Design and Dimensional Analysis of Mechanisms

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***Abstract* — Dimensional analysis has been used extensively to solve problems in a wide-ranging area, from structural mechanics to automobile suspension system to satellite modeling. Some other unusual areas covered by this technique include operations research and human population studies. Recent applications include a solution to dynamics of robotic systems. A novel combination of activity-based model reduction and dimensional analysis enable to assess the relative importance of each parameter in the system and neglect the least important ones, thereby providing more freedom than strict similitude-based scaling.**

Dimensional analysis has been used extensively to solve problems in wide -ranging areas, from structural mechanics to satellite to population studies. However, there appears to be very little attention paid to the use of the techniques of dimensional analyses for analysis of kinematic and dynamic analysis of mechanisms. This deals with deriving the terms needed for scaling various parameters like angular displacements, velocities, accelerations torques and frequencies. Extensive correlations are obtained for the above parameters for different configurations of slider crank and four-bar mechanisms. Comments on the use of this technique for solving more complex high-speed mechanisms are made.

Keywords— *dimensional analysis, switch gear mechanism four bar mechanism*

I. Introduction

Dimensional analysis has proved to be a very powerful methodology for solving many problems in the area of structural mechanics, biomechanics, control engineering,

robotics, mechanical reliability, metal forming, population studies, silicon rod processing etc. There are a large number of textbooks on the subject right from the year 1945 [1-6].

However, researchers in the field of mechanisms seem to be unaware of the tremendous potential of dimensional analysis. There have been very few investigations reported in the past in the use of this methodology. Very recently, Meek [7] has utilized dimensional analysis for the solution of torques in unbalanced slider crank, although the utilization being for. Earlier, a dimensional analysis was utilized by [8] for solving a mechanism used in automobiles. Other than this anse and very few other papers, there does not appear to be proper use of the Dimensional analysis

Buckingham Pi theorem connects the physical and material quantities taking part in a particular physical process through a relationship k between n number of physical and material quantities and the rank r of the dimensional matrix described by K= n-r The rank r is prescribed as r=2 for kinematic and static problems and k=r-3 for dynamic analysis.

High -speed mechanisms are utilized in many industrial systems and examples of these include textile machinery, slider crank mechanism in automobiles and switchgear mechanisms. While standard methods are available for kinematic analysis of simple mechanisms from literature, such methods do not appear to have been used for the analysis of high-speed mechanisms due to inherent limitations of the methods.

The best productivity, latest-technology system demanded by the mechanical industry need high operating speeds, superior reliability, accurate performance, high-precision and lightweight machinery. In order to overcome the problem of high-speed operation and increase efficiency, weights of many components in industrial robots and various machines are reduced. As operating speed increases and weights of components decrease, a rigid-body model of a mechanism is not enough anymore. So, these components cannot be treated as rigid links as they become flexible due to their high speeds of operation and associated large inertial forces. High-speed lightweight manipulators can be thought as an example of a flexible multi-body system. Modelling and analysis of flexible mechanisms have been researched since the early 1970s.

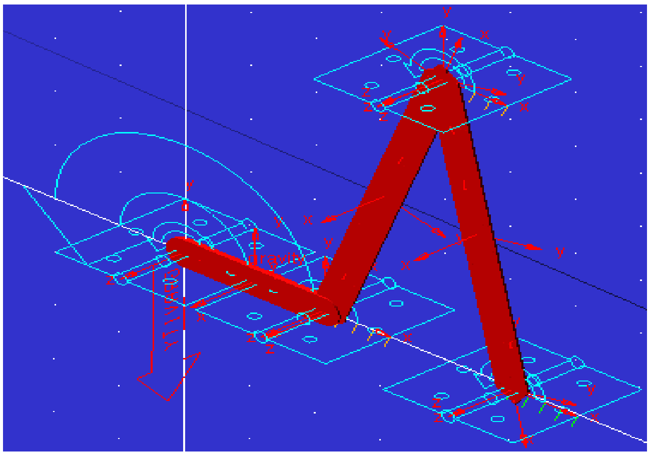


Fig. 1. FBM model in Adams software

II. ANALYSIS OF FOUR BAR MECHANISM

The full-size four bar mechanism (FBM) used for illustrating the correlations of the results obtained using the Pi terms determined earlier (eqns.) has the configuration given in Table I. The cross-sectional area and moment of inertia which are used in the dynamic analysis also mentioned.

