -\cosh\alpha \\ -\sin\alpha & \sinh\alpha & \cos\alpha & \cosh\alpha & \sin\alpha & -\sinh\alpha & -\cos\alpha & -\cosh\alpha \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\cos\lambda & \cosh\lambda & -\sin\lambda & \sinh\lambda \\ 0 & 0 & 0 & 0 & \sin\lambda & \sinh\lambda & -\cos\lambda & \cosh\lambda \\ \cos\alpha & \cosh\alpha & \sin\alpha & \sinh\alpha & -\cos\alpha & -\cosh\alpha & -\sin\alpha & -\sinh\alpha \\ -\cos\alpha & \cosh\alpha & -\sin\alpha & \sinh\alpha & \cos\alpha & -\cosh\alpha & \sin\alpha & -\sinh\alpha \\ \sin\alpha & \sinh\alpha & -\cos\alpha & \cosh\alpha & -\sin\alpha & -\sinh\alpha & \cos\alpha & -\cosh\alpha \\ -\cos\alpha & \cosh\alpha & -\sin\alpha & \sinh\alpha & 0 & 0 & 0 & 0 \end{vmatrix}$$

Where $\alpha = \lambda\gamma$

$$K = -\lambda \frac{\Delta_2}{\Delta_1}$$

The change in stiffness of cracked beams is given from the above equation and by using this we calculate the natural frequencies of the cracked beams.

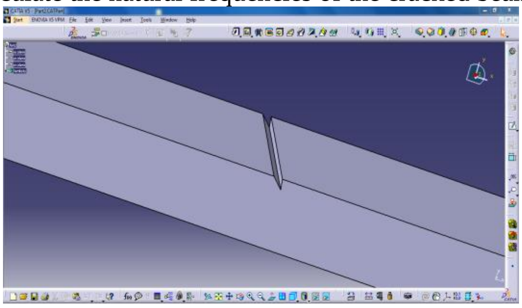


Fig 1: Magnified view of beam model with crack location at 0.20 m, crack depth 0.0009 m and crack angle 30° from fixed end.

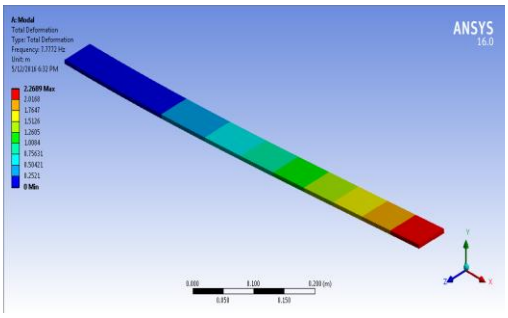
III. MODAL ANALYSIS USING ANSYS

Modal analysis is performed for beams having rectangular cross-section of dimensions 800 mm × 60 mm × 6 mm with inclined cracks of varying crack depths, crack angles and crack locations for cantilever beams with and without cracks [5,6]. Aluminium and copper alloy rectangular beams are considered with and without cracks and using ANSYS 16 Workbench analytical modal analysis was carried out. In ANSYS 16, ANSYS Workbench is used for simulation due to its interface with most of CAD packages. CATIAV5 R20 (Computer Aided Three-dimensional Interactive Application) is used as CAD package to develop geometric models for various cases of problem.

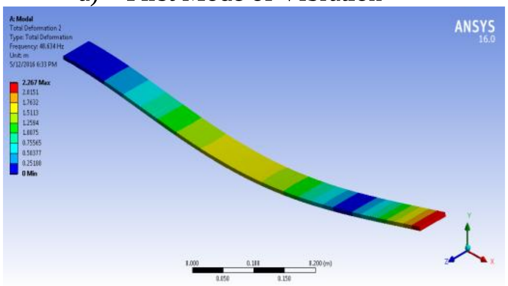
Table1. Material Properties of the Beam

Parameters	Aluminum Beam	Copper Alloy Beam
Young's Modulus (E)	70×10^9 N/m^2	110×10^9 N/m^2
Density (ρ)	2710 kg/m^3	8300 kg/m^3
Poisson ratio (ϑ)	0.346	0.34

