Theoretical and numerical modal analysis of aluminum and glass fiber reinforced polymer composite cracked plate

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Abstract— *This paper is concerned with analytical modelling and numerical analysis on the effects of cracks in structural plates and panels within aerospace systems such as aeroplane wing, and tail-plane structures, and as such, is part of a larger body of research into damage detection methodologies in such systems. Crack is a damage that often occurs on members of structures and may cause serious failure of the structures. In practice such a scenario is potentially extremely dangerous as it can lead to failure, with obvious consequences. The equation that is obtained by using Galerkin's approach in this case, the coefficients within the equation contain information about the geometrical and mass properties of the plate, the loading and boundary conditions, and the geometry, location, and potentially the orientation of the crack.*

In the present work different types of plates such as plate-1, plate-2, plate-3 i.e. (plate-1 l_1=1m, l_2=1m, t=10mm and half crack lengths a=0.01m, a=0.02m, a=0.05m, plate-2 l_1=0.5m, l_2=1m, t=10mm and half crack lengths a=0.01m, a=0.02m, a=0.05m, plate-3 l_1=1m, l_2=0.5m, t=10mm and half crack lengths a=0.01m, a=0.02m, a=0.05m) and different boundary conditions i.e. (boundary condition-1 Clamped-Clamped-Free-Free (CCFF), boundary condition-2 Clamped-Clamped-Simply Supported-Simply Supported, boundary condition-3 Simply Supported-Simply Supported- Simply Supported (SSSS) are studied for analytically and numerically using finite element analysis is performed to investigate the free vibration response of Aluminum and Glass Fiber Reinforced Polymer (GFRP) Composite Cracked plate. The finite element analysis ANSYS software is used to simulate both modal analysis and harmonic analysis. The modal analysis is carried out using ANSYS software and compared with theoretical values for different crack lengths and harmonic analysis is performed to study the variation of harmonic response for different crack lengths. In this work it is shown that different boundary conditions can be admitted for the plate and the modal natural frequencies are obtained for different crack lengths and it is observed that increase in the crack lengths leads to decrease in the value of natural frequencies for all the cases considered.

Keywords— *Thin Plate, Part Through crack, Natural Frequency, ANSYS, Modal Analysis, Harmonic Analysis, MATLAB.*

I. INTRODUCTION

Plates and beam structures are fundamental elements in engineering and are used in a variety of structural applications. Structures like aircraft wings, satellites, ships, steel bridges, sea platforms, helicopter rotor blades, spacecraft antennae, and subsystems of more complex structures can be modelled as isotropic plate elements. In this dissertation, only aircraft wing structures modelled as an isotropic plate are discussed. The plate panels on the tips of aircraft wings are mainly under transverse pressure, and are often subjected to normal and shear forces which act in the plane of the plate. The plate panels may not behave as intended if they contain even a small crack, or form of damage and such small disturbances can create a complete loss of equilibrium and cause failure [1, 2].

In engineering practice, however, many components of machines and structures are subjected to dynamic effects, produced by time-dependent external forces or displacements. Dynamic loads may be created by moving vehicles, wind gusts, seismic disturbances, unbalanced machine vibrations, flight loads, sound, etc. Dynamic effects of time-dependent loads on structures are studied in structural dynamics. Structural dynamics deals with time-dependent motions of structures, primarily, with vibration of structures, and analyses of the internal forces associated with them. Thus, its objective is to determine the effect of vibrations on the performance of the structure or machine.

To avoid structural damages caused by undesirable vibrations, it is important to determine:

1. Natural frequencies of the structure to avoid resonance;
2. Mode shapes to reinforce the most flexible points or to determine the right positions to reduce weight or to increase damping.

During operation, all structures are subjected to degenerative effects that may cause initiation of structural defects such as cracks which, as time progresses, lead to the catastrophic failure or breakdown of the structure. Thus, the importance of inspection in the quality assurance of manufactured products is well understood. Cracks or other defects in a structural element influence its dynamical behaviour and change its stiffness and damping properties. Consequently, the natural frequencies of the structure contain information about the location and dimensions of the damage.

II. Problem Formulation and Theoretical Modal Analysis

In this chapter, we derive the equation of motion for a given set of boundary conditions of an isotropic plate with an arbitrarily located part-through crack at the centre of the plate, consisting of a continuous line. The equilibrium principle is followed to derive the governing equation of motion in order to get a tractable solution to the vibration problem. Principally the effects of rotary inertia and through-thickness shear stress are neglected. Galerkin's method is applied to reformulate the governing equation of the cracked plates. The simplifying assumptions, and their validity, are described and when they are made during the derivation of the equations.

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) = -\rho h\frac{\partial^2 w}{\partial t^2} + n_x\frac{\partial^2 w}{\partial x^2} + n_y\frac{\partial^2 w}{\partial y^2} + 2n_{xy}\frac{\partial^2 w}{\partial x \partial y} + P_z$$

The above equation is the classical form of the governing equation of rectangular plate

The final form of equation is

$$\ddot{\psi}_{mn}(t) + \omega^2{}_{mn}\psi_{mn}(t) + \beta_{mn}\psi^3{}_{mn}(t) = \frac{\lambda_{mn}}{D}P_o(t)$$

$$\text{Where } \omega^2{}_{mn} = \frac{K_{mn}}{M_{mn}}$$

Therefore ω_{mn} is the natural frequency of the cracked rectangular plate. β_{mn} is the nonlinear cubic term and can be either a positive or a negative depending upon the system parameters.

At present in this analysis my main aim is to find the natural frequency of the cracked rectangular plate with three boundary conditions.

2.1 Boundary condition 1

Two adjacent edges are clamped while the other two edges are free – CCFF

$$X_m = \cos\left(\frac{\lambda_m x}{l_1}\right) - \cosh\left(\frac{\lambda_m x}{l_1}\right) - \gamma_m\left[\sin\left(\frac{\lambda_m x}{l_1}\right) - \sinh\left(\frac{\lambda_m x}{l_1}\right)\right]$$

$$Y_n = \cos\left(\frac{\lambda_n y}{l_2}\right) - \cosh\left(\frac{\lambda_n y}{l_2}\right) - \gamma_n\left[\sin\left(\frac{\lambda_n y}{l_2}\right) - \sinh\left(\frac{\lambda_n y}{l_2}\right)\right]$$

Whereas $\lambda_{m,n}$ and the $\gamma_{m,n}$ are the mode shape constants.