Name of link	Length (mm)	Cross sectional area (mm^2)	Moment of Inertia (mm^4)	Mass of the link (kg)	Young's modulus (GPa)	Density (kg/m^3)
Crank	108	(25.4x4) 107.7	166	0.3233	200	7801
Coupler	274	(25.4x1.6) 40.6	8.674	0.3153	70	2740
Follower	269	(25.4x1.6) 40.6	8.674	0.3053	70	2740
Ground link	254	--	--	--	--	--
Speed of crank (rpm)	(Full):3000		(Half):1500		(1/3):1000	

The velocities, accelerations, and torques are analyzed by using MATLAB programming. The stresses developed in links when the mechanism operate at different speeds are analyzed using Multi-body dynamics software Adams as it gives the stresses while the mechanism is in the dynamic condition. The frequencies at different crank positions are analyzed using Finite element analysis software ANSYS. The results of using the Pi terms discussed above together with those obtained by using the analysis are shown. shown in Figs. 2 and 3 respectively. Also, the crank torque is plotted in *Fig. 4*.

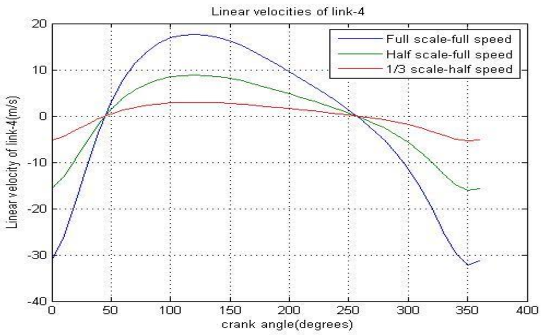


Fig. 2. FBM velocity of output link for different scales and speeds

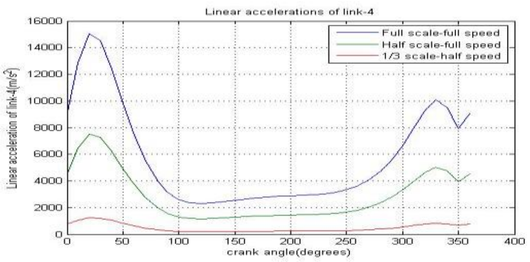


Fig. 3. FBM acceleration of output link for different scales and speeds

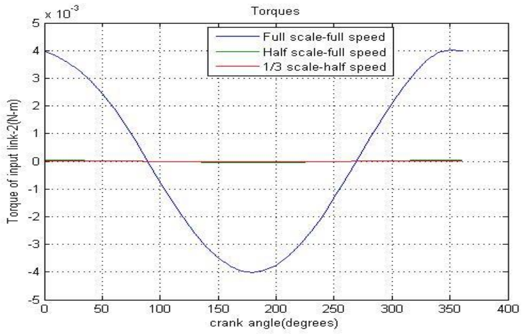


Fig.4. FBM torque of crank for different scales and speeds

The results obtained are correlated for velocity, acceleration, torque, stress and frequency by using Buckingham Pi theorem as shown below and the correlations are made and presented in Tables III

Buckingham π terms:

Velocity: As velocity is function of length (L) and time (T),

$$\text{Scale factor } S_{Vel}=\left(\frac{v_p}{v_m}\right)=\left(\frac{L_p}{L_m}\right)\left(\frac{T_p}{T_m}\right)$$

For 1/2 scale and full speed, π=(2)(1)=2.

At 0^0, v_p= -15.6283(2)=-31.2566

At 90^0; v_p=7.886 (2)= 15.772

At 180^0; v_p=6.3031 (2)= 12.6063

For 1/3 scale and 1/2 speed, π=(3)(2)=6

At 90^0; v_p=2.6287 (6)=15.7722

At 180^0; v_p=2.1010(6)=12.606.

Acceleration: As acceleration is function of length (L) and time $(T)^2$,

$$\text{Scale factor } S_{accn}=\left(\frac{v_p}{v_m}\right)=\left(\frac{L_p}{L_m}\right)\left(\frac{T_p}{T_m}\right)^2$$

For 1/2 scale and full speed, π=(2)$(1)^2$=2.

At 90^0; a_p= 1578.2 (2)= 3156.4

At 180^0; a_p= 1399.4(2)= 2798.8

For 1/3 scale and 1/2 speed, π=(3)$(2)^2$ =12

At 90^0; a_p= 12 (0.01)=0.12

At 180^0; a_p=12 (0.05)=0.60

Torque: As torque is function of mass (density x volume), length $(L)^2$ and time $(T)^2$,

$$\pi=\left(\frac{T_p}{T_m}\right)=\left(\frac{M_p}{M_m}\right)\left(\frac{L_p}{L_m}\right)^2\left(\frac{T_p}{T_m}\right)^2$$

For 1/2 scale and full speed, π= (25)$(2)^2$ $(1)^2$=100

At 90^0; T_p=0.0099e-4 (100)=0.00009

At 180^0; T_p=-0.4014e-4 (100)=0.004014

For 1/3 scale and 1/2 speed, π=(42.5)$(3)^2$ $(2)^2$ =1530

At 90^0; T_p=1530(0.0065e-5)=0.0009945

At 180^0; T_p=1530(0.2617e-5)=0.004004.

