Mathematical Modeling and Markov Chain Analysis of Corrosion Propagation in Pipelines to Predict the Failures

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ABSTRACT- *Pitting corrosion is localized corrosion that often causes leak and failure. This paper develops a basic foundation for a tool that can be used to predict the probability of leak occurring in oil pipeline due to pitting corrosion. The methodology is applicable to steel and infrastructure. The stochastic nature of pitting corrosion of metallic has been widely recognized. It is considered that this type of deterioration preserves no memory of the previous, so only current state of the damage influences its future development. By means of a simple Markov chain process, we formulate equations for probability distributions of a pit being in a defined set of corroded states. Each state represents a specific pit depth. By adjusting transition rates at different states we represent the corrosivity and mitigation conditions to which the oil pipeline is subjected. The transition rate models used here are flexible and capable of accommodating a wide range of corrosivity and mitigation scenarios. Mathematical relation is developed in order to predict the life of the pipelines by considering various parameters. We discuss hypothetical cases, such as increasing CO_2 content in oil causing gradual corrosion versus an episodic event causing rapid changes in the corrosivity conditions, demonstrating the ability to make adjustments to the model in order to simulate varying operational conditions.*

Index Terms- Corrosion, Failure, Markov Chains, Pitting, Lifetime.

I. INTRODUCTION

This work addresses the need for improved methods for corrosion risk management, life prediction and performance assessment for more effective corrosion control strategies and implementation. The reliability and safety of aging infrastructure is of huge importance. Further, corrosion costs to the United States were determined in 2002 to be $276 billion per year [6].Preventive strategies to reduce corrosion costs were described in the NACE International study, Corrosion Costs and Preventive Strategies in the United States. A major finding was the need for technical advances in methods for performance assessment and life prediction. Formal regulatory requirements have become more comprehensive and The Markov chain, although simplistic, serves as an excellent device to model the progression of corrosion. This paper uses a Markov chain to

Predict failures in a system. This process involves a finite number of states and the probabilities of moving from one state to the next. The key idea assumed is that each state depends only upon the current state. The events occurring prior to the previous state do not influence the current state [1]. In short, the probability of moving from state i to the state immediately following, state i+1, are the only probabilities considered. Considering the possibility of making a larger jump from one stage to a stage farther in the chain is something that could be explored in a future expansion of this model.

To begin the development of a useful tool which can be modified for any set of conditions, this paper lays out the initial concepts of the model predicting the probability of a leak in an oil pipeline due to corrosion, and the variables that are to be used throughout the remainder of the discussion. Modeling the movement from one state to another is introduced conceptually. Several case studies are considered to test and demonstrate the capabilities and output of the model being developed. Parameters are varied to explore various examples of conditions the pipeline could be subjected to and possible responses a facility could take to counteract the changes in corrosivity. It should be noted that this paper uses hypothetical estimations that are not based on actual data. This lack of data creates a limitation for the analysis presented in this paper since the model developed is not benchmarked against any results. Lastly, conclusions are drawn based on the outcomes of the case studies, and suggestions for future applications are given. For instance, other work could be done using this model to investigate possible responses a facility could take to counteract the changes in corrosivity. An example would be in [4], the topic of repairs or replacements is explored, which are examples of possible responses that could be determined by an expansion of the model developed in this paper. An exemplar of the increasing need is The Pipeline and Hazardous Materials Safety Administration (PHMSA) recent rule establishing integrity management requirements for gas distribution pipeline systems.

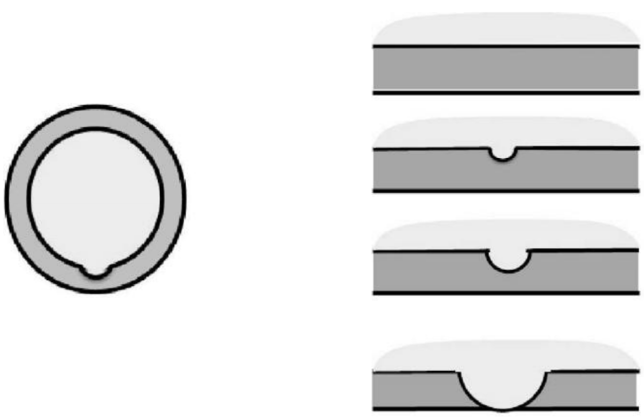
As part of the rule, the operator's IM program elements must include: identify threats; evaluate and rank risks; identify and implement measures to address risks; measure performance, monitor results and evaluate effectiveness [8]. Improved risk management tools such as the Markov chain based method here are essential to meet these needs. This paper develops a basic foundation for a tool that can be used to predict the probability of a leak occurring in an oil pipeline due to pitting corrosion. The methodology is directly applicable to other steel equipment and infrastructure in service in corrosive conditions.