2.2 Boundary condition 2

Two adjacent edges are clamped while the other two edges are simply supported–CCSS

$$X_m = \sum_{m=1}^{\infty} \sin\left(\frac{m\pi x}{l_1}\right)\sin\left(\frac{m\pi x}{2l_1}\right) = \frac{1}{2}\sum_{m=1}^{\infty}\left[\cos\left(\frac{m\pi x}{2l_1}\right) - \cos\left(\frac{3m\pi x}{2l_1}\right)\right]$$

$$Y_n = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi y}{l_2}\right)\sin\left(\frac{n\pi y}{2l_2}\right) = \frac{1}{2}\sum_{n=1}^{\infty}\left[\cos\left(\frac{n\pi y}{2l_2}\right) - \cos\left(\frac{3n\pi y}{2l_2}\right)\right]$$

2.3 Boundary condition 3

All sides are simply supported – SSSS

$$X_m = \sum_{m=1}^{\infty} \sin\left(\frac{m\pi x}{l_1}\right)$$

$$Y_n = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi y}{l_2}\right)$$

3. Theoretical calculations using Mat lab

Theoretical calculations for un-cracked and cracked Aluminum and GFRP composite plate is calculated using MATLAB for different boundary conditions.

3.1Theoretical Calculation for Natural Frequencies of Un-cracked and Cracked Aluminum Plate

The material used in the present work is Aluminum alloy and Glass Fiber Reinforced Polymer (GFRP) and their properties are given in the table 1

Table1 Material properties of Aluminum plate and GFRP Composite plate

Property	Aluminum Plate	GFRP Composite Plate
Young's Modulus (E) (N/m^2)	7.1×10^{10}	2.5×10^{10}
Density(ρ) (kg/m^3)	2770	1850
Poisson's Ratio (μ)	0.33	0.29

The theoretical calculation are performed using MATLAB for un-cracked and cracked Aluminum plate for different boundary conditions i.e. clamped – clamped – free – free (CCFF), clamped – clamped - simply supported – simply supported (CCSS), and all sides simply supported (SSSS) and crack geometry as shown below

For Half Crack 10 mm (l=10mm, b=2.5mm, d=10mm)

For Half Crack 20 mm (l=20mm, b=5mm, d=10mm)

For Half Crack50 mm (l=50mm, b=12.5mm, d=10mm)

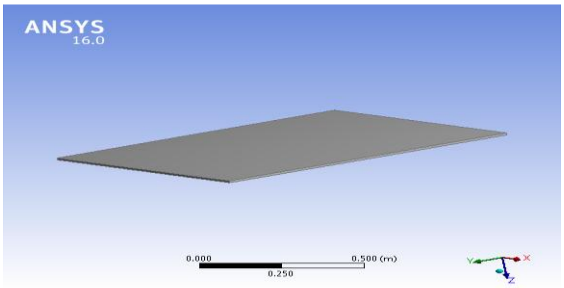
3.2 Specifications of the Test Plate

a. **Plate-1**: For this type of plate the length l_1=1m along x-axis and length l_2=1m along y-axis and the plate consists of a centre line crack of length l=2a, where a is the half crack length. We have considered uncracked plate and cracked plate of different half crack lengths such as a=0.01m, a=0.02m, a=0.05m, and thickness of plate is 0.01m.

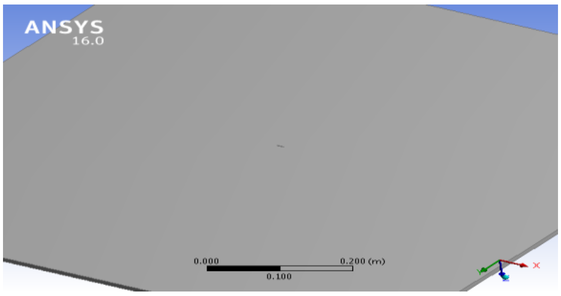
b. **Plate-2**: For this type of plate the length l_1=0.5m along x-axis and length l_2=1m along y-axis and the plate consists of a centre line crack of length l=2a, where a is the half crack length. We have considered uncracked plate and cracked plate of different half crack lengths such as a=0.01m, a=0.02m, a=0.05m, and thickness of plate is 0.01m.

c. **Plate-3**: For this type of plate the length l_1=1m along x-axis and length l_2=0.5m along y-axis and the plate consists of a centre line crack of length l=2a, where a is the half crack length. We have considered uncracked plate and cracked plate of different half crack lengths such as a=0.01m, a=0.02m, a=0.05m, and thickness of plate is 0.01m.