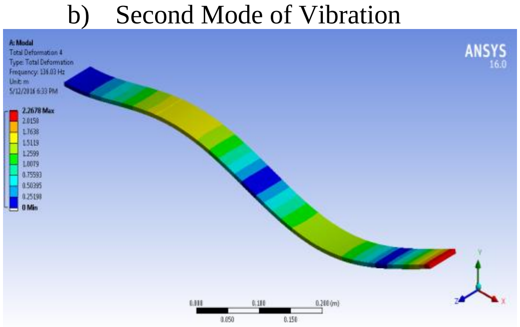
The sample case considered in this analysis was cracked cantilever beam of crack depth 0.0009 m, crack angle 15° and crack location at 0.20 m shown in figure 2. Similarly, for all remaining cases of the cantilever beam model obtained results are tabulated.



a) First Mode of Vibration



b) Second Mode of Vibration



c) Third Mode of Vibration

Fig 2: Mode Shapes of Cracked Aluminium Cantilever Beam.

IV. MODAL ANALYSIS BY THEORETICAL APPROACH

Un-cracked beams Natural frequencies are obtained by using the below given formula11. Where, λ_i = 1.875, 4.694, 7.855 for the first three natural frequencies of Un-cracked Cantilever Beams for both Aluminum and Copper Alloy material [4,7]. λ_i is called the Frequency parameter. The values obtained are tabulated in table 2.

$$\omega_n = \lambda_i^2 \sqrt{\frac{EI}{\rho A}}$$

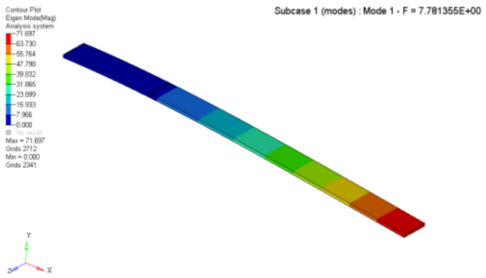
Table 2. Natural frequencies of Un-cracked beam

Material	FNF (Hz)	SNF(Hz)	TNF(Hz)
Aluminium	7.69	48.20	136.05
Copper Alloy	5.5405	34.720	97.9821

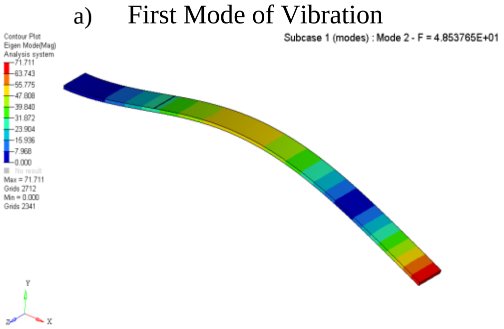
For cracked beams with varying crack depths, crack location and crack angles the change in stiffness of the beam is used to calculate the natural frequencies of the beams [8]. The change in stiffness is calculated from the matrices given in chapter 2 of this paper.

V. MODAL ANALYSIS USING HYPERMESH

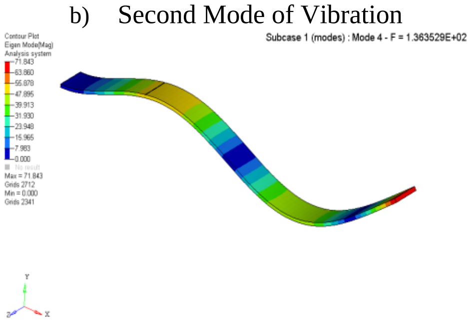
HyperMesh v 11.0 is used to find the Natural frequencies of Un-cracked and cracked cantilever beams. OptiStruct is the solver used in this software. The .iges file done in CATIA is imported into this software. The sample case considered in this analysis was cracked cantilever beam of crack depth 0.0009 m, crack angle 15° and crack location at 0.20 m. Similarly, for all remaining cases of the cantilever beam model obtained results are tabulated in tables 3 and 4.