II. CONCEPTUAL FRAMEWORK OF THE MODEL

According to Fang [3], the typical appearance of corroding pits in pipelines is hemispherical; therefore, hemispherical pits are the type that are accounted for by this model. The scenario the model captures is demonstrated in Figure 2.1A and Figure 2.1B. In Figure 2.1A, the cut out of a pipe is shown with a pit developing on the inner side of the pipe. See Figure 2.1B for a demonstration of the pitting progression occurring in various stages[1].

This model considers $n = 5$ stages of corrosion. Stage 1 represents a pipe where no corrosion has occurred. Stages 2 through 5 represent the gradual development of the corrosion in the pipe; Stage 5 being the last before an actual failure, meaning a leak in the pipeline. The probability of being in each of the stages is represented by P_i; see Table 2.1. Using the thickness of the pipe, H, and n stages, we define the depth of pitting between each stage, $\frac{H}{n}$.

We restrict pipeline transitions from one state to the state immediately following.



A B

Figure 2.1: (A) Cross-sectional view of a pipe with a small hemispherical pit. (B) Demonstration of the various stages of corrosion occurring in a pipe's wall.

Any preventative measures taken in response to corrosive conditions serve only to hinder the further corrosion of the pipeline, but do not return the pipeline to previous to a previous state with less corrosion. Repairs are not accounted for in this paper. The transition rate from one state to the next, λ, is a function of expressions which are discussed and developed later in the paper.

The function λ represents the rate at which pits in an oil pipeline move from one stage of corrosion to the next in a defined area of a pipe, say a kilometer in length. λ is only a function of time and a parameter b, which reflects corrosivity and mitigation conditions. Fully understanding b in terms of corrosivity and mitigation conditions is left to future work. The main goal is to develop and evaluate models for λ that allow us to make predictions on how long an oil pipeline will operate before a leak occurs in the section of pipe being examined.

Description	**Variable**	**Function of**
Time(years)	$t \in [0,50]$	Independent
Corrosivity	b	t, Corrosivity conditions, Materials of the pipe, Flow rate, Phase of the substance(gas, oil, water), etc.
Rate at which pipe moves to next state ($\frac{1}{time}$)	λ	t, b
Probability of being in state i	$P_i \in [0,1]$	t

Table 2.1: List of variables used throughout the paper.

The purpose of this paper is to demonstrate the flexibility of λ and the ability to adjust the model to simulate various scenarios.

III. MODEL FORMULATION

The equations demonstrating the Markov process [1] are defined as follows:

$$\frac{dP_1}{dt} = -\lambda_1 P_1 \tag{3.1}$$

Three variations of the λ_i's are considered. Refer to Table 3.1. In the simplest scenario, all λ_i's equal the same constant, b. In the second case, $\lambda_i = b + 0.05(i - 1)$ for $i = 1,2,...$; therefore, $\lambda_1 < \lambda_2 < ... < \lambda_i < ... < \lambda_N$. The final consideration is λ defined as

$$\lambda_i = \frac{bi[a^2 + t]}{[a^2 + t^2]}, \quad i = 1, 2, \ldots \tag{3.2}$$

where a is used to define the period of time where conditions remain constant within the pipe.

Case	Description	Equation
(1)	All All λ_i's are equal to the same function	$\lambda_i = b$, for i=1,2,...
(2)	$\lambda_1 < \lambda_2 < \lambda_3 < ...$	$\lambda_i = b + 0.05(i - 1)$, for i=1,2,...