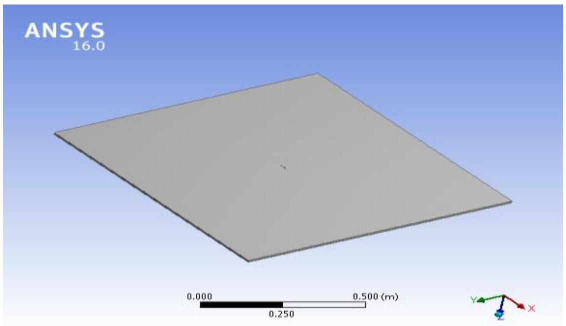
Plate-1



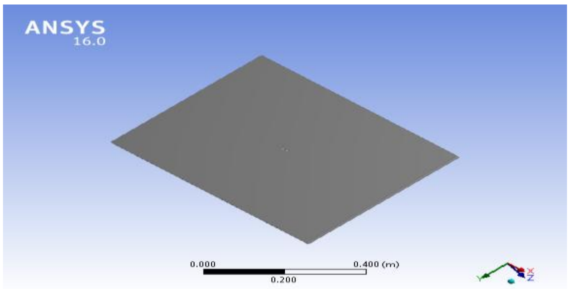
(a) Un-cracked Plate



(b) Crack length (l)=2a=0.02m



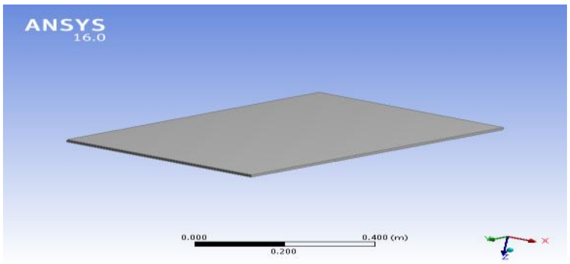
(c) Crack length (l) =2a=0.04m



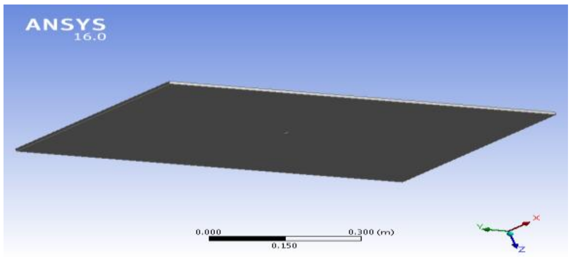
(d) Crack length (l) =2a=0.1m

Figure 1 Specifications of Plate-1

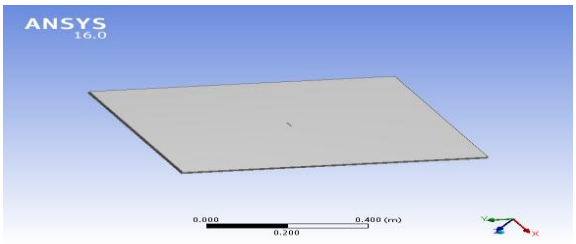
Plate-2



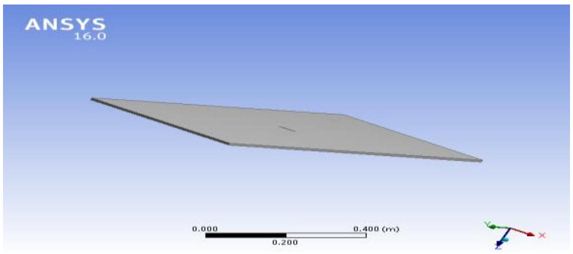
(a) Un-cracked Plate



(b) Crack length (l)=2a=0.02m



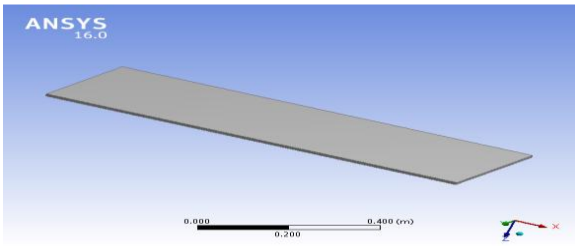
(c) Crack length (l) =2a=0.04m



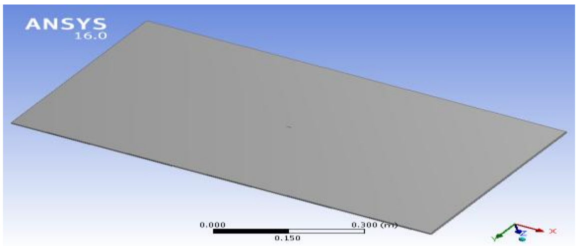
(d) Crack length (l) =2a=0.1m

Figure 2 Specifications of Plate-2

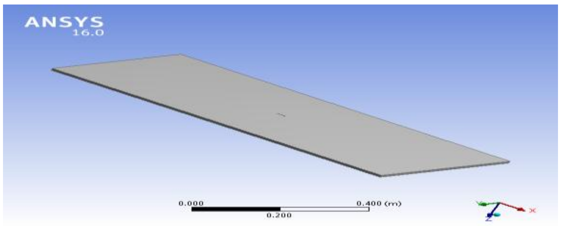
Plate-3



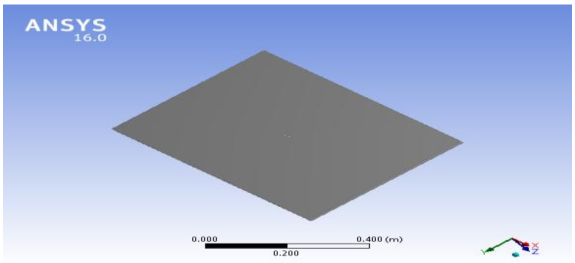
(a) Un-cracked Plate



(b) Crack length (l)=2a=0.02m



(c) Crack length (l) =2a=0.04m



(d) Crack length (l) =2a=0.1m

Figure 3 Specifications of Plate-3

Natural frequency of rectangular plate is given by

$$\omega^2{}_{mn} = \frac{K_{mn}}{M_{mn}}, \qquad f_n = \frac{\omega_{mn}}{2\pi}$$

$$K_{mn} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{mn}\int_0^1\int_0^1 \left[X^{iv}{}_m Y_n + 2X''{}_m Y''{}_n + Y^{iv}{}_n X_m - \frac{2a(\mu X''{}_m Y''{}_n + Y^{iv}{}_n X_m)}{3\left(\frac{\alpha^0{}_{tb}}{6} + \alpha^0{}_{bb}\right)(3+\mu) + (1-\mu)h + 2a}\right] X_m Y_n\,dxdy$$

$$M_{mn} = \frac{\rho h}{D}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{mn}\int_0^1\int_0^1 X^2{}_m\, Y^2{}_n\,dxdy$$

$$D = Eh^3/12(1-\mu^2)$$

The natural frequencies of un-cracked and cracked plate models for different boundary conditions for Aluminum plates are tabulated as shown in table 2

Table 2 Natural Frequency of Un-cracked and Cracked Aluminum plates for different boundary conditions