a) First Mode of Vibration



b) Second Mode of Vibration



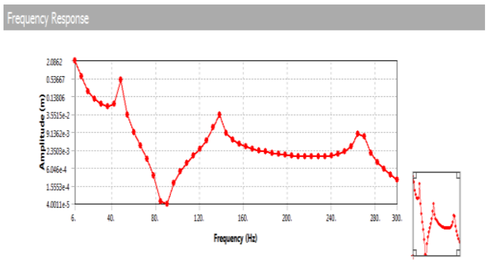
c) Third Mode of Vibration

Fig 3: Mode Shapes of Cracked Aluminium Cantilever Beam

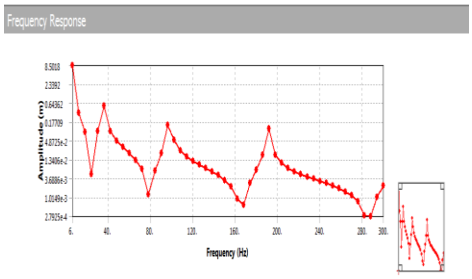
VI. HARMONIC ANALYSIS

A harmonic, or frequency-response, analysis considers loading at one frequency only. Loads may be out-of phase with one another, but the excitation is at a known frequency. In a harmonic analysis, Young's Modulus, Poisson's Ratio, and Mass Density are required input. All other material properties can be specified but are not used in a harmonic analysis because of the fact that modal coordinates are used; a harmonic solution using the Mode Superposition method will automatically perform a modal analysis first. Although a free vibration analysis is performed first, the harmonic analysis portion is very quick and efficient. Hence, the Mode Superposition method is usually much faster overall than the Full method. Since a free vibration analysis is performed, Simulation will know what the natural frequencies of the structure are. In a harmonic analysis, the peak response will correspond with the natural frequencies of the structure.

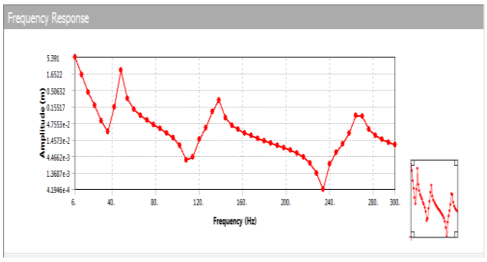
All applied loads vary harmonically at the specified frequency. Specified in cycles per second (Hertz) by a frequency range and number of sub-steps within that range using Analysis Settings option. For example, a range of 0-500 Hz with 100 sub-steps gives solutions at frequencies of 5, 10,..500Hz. Same range with 1 sub-step gives one solution at 5 Hz. The In our problem Frequency range of rectangular cross sections beam models is 0Hz to 300Hz. Full Method is implemented to perform Harmonic Analysis and it was set using Analysis Settings option. The Harmonic Analysis was performed beams of rectangular cross section cantilever beam conditions. Boundary Condition for Cantilever beam is a fixed support was placed at left end of the beam. A Force of 500 N is applied along Y-direction for all cases of beam models [4].



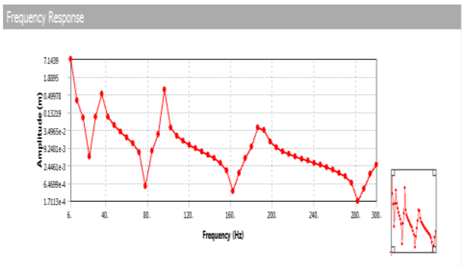
a) Frequency response of un-cracked Aluminium Cantilever Beam



b) Frequency response of un-cracked Copper Alloy Cantilever Beam



c) Frequency response of cracked Aluminum Beam of crack depth 0.0009 m, crack location 0.25m and crack angle 15°



d) Frequency response of cracked Copper Alloy Beam of crack depth 0.0030 m, crack location 0.20m and crack angle 15°

Fig 4: Harmonic analysis of un-cracked and cracked beam

VII. RESULTS

The natural frequencies obtained for un-cracked and cracked beams using different methods are tabulated. The un-cracked and cracked cases for aluminium and copper alloy beam using ANSYS, theoretical calculations and HyperMesh are tabulated below. The aluminium cantilever beam cracked and un-cracked cases are given in table 3 and for Copper alloy it is given in table 4.