Stress: As stress is function of mass (density x volume), length (L) and time $(T)^2$,

$$S_\sigma = \left(\frac{\sigma_p}{\sigma_m}\right) = \left(\frac{M_p}{M_m}\right)\left(\frac{L_p}{L_m}\right)\left(\frac{T_p}{T_m}\right)^2$$

For 1/2 scale and full speed, $\pi = (2)(1)^2 = 2$

At 0^0; $\sigma_p = 2(18.35) = 36.70$

At 72^0; $\sigma_p = 2(8.15) = 16.30$

For 1/3 scale and 1/2 speed, $\pi = (3)(2)^2 = 12$

At 0^0; $\sigma_p = 12(3.026) = 36.31$

At 72^0; $\sigma_p = 12(1.37) = 16.44$

Fundamental frequency: As frequency is function of length (L) and time (T)

For 1/2 scale, and full speed $\pi = 1/2$,

At 0^0, F=(1/2)(28.349)=14.175

At 72^0, F=(1/2)(29.644)=14.822

For 1/3 scale and half speed $\pi = 1/6$,

At 0^0, F=(1/6)(88.06)=14.678

At 72^0, F=(1/3)(88.948)=14.8

TABLE III.

Correlations between results of the full-sized and 1:2 scaled down FBM mechanisms with the scaled mechanism operating at one-full speed of the full-sized mechanism

Angle (deg)	**Linear velocityof centre mass**		**Linear accelarstionof centre mass**		**Crank torque**	
	From current analysis	***From Pi theorem***	***From current analysis***	***From Pi theorem***	***From current analysis***	***From Pi theorem***
0	31.2567	-31.2566	9039	9039	0.0040	0.00397
90	15.772	15.772	3156	3156.0	0.0001	0.00009
180	12.6063	12.6063	2799	2798.4	0.0040	0.00401
270	-2.8328	-2.8328	4075	4075.2	0.0001	0.00009

Angle (deg)	**Von-mises stress (N/mm^2)**		**Fundamental frequency (HZ)**	
	From current analysis	From pi therom	From current analysis	From pi therom
0	16.30	14.85	14.88	36.70
72	24.56	15.82	15.96	16.30
216	30.22	14.04	14.05	24.36
324	30.22	14.04	14.05	30.22

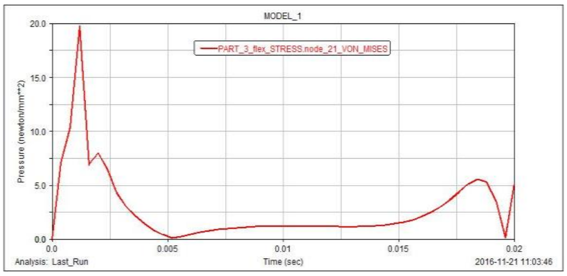
TABLE.IV.

Correlations between results of the full-sized and 1:3 scaled down FBM mechanisms with the scaled mechanism operating at one-half speed of the full-sized mechanism

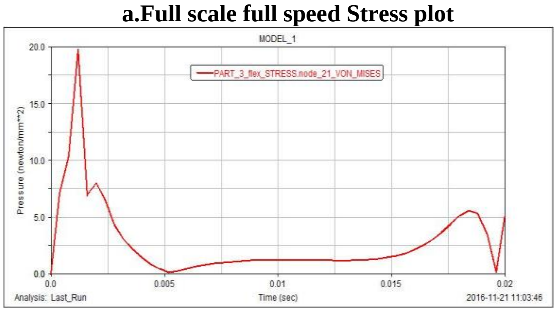
Angle (deg)	**Linear Velocity of centre of mass output lever link (m/s)**		**Linear acceleration of centre of mass of output lever link. (m/s^2)**	
	From current analysis	*From Pi theorem*	*From current analysis*	From pi theorem
0	31.256	31.256	9039	0.0040
90	15.772	15.772	3156	0.0001
180	12.606	12.606	2799	0.0040
270	2.8328	2.833	4075	0.0001

Angle (deg)	**Von-mises stress (N/mm^2)**		**Fundamental frequency (HZ)**	
	From current analysis	***From Pi theorem***	***From current analysis***	***From Pi theorem***
0	36.30	36.31	14.54	14.69
72	16.20	16.44	14.85	14.83
216	24.62	24.64	15.82	15.97
324	30.20	30.29	14.04	14.06