Type of Plate	Natural Frequency in Hz			
	Boundary Condition-1			
	Un-Cracked	**Cracked**		
		a=0.01m	**a=0.02m**	**a=0.05m**
Plate-1	11.90	11.42	11.07	10.43
Plate-2	33.35	33.12	32.62	32.38
Plate-3	33.43	33.14	32.93	32.49
Type of Plate	**Boundary Condition-2**			
	Un-Cracked	**Cracked**		
		a=0.01m	**a=0.02m**	**a=0.05m**
Plate-1	69.51	67.18	65.51	62.45
Plate-2	181.10	179.97	178.85	177.19
Plate-3	181.10	168.41	158.91	140.65
Type of Plate	**Boundary Condition-3**			
	Un-Cracked	**Cracked**		
		a=0.01m	**a=0.02m**	**a=0.05m**
Plate-1	15.30	14.84	14.52	13.92
Plate-2	31.35	30.96	30.69	30.21
Plate-3	33.87	33.43	33.11	32.55

The natural frequencies of un-cracked and cracked plate models for different boundary conditions for GFRP Composite plates are tabulated as shown in table 3

Table 3 Natural Frequency of Un-cracked and Cracked GFRP Composite plate for different boundary conditions

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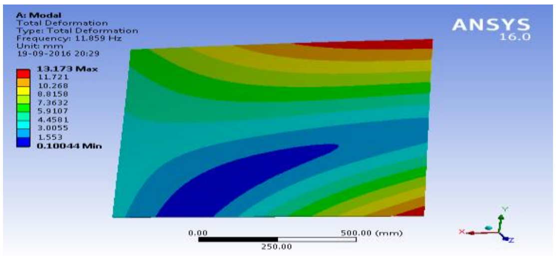
Type of Plate	Natural Frequency in Hz			
	Boundary Condition-1			
	Un-Cracked	**Cracked**		
		a=0.01m	**a=0.02m**	**a=0.05m**
Plate-1	8.56	8.23	7.98	7.50
Plate-2	24.03	23.97	23.94	23.80
Plate-3	24.06	23.85	23.83	22.97
Type of Plate	**Boundary Condition-2**			
	Un-Cracked	**Cracked**		
		a=0.01m	**a=0.02m**	**a=0.05m**
Plate-1	50.02	48.46	47.30	45.17
Plate-2	130.32	129.53	128.84	127.73
Plate-3	130.32	121.62	115.01	102.14
Type of Plate	**Boundary Condition-3**			
	Un-Cracked	**Cracked**		
		a=0.01m	**a=0.02m**	**a=0.05m**
Plate-1	11.01	10.71	10.48	10.07
Plate-2	22.56	22.31	22.14	21.82
Plate-3	24.39	24.09	23.88	23.50

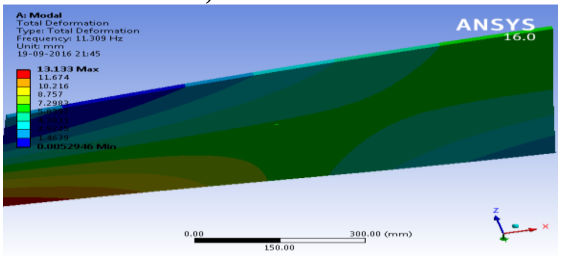
4. Modal analysis using ANSYS

Modal Analysis [3] of Aluminum plate for different crack lengths and different boundary conditions are performed using ANSYS 16 software and the mode shapes for different boundary conditions are shown below

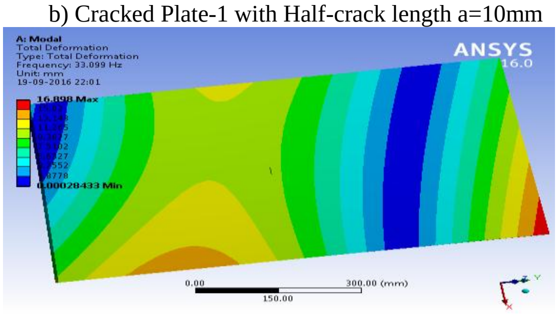
4.1 Boundary Condition -1



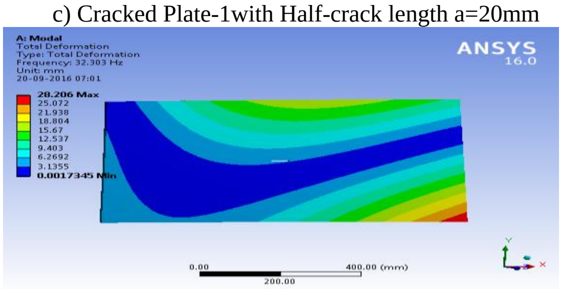
a) Uncracked Plate-1



b) Cracked Plate-1 with Half-crack length a=10mm



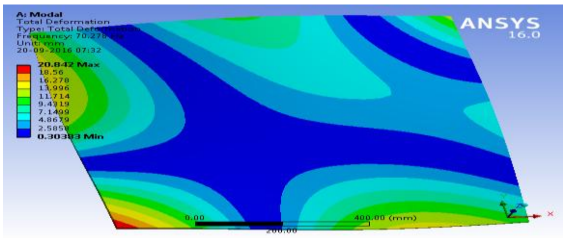
c) Cracked Plate-1with Half-crack length a=20mm



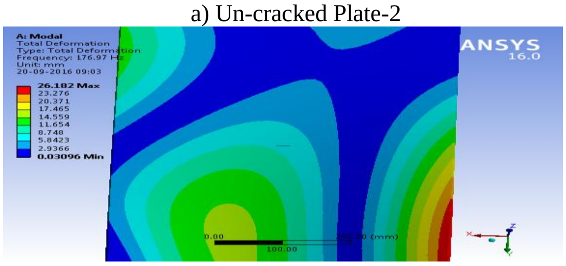
d) Cracked Plate-1 with Half-crack length a=50mm

Figure 4 First mode natural frequencies of aluminum plates for boundary condition-1