Table 3. Natural frequencies of aluminium cantilever beam

Crack Location (m)	Crack Depth (m)	FNF (f_1) using (ANSYS)	SNF (f_2) using (ANSYS)	TNF (f_3) using (ANSYS)	FNF (f_1) using (Theoretical calculations)	SNF (f_2) using (Theoretical calculations)	TNF (f_3) using (Theoretical calculations)	FNF (f_1) using (Hyper Mesh)	SNF (f_2) using (Hyper Mesh)	TNF (f_3) using (Hyper Mesh)
Un-cracked		7.761	48.619	136.15	7.69	48.2	136.05	7.7477	48.528	135.89
Crack inclination 15^0										
0.20	0.0009	7.7772	48.634	136.03	7.6899	48.2002	134.234	7.7813	48.537	136.35
	0.0015	7.7634	48.643	135.85	7.67	48.0755	133.887	7.7652	48.534	136.14
	0.0021	7.7334	48.641	135.51	7.6694	48.0718	133.877	7.7395	48.529	135.81
	0.0030	7.6588	48.595	134.48	7.665	48.0442	133.800	7.6684	48.514	134.87
0.28	0.0009	7.7512	48.567	135.95	7.6896	48.1984	134.229	7.7766	48.671	136.59
	0.0015	7.7464	48.56	135.84	7.666	48.0504	133.817	7.7717	48.649	136.48
	0.0021	7.7239	48.451	135.31	7.643	47.9063	133.416	7.7631	48.609	136.27
	0.0030	7.675	48.204	134.2	7.61	47.6994	132.840	7.7435	48.515	135.80
0.40	0.0009	7.7544	48.542	136.07	7.6895	48.1977	134.227	7.7487	48.555	135.89
	0.0015	7.7545	48.47	135.9	7.663	48.0316	133.765	7.7441	48.430	135.89
	0.0021	7.7482	48.292	135.71	7.64	47.8875	133.363	7.7365	48.224	135.89
	0.0030	7.7246	47.747	135.12	7.625	47.7935	133.102	7.7158	47.677	135.89
0.48	0.0009	7.7552	48.562	136.12	7.6894	48.1971	134.226	7.7474	48.500	135.84
	0.0015	7.7538	48.478	135.99	7.653	47.969	133.590	7.7467	48.458	135.78
	0.0021	7.749	48.280	135.64	7.612	47.7120	132.875	7.7448	48.343	135.59
	0.0030	7.7393	47.753	134.8	7.597	47.618	132.613	7.7376	47.944	134.94
0.56	0.0009	7.7597	48.580	136.07	7.6892	48.1959	134.222	7.7487	48.586	136.23
	0.0015	7.7593	48.519	135.72	7.643	47.9063	133.416	7.7486	48.551	136.03
	0.0021	7.7551	48.413	135.19	7.619	47.7558	132.997	7.7481	48.482	135.63
	0.0030	7.7525	48.202	134.02	7.591	47.5803	132.508	7.7464	48.289	134.54
Crack inclination 30^0										
0.20	0.0009	7.7806	48.648	136.06	7.675	48.1069	133.974	7.7488	48.529	135.92
	0.0015	7.7617	48.644	135.82	7.652	47.9627	133.573	7.7415	48.528	135.83
	0.0021	7.7317	48.625	135.47	7.64	47.8875	133.363	7.7261	48.525	135.62
	0.0030	7.672	48.61	134.66	7.629	47.8185	133.171	7.6853	48.516	135.08