Table 3.1: Functions used for λ_i, to model the rate at which a pit moves from one state of corrosion to the next.

Simple variations of λ that are being used in this paper, such as $\lambda_1 = \lambda_2 = ... = \lambda_i = ... = \lambda_N$; or λ_i's equal to the same function, with each λ_i only varying by a factor, have been explored and used as

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a basis for further expansions in numerous papers [1, 2, 3]. Provan and Rodriguez fit equation (3.3) to exponential data[4]. We modify their model as in equation (3.2) to be used for the third consideration of λ [1,3],

$$\lambda_i = \frac{bi[1+bt]}{[1+bt^k]}, i = 1, 2, \ldots \qquad (3.3)$$

Let $D(t)$ represent the depth of a pit. In [3], $D = t^b$ is used to model pit depth. The damage rate, b, can be a function of various parameters such as pipe material, radius of the pipe, corrosivity of the environment (CO_2, pH, chloride concentration), and flow rate within the pipe. It can also change with time as corrosivity and/or mitigation conditions change. Taking the natural logarithm of both sides we find

$$\ln(D) = b \cdot \ln(t). \qquad (3.4)$$

(3.4)

Upon differentiation we obtain

$$\frac{1}{D}\frac{dD}{dt} = \frac{b}{t} + \frac{db(t)}{dt} \cdot \ln(t). \qquad (3.5)$$

Equation (3.5) represents the rate of change of pit depth with respect to time. The unit illustration is as follows:

[(Depth/Time)/Time]=1/Time= $\boldsymbol{\lambda}$ (3.6)

Equation (3.5) could represent λ. Notice, in equation (3.5), db(t)/dt>0 represents a threat occurring, meaning an event takes place that increases the conditions for corrosion; db(t)/dt <0 means a mitigation scheme is applied, decreasing the corrosivity conditions.

These ideas are captured by the model (3.2), where using equation (3.3), the variable a gives us the opportunity to account for periods of time where the conditions of the pipe are constant. The power k=2 is chosen. [5,6,7] When smaller values of a are chosen, and at larger values of t, λ behaves like $\frac{b}{t}$ as seen in equation (3.2) and Figure 3.1. Increasing λ indicates that the conditions on the pipe are more corrosive; therefore, the rate at which the pipe moves to more damaged states increases. As λ decreases, either conditions are less corrosive or mitigation schemes have been applied. Hence, the rate at which the pipe becomes more damaged slows down. Functions and values for the corrosivity conditions relative to time, represented by b, are explored in following sections.

IV. DEMONSTRATION OF PROPOSED TRANSITION RATES

This chapter demonstrates the ability of the proposed model to accommodate various scenarios, such as changes in corrosivity or mitigation conditions. By exploring different expressions for b, which controls the rate of pit growth, we can simulate these changes and see how they affect the probabilities of a failure occurring in the system. Within each of the three forms for λ (Table 3.1), we examine subcases, summarized in Table4.1.

The first subcase is λ_i=const, resulting in constant corrosivity conditions. This means that the conditions the pipeline is subjected to are neither becoming more severe nor more benign, simulating a bare steel pipe in an unchanging environment. See Figure 6.6A for the graph of this λ.

Sub case	Scenario	Possible Cause	Function
s-1	Constant corrosivity	Bare steel pipe	$b = c$, c=0.33
S-2	Increased corrosivity Remaining constant	Loss of inhibiter effectiveness	b=c, c=0.33,t∈[0,8) c=0.5,t∈[8,50]
S-3	Gradual change in coorrosivity	Water cut; souring	b=c+mt, c=0.33,m=0.02/yr
S-4	Episodic increase in corrosivity	Inhibition injection failure	b=c, c=0.33,t∈[0,8) c=1.0,t∈[8,8.25] c=0.33, t∈(8.25,50]

Table 4.1: Subcases varying the equation for b, to explore different scenarios

The second subcase is λ_i's are equal to a piecewise constant function. Refer to Figure 6.6A for the graph of λ. At the beginning, the corrosivity conditions are constant. Nothing is changing, until after eight years (chosen for illustration), a step change in b occurs, indicating that a sudden event occurred - a loss of inhibitor effectiveness or step increases in CO_2 concentration caused by a change in the fluid flowing through the pipe, for example. An increase in b means the corrosivity conditions on the pipe are getting worse; therefore the pipe is more likely to experience pitting. No Table 4.1: Subcases varying the equation for b, to explore different scenarios mitigation schemes are applied, so after the event the corrosivity conditions remain constant, resulting in a constant b after the upset at eight years.