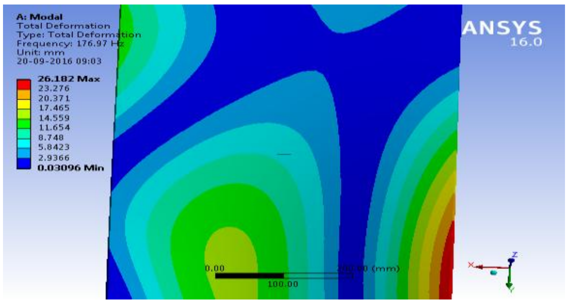
4.2 Boundary Condition -2



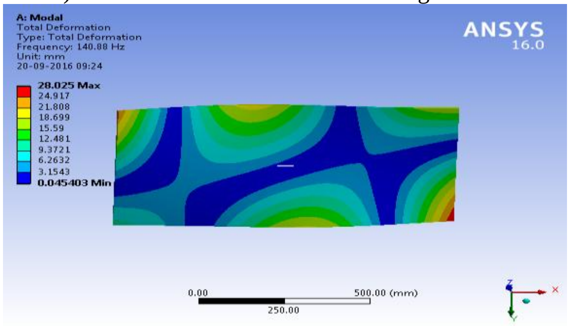
a) Un-cracked Plate-2



b) Cracked Plate-2 with Half-crack length a=10mm



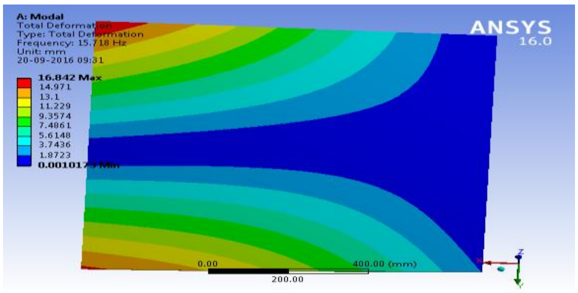
c) Cracked Plate-2with Half-crack length a=20mm



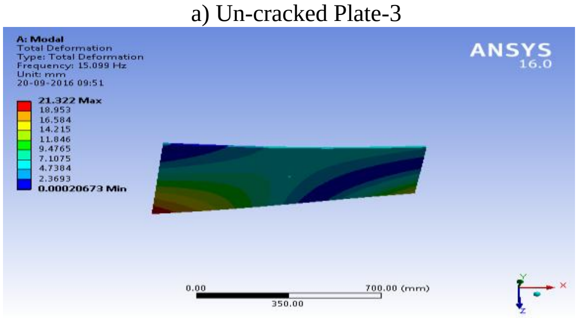
d) Cracked Plate-2 with Half-crack length a=50mm

Figure 5 First mode natural frequencies of aluminum plates for boundary condition-2

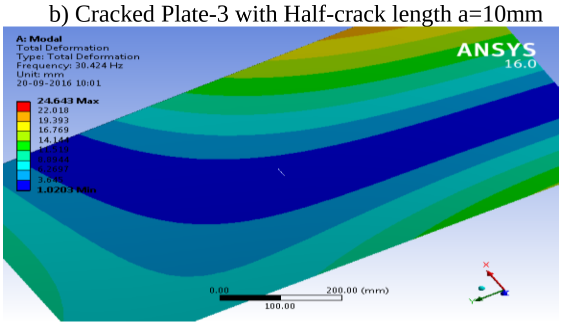
4.3 Boundary Condition -3



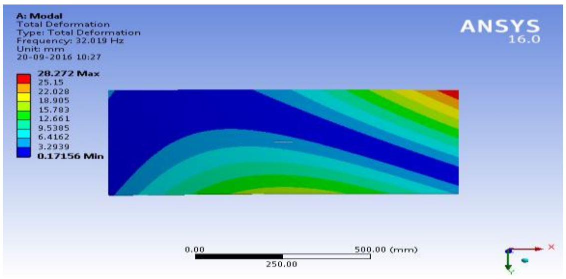
a) Un-cracked Plate-3



b) Cracked Plate-3 with Half-crack length a=10mm



c) Cracked Plate-3with Half-crack length a=20mm



d) Cracked Plate-3 with Half-crack length a=50mm

Figure 6 First mode natural frequencies of aluminum plates for boundary condition-3

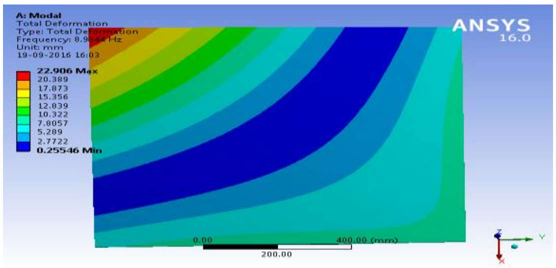
The natural frequencies of un-cracked and cracked plate [4,5,6] models for different boundary conditions for Aluminum plates are tabulated as shown in table 4

Table 4 Natural Frequency of Un-cracked and Cracked Aluminum plates for different boundary conditions

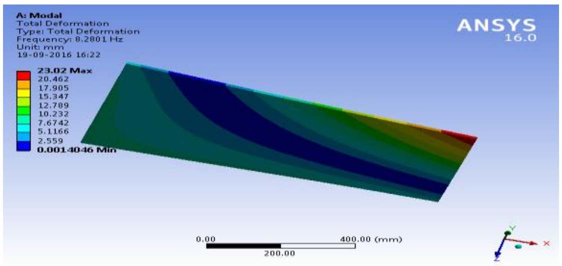
Type of Plate	Natural Frequency in Hz			
	Boundary Condition-1			
	Un-Cracked	Cracked		
		a=0.01m	a=0.02m	a=0.05m
Plate-1	11.85	11.30	11.28	11.23
Plate-2	33.39	33.10	33.08	32.42
Plate-3	33.44	32.95	32.45	32.30
Type of Plate	Boundary Condition-2			
	Un-Cracked	Cracked		
		a=0.01m	a=0.02m	a=0.05m
Plate-1	70.27	66.17	65.49	62.41
Plate-2	181.07	179.92	176.96	173.12
Plate-3	180.32	167.44	160.17	140.87
Type of Plate	Boundary Condition-3			
	Un-Cracked	Cracked		
		a=0.01m	a=0.02m	a=0.05m
Plate-1	15.71	15.09	13.99	13.94
Plate-2	34.32	32.44	30.42	29.25
Plate-3	34.32	33.14	32.45	32.01