0.28	0.0009	7.7541	48.595	136.02	7.663	48.0316	133.765	7.7498	48.537	135.95
	0.0015	7.7399	48.527	135.68	7.641	47.8937	133.381	7.7451	48.517	135.84
	0.0021	7.7228	48.446	135.29	7.63	47.8248	133.189	7.7336	48.463	135.57
	0.0030	7.6776	48.209	134.25	7.621	47.7684	133.032	7.6969	48.284	134.71
0.40	0.0009	7.755	48.535	136.03	7.659	48.0066	133.695	7.7566	48.772	135.90
	0.0015	7.7541	48.458	136.1	7.642	47.9000	133.398	7.7505	48.600	135.90
	0.0021	7.7474	48.275	136.09	7.629	47.8185	133.171	7.7454	48.461	135.89
	0.0030	7.7223	47.667	136.06	7.612	47.7120	132.875	7.7334	48.130	135.89
0.48	0.0009	7.755	48.548	136.1	7.639	47.8812	133.346	7.7489	48.585	135.98
	0.0015	7.7531	48.436	135.91	7.631	47.8311	133.206	7.7479	48.525	135.88
	0.0021	7.7493	48.283	135.66	7.61	47.6994	132.840	7.7458	48.402	135.68
	0.0030	7.7375	47.636	134.6	7.586	47.5490	132.421	7.7399	48.068	135.13
0.56	0.0009	7.7588	48.572	136.03	7.635	47.8561	133.276	7.7485	48.566	136.10
	0.0015	7.7592	48.512	135.68	7.629	47.8185	133.171	7.7483	48.521	135.84
	0.0021	7.7551	48.412	135.18	7.595	47.6054	132.578	7.7477	48.442	135.39
	0.0030	7.7518	48.174	133.85	7.582	47.5239	132.351	7.7459	48.236	134.22
Crack inclination 45^0										
0.20	0.0009	7.747	48.586	135.96	7.669	48.0692	133.870	7.7794	48.537	136.33
	0.0015	7.7297	48.576	135.7	7.647	47.9314	133.486	7.7620	48.534	136.10
	0.0021	7.690	48.566	135.16	7.641	47.8937	133.381	7.7345	48.528	135.74
	0.0030	7.6424	48.561	134.56	7.627	47.8060	133.136	7.6689	48.511	134.85
0.28	0.0009	7.7535	48.576	135.99	7.654	47.9752	133.608	7.7495	48.535	135.94
	0.0015	7.7458	48.53	135.78	7.638	47.8749	133.328	7.7438	48.510	135.81
	0.0021	7.7252	48.431	135.29	7.627	47.8060	133.136	7.7311	48.449	135.51
	0.0030	7.6766	48.197	134.23	7.611	47.7057	132.857	7.6899	48.243	134.55
0.40	0.0009	7.7556	48.522	135.97	7.643	47.9063	133.416	7.7565	48.770	135.90
	0.0015	7.7428	48.277	135.97	7.637	47.8687	133.311	7.7543	48.707	135.90
	0.0021	7.7348	48.063	136.0	7.621	47.7684	133.032	7.7489	48.557	135.90
	0.0030	7.7237	47.743	136.0	7.609	47.6932	132.822	7.7434	48.405	135.89
0.48	0.0009	7.7529	48.526	135.94	7.632	47.8373	133.224	7.7488	48.581	135.98
	0.0015	7.7522	48.433	135.76	7.626	47.7997	133.119	7.7479	48.520	135.87
	0.0021	7.7516	48.254	135.47	7.603	47.6556	132.718	7.7458	48.397	135.67
	0.0030	7.7387	47.726	134.57	7.574	47.4738	132.211	7.7405	48.095	135.15
0.56	0.0009	7.7531	48.537	135.81	7.629	47.8185	133.171	7.7485	48.568	136.11
	0.0015	7.7565	48.492	135.51	7.615	47.7308	132.927	7.7483	48.528	135.88
	0.0021	7.7556	48.384	134.89	7.584	47.5365	132.386	7.7478	48.458	135.48
	0.0030	7.7486	48.038	133.0	7.576	47.4863	132.246	7.7465	48.291	134.50

Table 4. Natural frequencies of copper alloy cantilever beam

Crack Location (m)	Crack Depth (m)	FNF (f_1) using (ANSYS)	SNF (f_2) using (ANSYS)	TNF (f_3) using (ANSYS)	FNF (f_1) using (Theoretical calculations)	SNF (f_2) using (Theoretical calculations)	TNF (f_3) using (Theoretical calculations)	FNF (f_1) using (Hyper Mesh)	SNF (f_2) using (Hyper Mesh)	TNF (f_3) using (Hyper Mesh)
Un-cracked		5.5574	34.814	97.492	5.5405	34.720	97.9821	5.5483	34.752	97.318
Crack inclination 15^0										
	0.0009	5.5675	34.858	97.494	5.51	34.5366	96.1825	5.5724	34.759	97.645
	0.0015	5.5593	34.834	97.039	5.497	34.4552	95.9556	5.5609	34.757	97.497