The third subcase is λ_i's are equal to a linear function with a gradual slope. Refer to Figure 6.6A for a graph of λ. A gradual increase in CO_2 in the pipeline, souring due to H_2S content, or an increase in the water/oil ratio present in the material flowing through the pipeline are examples of possible causes of a gradual change in corrosivity conditions. As b increases over time, the corrosivity conditions are becoming more aggressive. In this scenario, no mitigation schemes are applied so the conditions only continue to worsen with no action taken to prevent further damage.

The fourth subcase is λ_i's are equal to a piecewise constant function incurring two step changes. The difference between this scenario and the second subcase is that in the second subcase the event caused an increase in the corrosivity conditions and a response was never made to mitigate the event. In this scenario, an event occurs, after eight years, which rapidly increases the corrosivity conditions; perhaps a loss of inhibition. Three months later, a mitigation scheme (for example, restoration of an inhibitor occurs) which decreases the corrosivity conditions. The decrease in b decreases the rate at which the pipe will move into a more damaged state, therefore, prolonging the life of the pipe. See Figure 6.6A for a graph of λ.

The fifth and final subcase is the scenario where λ_i's are equal to a constant, similar to the first subcase discussed. In this scenario, however, after eight years, an in-line inspection is conducted. The results from the in-line inspection give the number and depths of the pits present in the pipe. Using this information on the pit depth distribution, we define probabilities of a pit in the pipe being in one of the stages of corrosion. These probabilities are then compared to the predicted probabilities at t=8 years, and conclusions can be made as to whether or not the model was underestimating or overestimating the condition of the pipe. We demonstrate how uncertainties of the transition rates create a wide spread in predictive values.

The actual probabilities from the in-line inspection results are thus used to reset the initial conditions so that predictions about the conditions of the pipe are recalculated starting at t=8 years. We demonstrate how the uncertainty of the predictions decreases when compared to the original predictions, and so the spread of values over time narrows. This allows for reduced uncertainty, more accurate predictions, and more precise decisions can be made as to when the pipe should be repaired, replaced, or mitigation schemes applied, see Figure 6.7.

V. TOOL VALIDATION

A MATLAB code, was written to solve the system conditions based on equations (3.1). The ordinary differential equation solver ode45 was used, with absolute and relative tolerances set to $1\text{x}10^{-10}$. We solved equation (3.1) analytically for the case when λ_i's are equal to the same constant. The initial conditions are assumed to be $P_1 = 1$ and $P_i = 0$ for $i = 2,3,...$, meaning that the probability of being in a stage where no corrosion has occurred at $t = 0$ is 1. The results were compared to those output by the code, as a validation. Equation (3.1) is a first order linear ordinary differential equation which has as its solution

$$P_1 = ce^{-\int_0^t \lambda d\tau}. \qquad (5.1)$$

We utilize the initial condition $P_1(0) = 1$ to find $c = 1$, therefore

$$P_1 = e^{-\lambda t}. \qquad (5.2)$$

Solving for the remaining probabilities where the initial condition $P_i(0) = 0$ for $i = 2,3...$ yields

$$P_i = \frac{\lambda^{i-1}}{(i-1)!}t^{i-1}e^{-\lambda t}. \qquad (5.3)$$

The solutions output by the code graphed along with the analytic solutions found in equation (5.2) and equation (5.3) can be seen in Figure 5.1. The plots of the code output and analytic solutions agree; hence, we proceed with using the developed MATLAB code as a tool for solving the underlying equations of this problem. The solutions give us the probabilities of being in each stage of corrosion, P_i, and the summation of these yield the probability of no leak in the pipe due to a corroded pit. One minus this summation is the probability of failure. Now that we have an accurate and reliable computational tool, we can present in the next chapter the purpose of this paper, which is to apply "what-if" scenarios to the model and make adjustments accordingly to demonstrate its flexibility.