Modal Analysis of GFRP Composite plate for different crack lengths and different boundary conditions are performed using ANSYS 16 software and the mode shapes for different boundary conditions are shown

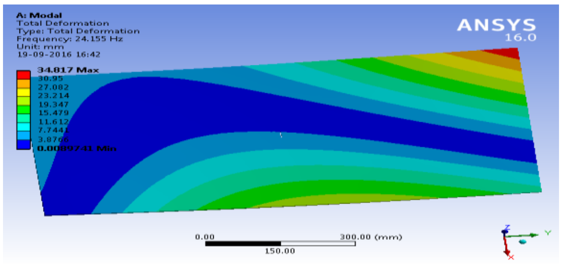
4.4 Boundary Condition -1



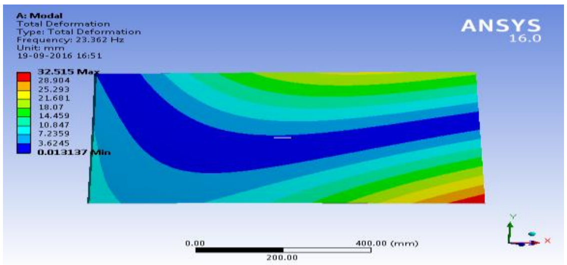
a) Uncracked Plate-1



b) Cracked Plate-1 with Half-crack length a=10mm



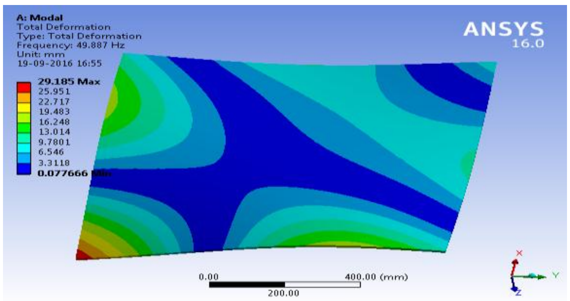
c) Cracked Plate-1with Half-crack length a=20mm



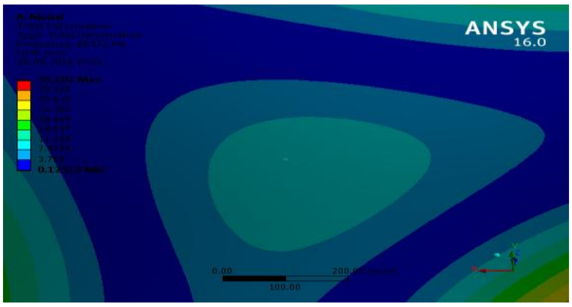
d) Cracked Plate-1 with Half-crack length a=50mm

Figure 7 First mode natural frequency of GFRP composite plate for boundary condition-1

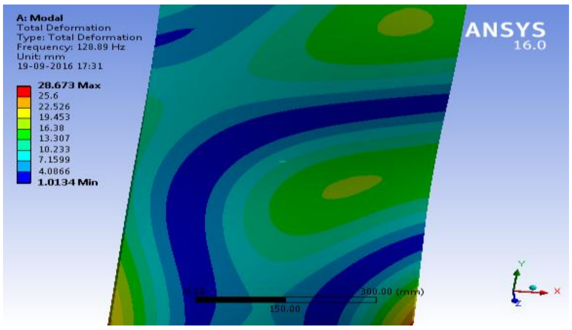
4.5 Boundary Condition -2



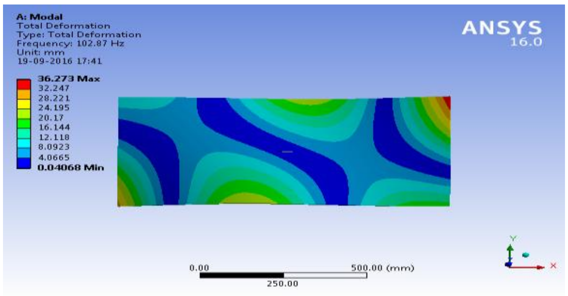
a) Un-cracked Plate-2



b) Cracked Plate-2 with Half-crack length a=10mm



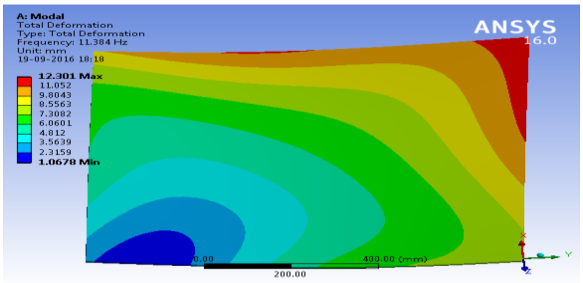
c) Cracked Plate-2with Half-crack length a=20mm



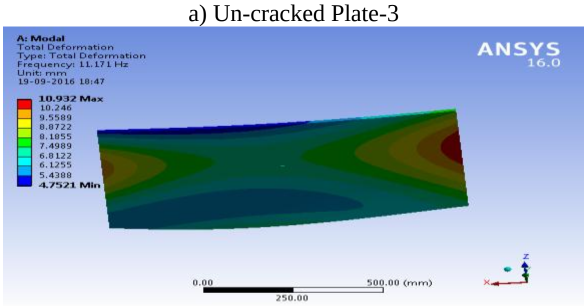
d) Cracked Plate-2 with Half-crack length a=50mm

Figure 8 First mode natural frequency of GFRP composite plate for boundary condition-2