0.20	0.0021	5.5378	34.833	97.283	5.483	34.3674	95.7112	5.5425	34.753	97.257
	0.0030	5.4842	34.799	96.3	5.478	34.3361	95.6239	5.4915	34.742	96.587
0.28	0.0009	5.5506	34.778	97.353	5.507	34.5178	96.1301	5.5690	34.855	97.821
	0.0015	5.5474	34.775	97.28	5.485	34.3799	95.7461	5.5655	34.839	97.738
	0.0021	5.5309	34.694	96.892	5.479	34.3423	95.6414	5.5593	34.810	97.588
	0.0030	5.4958	34.517	96.09	5.47	34.2859	95.4843	5.5453	34.743	97.250
0.40	0.0009	5.5528	34.759	97.434	5.502	34.4865	96.0429	5.5491	34.772	97.320
	0.0015	5.5528	34.708	97.452	5.481	34.3549	95.6763	5.5458	34.682	97.319
	0.0021	5.5482	34.58	97.461	5.474	34.3110	95.5541	5.5403	34.535	97.319
	0.0030	5.5315	34.19	97.42	5.468	34.2734	95.4494	5.5255	34.142	97.317
0.48	0.0009	5.5535	34.776	97.474	5.495	34.4426	95.9207	5.5481	34.732	97.283
	0.0015	5.5525	34.713	97.375	5.48	34.3486	95.6588	5.5477	34.702	97.235
	0.0021	5.5491	34.573	97.131	5.469	34.2796	95.4668	5.5463	34.619	97.100
	0.0030	5.5421	34.194	96.522	5.462	34.2358	95.3446	5.5411	34.333	96.638
0.56	0.0009	5.5567	34.788	97.439	5.491	34.4175	95.8509	5.5491	34.794	97.558
	0.0015	5.5564	34.744	97.188	5.476	34.3235	95.5890	5.5490	34.769	97.413
	0.0021	5.5534	34.668	96.802	5.465	34.2546	95.3970	5.5486	34.719	97.131
	0.0030	5.5516	34.516	96.964	5.459	34.2170	95.2923	5.5474	34.581	96.349
Crack inclination 30^0										
0.20	0.0009	5.5677	34.856	97.501	5.507	34.5178	96.1301	5.5491	34.753	97.335
	0.0015	5.5523	34.855	97.427	5.492	34.4238	95.8683	5.5439	34.753	97.270
	0.0021	5.5375	34.867	97.359	5.48	34.3486	95.6588	5.5328	34.751	97.125
	0.0030	5.4957	34.932	97.13	5.472	34.2985	95.5192	5.5034	34.744	96.731
0.28	0.0009	5.5526	34.798	96.131	5.498	34.4614	95.9730	5.5498	34.758	97.357
	0.0015	5.5424	34.749	96.872	5.483	34.3674	95.7112	5.5464	34.744	97.277
	0.0021	5.530	34.69	97.16	5.472	34.2985	95.5192	5.5382	34.705	97.083
	0.0030	5.4976	34.52	97.399	5.467	34.2671	95.4319	5.5119	34.577	96.471
0.40	0.0009	5.5533	34.756	97.411	5.493	34.4301	95.8858	5.5547	34.927	97.322
	0.0015	5.5524	34.696	97.455	5.48	34.3486	95.6588	5.5504	34.804	97.321
	0.0021	5.5477	34.568	97.452	5.472	34.2985	95.5192	5.5467	34.704	97.321
	0.0030	5.5304	34.148	97.432	5.465	34.2546	95.3970	5.5381	34.467	97.32
0.48	0.0009	5.5531	34.758	96.383	5.487	34.3925	95.7810	5.5492	34.793	97.382
	0.0015	5.552	34.685	97.148	5.475	34.3173	95.5716	5.5485	34.750	97.312
	0.0021	5.5493	34.575	97.325	5.463	34.2420	95.3621	5.5470	34.662	97.166
	0.0030	5.5408	34.11	97.418	5.456	34.1982	95.2399	5.5428	34.422	96.768
0.56	0.0009	5.556	34.783	97.408	5.488	34.3987	95.7985	5.5489	34.780	97.466
	0.0015	5.5563	34.739	97.161	5.472	34.2985	95.5192	5.5488	34.748	97.284
	0.0021	5.5534	34.667	96.793	5.46	34.2232	95.3097	5.5484	34.691	96.960
	0.0030	5.5511	34.496	95.841	5.453	34.1794	95.1875	5.5470	34.543	96.121
Crack inclination 45^0										
0.20	0.0009	5.5677	34.856	96.351	5.498	34.4614	95.9730	5.5710	34.759	97.630
	0.0015	5.5523	34.855	96.787	5.488	34.3987	95.7985	5.5586	34.757	97.470
	0.0021	5.5375	34.867	97.174	5.476	34.3235	95.5890	5.5389	34.753	97.209
	0.0030	5.4957	34.932	97.359	5.469	34.2796	95.4668	5.4918	34.740	96.573
0.28	0.0009	5.5526	34.798	97.381	5.487	34.3925	95.7810	5.5496	34.757	97.351
	0.0015	5.5424	34.749	97.232	5.477	34.3298	95.6065	5.5455	34.739	97.256
	0.0021	5.53	34.69	96.907	5.468	34.2734	95.4494	5.5364	34.696	97.040