VI. CASE STUDIES

Here we discuss two possible scenarios, summarized in Table 3.1 that the proposed model can accommodate. Within each section, the variations shown in Table 4.1 are presented.

6.1 Case 1: $\lambda_i = b$, for i=1,2,...,n

For Case 1, all λ_i's are equal. This represents a scenario in which the corrosion conditions remain fixed.

6.1.1 Subcase 1

The first scenario is a basic example where all λ_i's are equal and b is a fixed constant. This could serve as a model for a bare steel pipe with a constant corrosivity of fluids. A value of 0.33 is commonly chosen for b in cases such as this[4].

Figure 6.1 represents the various probabilities of being in each of the 5 states prior to failure. The probability of being in a more damaged state is initially zero and then gradually increases as time passes. This makes practical sense since as a pipe begins to corrode the probability of moving into a more damaged state increases, until it eventually moves into the next state, which then makes the probability of being in its previous state decrease. The results show that after six years the probability of a leak occurring begins to rapidly increase; with a 70% chance of failure occurring around seventeen years. For this scenario, we select 70% as the corrosion limit where corrective action is required. Various corrective actions could be considered, e.g. make repairs or replace the pipe. The value for the "corrective action limit" depends on outcomes of risk analysis. For the remaining cases, we use 70% as benchmark for the corrective action limit.

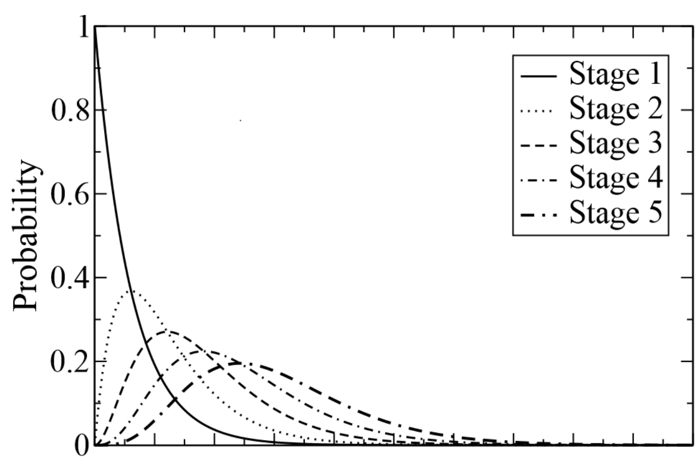

Figure6.1 Probability of stages in corrosion Subcase1

6.1.2 Subcase 2

The second case is the occurrence of a sudden increase in corrosivity, represented by a step change in a piecewise function. Both pieces are constant values before and after the increase in corrosivity. The initial value of b still equals 0.33, but after eight years suddenly increases to 0.5.

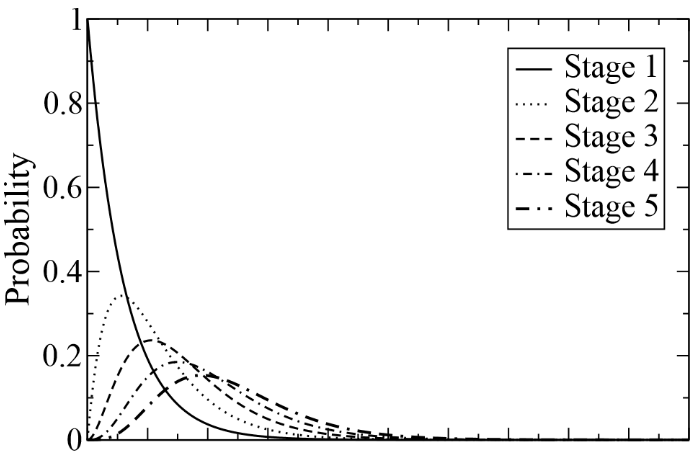

Figure6.2 Probability of stages in corrosion Subcase 2

6.2 Case 2: $\lambda_i = b_i$, for i=1,2,...,n, where $b_i = b + 0.05(i - 1)$

For Case 2, the λ_i's equal different functions, $\lambda_i = b_i = b+0.05(i-1)$, for i=1,2,... This represents a scenario in which the corrosion rate increases as the damage increases. The value of b is varied in the subsections to follow.