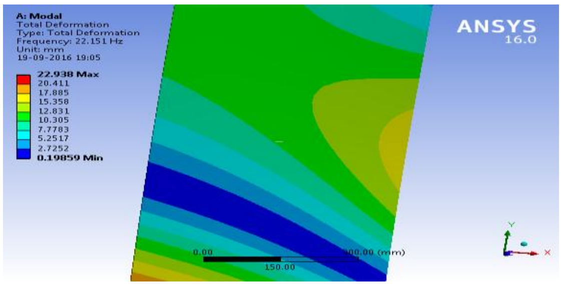
4.6 Boundary Condition -3



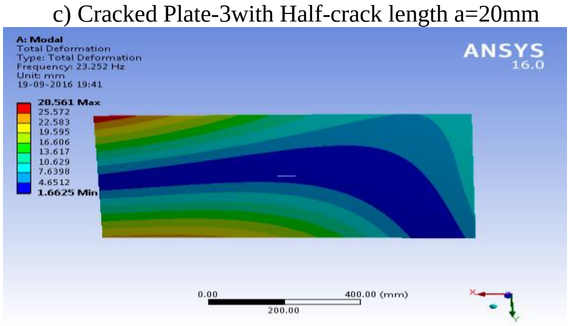
a) Un-cracked Plate-3



b) Cracked Plate-3 with Half-crack length a=10mm



c) Cracked Plate-3with Half-crack length a=20mm



d) Cracked Plate-3 with Half-crack length a=50mm

Figure 9 First mode natural frequency of GFRP composite plate for boundary condition-3

The natural frequencies of un-cracked and cracked plate models for different boundary conditions for GFRP Composite plates are tabulated as shown in table 5

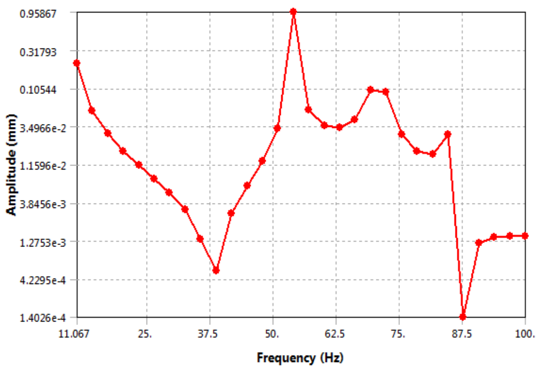
Table 5 Natural Frequency of Un-cracked and Cracked GFRP Composite plates for different boundary conditions

Type of Plate	Natural Frequency in Hz			
	Boundary Condition-1			
	Un-Cracked	Cracked		
		a=0.01m	a=0.02m	a=0.05m
Plate-1	8.89	8.27	8.03	7.88
Plate-2	24.56	24.28	24.15	23.86
Plate-3	24.69	24.16	24.02	23.36
Type of Plate	Boundary Condition-2			
	Un-Cracked	Cracked		
		a=0.01m	a=0.02m	a=0.05m
Plate-1	49.89	48.57	47.21	45.25
Plate-2	130.84	129.94	128.88	127.41
Plate-3	130.21	121.15	114.56	102.86
Type of Plate	Boundary Condition-3			
	Un-Cracked	Cracked		
		a=0.01m	a=0.02m	a=0.05m

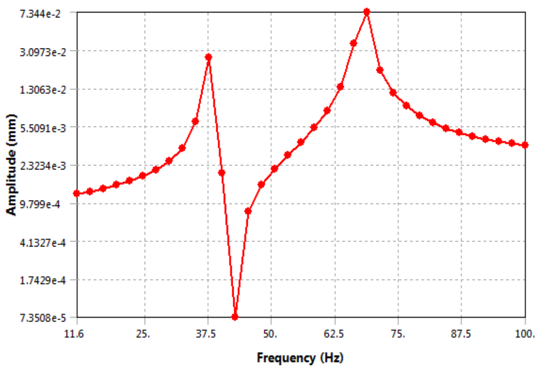
Plate-1	11.38	11.16	10.86	9.97
Plate-2	23.11	22.72	22.14	21.74
Plate-3	24.84	24.34	23.83	23.25

5. Harmonic analysis

The natural frequencies obtained in Modal Analysis are verified by the peaks obtained in the Harmonic Analysis. Harmonic Analysis of Aluminum plates and GFRP Composite plates for different crack lengths and different boundary conditions are shown in figure 10-11.

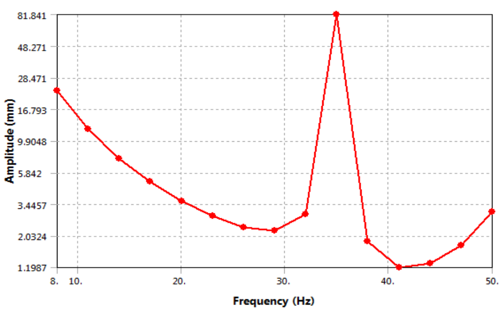


a) Uncracked Plate-1 for BD-1

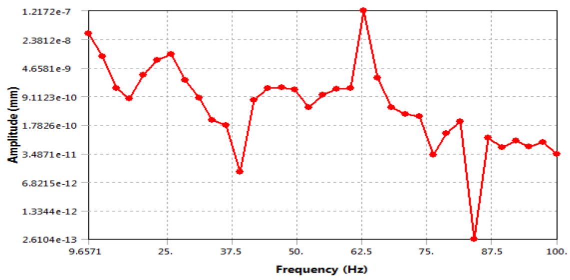


b) Plate-2 with Half-crack length 10mm for BD-1

Figure 10 Frequency responses of aluminum plates



a) Uncracked Plate-1 for BD-1



b) Plate-2 with Half-crack length 10mm for BD-1

Figure 11 Frequency responses of GFRP Composite plates

In harmonic analysis from figures 10 and 11 we can conclude that the peaks obtained in frequency response curves will match to the natural frequencies obtained in modal analysis.

6. Conclusion

In Damage will alter the present and future performance of the system. Therefore it is necessary to identify the damage well in advance, there are so many methods for identifying damage out of which changes in the frequency is one of the method.

In the present work frequency change is used to identify the damage in the plate [7,8] structures. In this work three types of plates are considered and it is denoted as plate-1, plate-2, and plate-3.

For all the three plates [9-11] modal analysis is conducted theoretically and numerically by considering the three boundary conditions i.e. Clamped – Clamped – Free – Free (CCFF), Clamped – Clamped - Simply Supported – Simply Supported (CCSS), and All sides Simply Supported (SSSS).