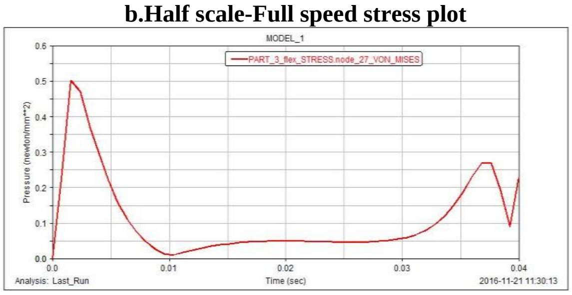
The stress variation at a point on link of FBM obtained using Adams software for one rotation in Full, half and 1/3 scale when crank operates at uniform speed of 3000 rpm are shown in *Fig. 2 a, b* and *c* respectively



a.Full scale full speed Stress plot

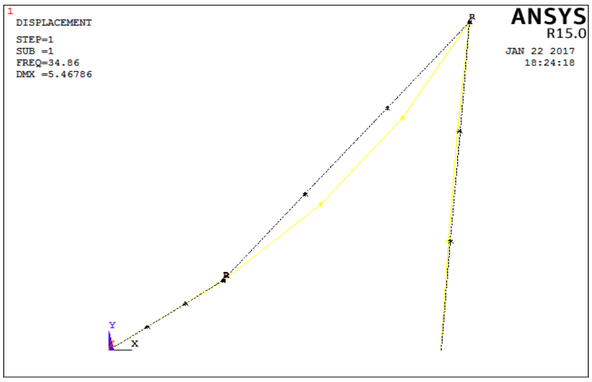


b.Half scale-Full speed stress plot

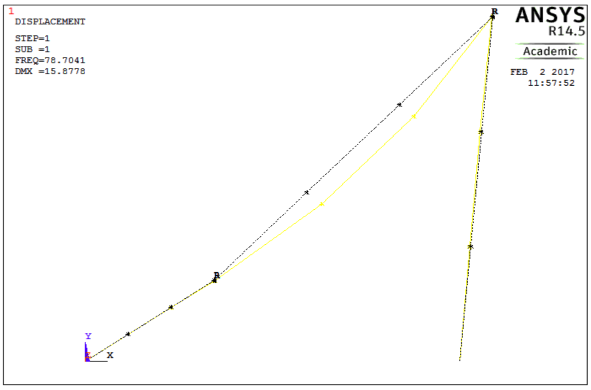


c.1/3 scale-half speed stress plot

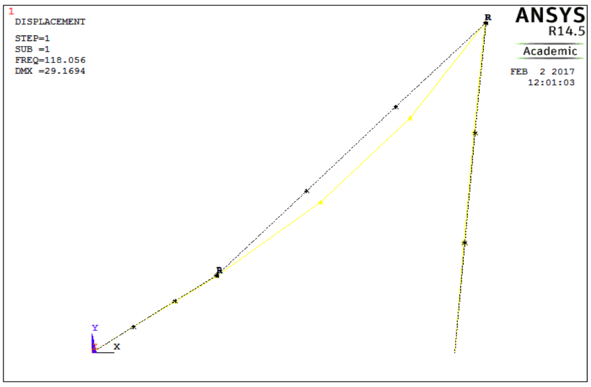
Fig. 5. FBM stress plot for different scales and speeds



a. Full scale



b. Half scale



c.1/3 scale

Fig. 6. FBM fundamental frequency at crank angle 36^0 with different scales

III. RESULTS AND DISCUSSION

It is observed that, the results shown from Table-III, for the given mechanism at a considered scale, the parameters can be correlated by using Pi terms. For example, the values of frequencies are related by the pi term is 2 and 3 for half and 1/3 scales respectively. Similarly for other parameters of velocity, acceleration, torque and stress values, the correlations are made. Hence the mechanisms required parameters of actual size and under actual working conditions, can be estimated by using pi terms with the help of scaled models by running them at possible available working conditions.

IV. CONCLUSION

Pi terms for various response output parameters of the four bar mechanism and their use in predicting the results from the results of the corresponding scaled mechanism are detailed. Illustration using configurations of these mechanisms yielded good correlations of results in comparison.

The use of the Pi terms is not in the comparison between two configurations of a four bar mechanism per se but in their applicability in a testing of very complex high-speed mechanisms like automotive engine mechanisms and high-speed switchgear using scaled down models prior to finalizing the design as reported.

As the correlations for FBM are proven by using pi terms, the same procedure can be implemented for complex mechanisms e.g. switchgear mechanisms for which the testing is difficult to conduct at its actual working conditions and those can be evaluated by using scaled models with Buckingham pi theorem appropriately.

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