6.2.1 Subcase 1

Similar trends appear in Figure 6.3 when compared to Figure 6.1 from Case 1, because b is the same; it is only the λ_i's that change between these cases. In this case, the probabilities of being in more corroded stages increase and peak a few years later because the λ_i's increase in this scenario instead of all being equal like in Case 1.

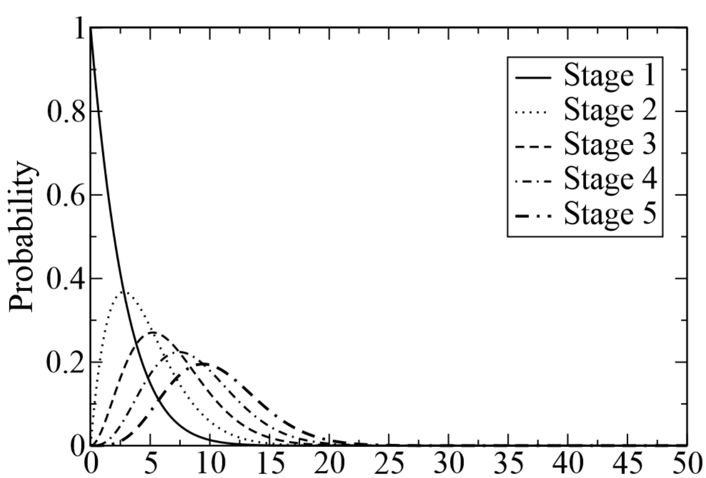

Figure6.3 Probability of stages in corrosion Subcase1

6.2.2 Subcase 2

Figure 6.4 shows a very slight increase in the probability of a leak occurring because the probabilities of being in more corroded states increase due to the step change in b from 0.33 to 0.5.

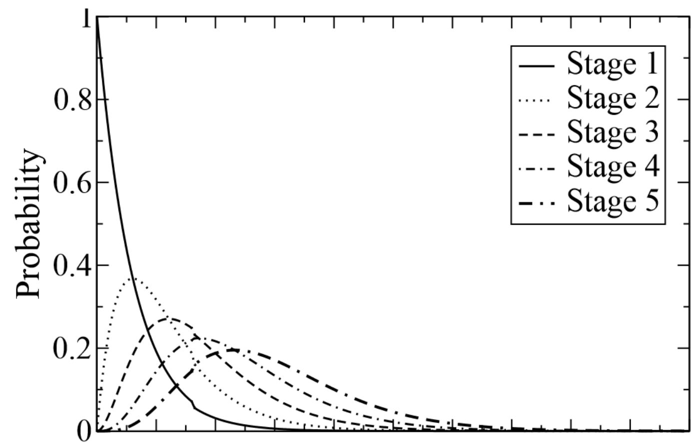

Figure6.4 Probability of stages in corrosion Subcase 2

VII. CONCLUSION

A model was developed to predict the probability of an oil pipeline experiencing a leak due to pitting under corrosive conditions using an n-stage Markov model. The MATLAB code computes the probabilities at any given time of an oil pipeline leaking. We discuss possible scenarios of transition rates in the model and estimate parameters one could use to simulate such scenarios, to demonstrate the ability of the model to be adjusted to fit the conditions of the pipeline at any time. A primary focus of this paper is to examine the variations in probabilities of failure due to changes in modeling corrosion damage evolution. We consider three versions of the function modeling the transition rate for the pipe moving from one corroded state to the next, we also consider different models for the corrosivity conditions. Hypothetical scenarios are discussed, such as a pipeline coating failing, an inhibitor losing effectiveness, or a water slug occurring. Parameters are then estimated, and changes in probabilities of a failure are analyzed for the different conditions simulated. The main purpose of discussing a wide range of corrosivity conditions and mitigation schemes is to demonstrate the ability to adjust the model for any conditions. The approach is useful for increasing the reliability and safety of aging infrastructure. The effectiveness of improved risk management tools, such as the Markov chain based method here, are essential to meet these needs for formal Integrity Management programs.

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