The following are the conclusions drawn from the results obtained

Theoretical modal analysis for Aluminum and GFRP [15-16] composite plates\

Plate-1: The theoretical natural frequencies of uncracked and cracked plate-1 is obtained from the formulae with the help of MATLAB software for different crack lengths and boundary conditions, it is observed that the values of natural frequencies are decreasing in a small amount with increasing the crack lengths.

Plate-2: The theoretical natural frequencies of uncracked and cracked plate-2 is obtained from the formulae with the help of MATLAB software for different crack lengths and boundary conditions, it is observed that the values of natural frequencies are decreasing in a small amount with increasing the crack lengths.

Plate-3: The theoretical natural frequencies of uncracked and cracked plate-3 is obtained from the formulae with the help of MATLAB software for different crack lengths and boundary conditions, it is observed that the values of natural frequencies are decreasing in a small amount with increasing the crack lengths.

Theoretical modal analysis for Aluminum and GFRP composite plates

Plate-1:The numerical natural frequencies of uncracked and cracked plate-1 is obtained with the help of ANSYS software for different crack lengths and boundary conditions, it is observed that the values of natural frequencies are decreasing in a small amount with increasing the crack lengths.

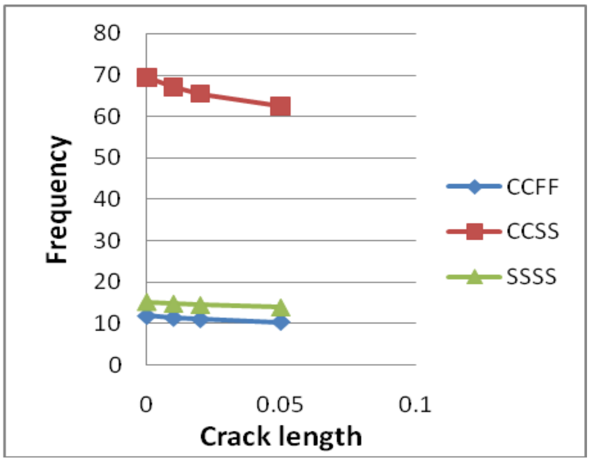
Plate-2: The numerical natural frequencies of uncracked and cracked plate-2 is obtained with the help of ANSYS software for different crack lengths and boundary conditions, it is observed that the values of natural frequencies are decreasing in a small amount with increasing the crack lengths.

Plate-3: The numerical natural frequencies of uncracked and cracked plate-3 is obtained with the help of ANSYS software for different crack lengths and boundary conditions, it is observed that the values of natural frequencies are decreasing in a small amount with increasing the crack lengths.

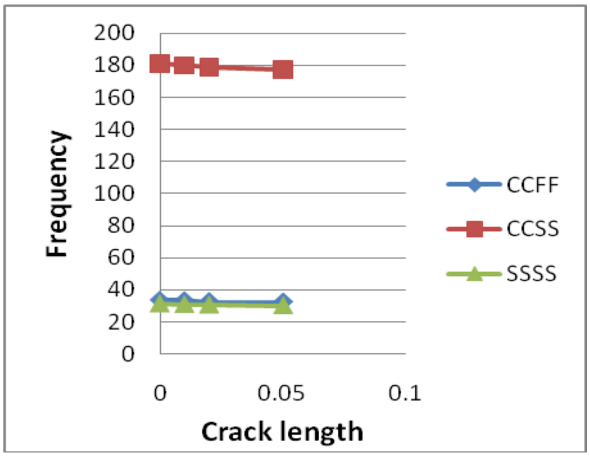
It is observed from the results the theoretical modal frequencies are almost coinciding with ANSYS results for all the cases considered in this work.

From the graphs obtained by varying the theoretical and numerical natural frequencies with crack lengths it is clearly shown that decreasing in the natural frequencies for increasing in the crack lengths for different boundary conditions.

Therefore finally it is concluded that this method is used to detect the damage in plate structures by observing the changes in the natural frequency.

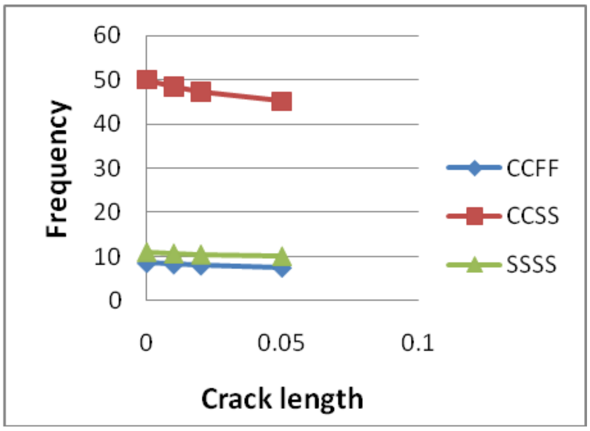


a) Variations of Natural frequency vs Crack length for Plate-1

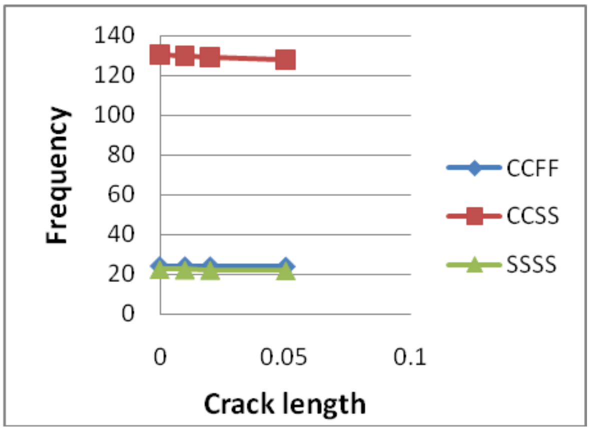


b) Variations of Natural frequency vs Crack length for Plate-2

Figure 12 Variations of Natural frequency vs Crack length for Aluminum plates



a) Variations of Natural frequency vs Crack length for Plate-1



b) Variations of Natural frequency vs Crack length for Plate-2

Figure 13 Variations of Natural frequency vs Crack length for GFRP Composite plates

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