	0.0030	5.4976	34.52	96.116	5.46	34.2232	95.3097	5.5069	34.548	96.357
0.40	**0.0009**	5.5533	34.756	97.371	5.489	34.4050	95.8159	5.5547	34.926	97.322
	0.0015	5.5524	34.696	97.371	5.476	34.3235	95.5890	5.5531	34.881	97.322
	0.0021	5.5477	34.568	97.391	5.465	34.2546	95.3970	5.5492	34.773	97.321
	0.0030	5.5304	34.148	97.393	5.455	34.1919	95.2224	5.5452	34.663	97.320
0.48	**0.0009**	5.5531	34.758	97.344	5.476	34.3235	95.5890	5.5492	34.790	97.379
	0.0015	5.552	34.685	97.215	5.465	34.2546	95.3970	5.5485	34.746	97.305
	0.0021	5.5493	34.575	97.011	5.453	34.1794	95.1875	5.5470	34.658	97.158
	0.0030	5.5408	34.11	96.336	5.442	34.1104	94.9955	5.5432	34.442	96.785
0.56	**0.0009**	5.556	34.783	97.27	5.469	34.2796	95.4668	5.5489	34.781	97.474
	0.0015	5.5563	34.739	97.038	5.455	34.1919	95.2224	5.5488	34.753	97.311
	0.0021	5.5534	34.667	96.594	5.442	34.1104	94.9955	5.5485	34.702	97.020
	0.0030	5.5511	34.496	95.231	5.431	34.0415	94.8035	5.5475	34.582	96.324

VIII. CONCLUSIONS

Based on the results obtained from various techniques for identification of inclined crack on the cantilever beam structure, the following conclusions are drawn:

Due to the changes in the crack location, crack depth and crack angle there is always a significant change in the vibration parameters (natural frequencies and mode shapes).

It is observed that the values obtained from ANSYS, MATLAB and HyperMesh are in good agreement. The Natural frequencies obtained from above methods are verified using Harmonic analysis. The peaks in the curves obtained in the Harmonic Analysis shows the Resonating peaks and the lower most points in the curves shows the anti-resonating frequencies.

From the inspection of the mode shapes of the inclined cracked cantilever beam with different crack location, crack depth and crack inclination, the magnitude of deviation in mode shapes increases with increase in crack depths.

For, constant crack location and crack inclination, but the crack depth increases:

- The natural frequency of the cracked beam decreases with increase in the crack depth. The amplitude at crack location decreases with increase the crack depth for each mode shape.

For, constant crack depth and crack inclination, but crack location increases from cantilever end:

- When the crack location increases, the natural frequency also increases. At particular crack location of a beam, the amplitude is minimum with respect to other beams having a different crack location.
- The natural frequencies decrease as the inclination of the crack increases.

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