Vibration Analysis of an Un-cracked & Cracked Fixed Beam by using analytical and FEM

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Abstract: *The* existence *of cracks causes changes in the physical properties of a structure which presents flexibility, and thus reducing the rigidity by lowering stiffness of the structure with an inherent reduction in modal natural frequencies. Consequently it brings to the change in the dynamic characteristics of the beam. This paper* concentrates *on the theoretical analysis of transverse free vibration of a fixed –fixed beam and analyzes the relation between the modal natural frequencies with crack depth, modal natural frequency with crack location and influence of mode shape on crack. All the theoretical values are* examined *with the numerical method by using ANSYS software and correlate the theoretical values with the numerical values to find out percentage error between them. Also in this paper, investigates the numerical studies for damage detection in beam structure with an open edge crack has been presented. Variations of natural frequencies due to crack at various locations and with varying crack depths have been studied. The analysis was performed using ANSYS software*

Keywords: *Fixed beam, Mode Shape Natural Frequency, Crack, ANSYS.*

I. Introduction

Most of the members of engineering structures operate under loading conditions, which may cause damages or cracks in overstressed zones. The presence of cracks in a structural member, such as a beam and plate, causes local variations in stiffness, the magnitude of which mainly depends on the location and depth of the cracks. The existence of damage causes changes in the physical properties of a structure member which in turn alter its dynamic response characteristics are natural frequency and mode shape. This property leads to detect existence of crack and its location and depth in the structural members.

The vibration behaviour of cracked structures has been investigated by many researchers. The majority of published studies various techniques for structural damage detection involving modal parameters have widely been used over the past few decades. As such, the modal parameters of a structure are easy to obtain from forced, free or ambient vibration measurements. In many of these techniques, mode shapes or the data derived from mode shapes have been used for location-detection of damage. From damage sensitivity perspective, the data obtained from the mode shape in curvature form seemsfar promising than the one from mode shapes. In this paper, the natural frequencies of cracked and un-cracked beams have been calculated using finite element software ANSYS and up to fifth mode and compared with theoretical values and the mode shape curvatures from damaged modes are analyzed with wavelet transform, this data is studied at three stages i.e. mode shapes; its curvature and subsequent wavelet transform to increase the sensitivity.

Liew and Wang [1] proposed an application of spatial wavelet theory to damage identification in structures. They calculated the wavelet coefficients along the length of the beam based on the numerical solution for the deflection of the beam, the damage location was then indicated by a peak in the coefficients of the wavelets along the length of the beam. In the investigation of existence of crack and its localization and depth of damage the pioneer of the work Dimarogonas [2] the crack was modelled as a local flexibility and the equivalent stiffness was computed using fracture mechanics methods. Adams and Cawley [3] developed an experimental technique to estimate the location and depth of a crack from changes in natural frequencies. Rizos PF[4] proposed method o identification of crack location and magnitude in a cantilever from fundamental vibration modes. Pandey A.K., M. Biswas, and M. M. Samman [5] proposed that fundamental mode shape curvature to be a sensitive parameter for damage localization. Curvature was calculated and utilized for damage localization of a simulated beam discredited into a number of finite elements . Douka et al [6] investigated experimental and analytical identification of crack in cantilever beam depending on wavelet analysis. The size and the location of the crack is determined using wavelet transform for fundamental mode of vibration. Due to the rapid changes in the spatial difference of the response, the crack location is determined . Patil and Maiti [7] have proposed a method for prediction of location and size of multiple cracks based on measurement of natural frequencies for slender cantilever beams. Abdel Wahab and De Roeck [8] apply curvature mode based damage detection methods in a continuous beam by averaged modal-curvature difference arising from pairs of damaged and intact mode shapes. The vibration methods are based on the occurrence that damage in a structure produces a local increase in flexibility which induces changes in the dynamic properties of the structure. The analysis of these changes can be used for damage identification. In recent years, the wavelet transform (WT) has been proposed as a promising mathematical tool for damage detection and localization, in light of its ability to locally analyze a signal. Q. Wang and X. Deng [9] applied the WT to spatial problems, specifically to identify cracks in structures: using free vibrations of cracked beams with a local reduction of stiffness, they showed that the wavelet coefficients calculated along the beam presented a maximum precisely at the crack location. Further they dealing with beams, plates or frame structures have validated this technique as a promising research tool.S.T. Quek, Q. Wang, L. Zhang, K.K. Ang [10] studied on wavelet transforms in the one-dimensional case is very extensive and applicability of various wavelets in detection of cracks in beams.Koushik, and Samit Ray [11] studied the cantilever shear beam, discretized into a large number of elements. It is demonstrated that the change in the fundamental mode shape due to any damage is an excellent indicator of damage localization as it is found to be discontinuous at the location of damage. Further, the change in higher derivatives (i.e., slope and curvature) of the fundamental mode shape is shown to be sensitive enough in damage localization.Y.F.Xu , W.D.Zhu , J.Liu , Y.M.Shao [12] proposed two new non-model-based methods that use measured mode shapes to identify embedded horizontal cracks in beams. M. Rucka, K. Wilde [13] proposed a method for estimating the damage location in beam and plate structures a Plexiglas cantilever beam and a steel plate with four fixed boundary conditions are tested experimentally by using the one-dimensional continuous wavelet transform. Chih-Chieh Chang, Lien-Wen Chen [14] presents a technique for structure damage detection based on the spatially distributed signals without experimental data. W.L. Bayissa, N. Haritosa, S. Thelandersson [15] have done work that the wavelet analysis coefficients be employed in the space domain of the structure to detect and localize single as well as multiple damage states and the damage identification results are compared with those obtained from existing and well-established methods. A.Bagheri, G.Ghodrati Amiri, S. A. Seyed Razzaghi[16] has been employed discrete curvelet transform using unequally-spaced fast Fourier transforms to identify damage location in plate structures. In addition, the performance and sensitivity of the proposed method have been investigated using numerical and experimental data. Mario Solis, Mario Algaba, Pedro Galvin [18] presents a new damage detection methodology for analyses the severity threshold of damage in beams by applying continuous wavelet analysis.F. Bakhtiari-Nejad [19] presents analytical estimation based on the Rayleigh's method to find out natural frequencies and mode shape for a beam having one or two cracks. In addition that investigates the upper limit of crack depth in which natural frequencies and mode shapes have error less than 5% and 7% respectively obtained by analytical estimation in compare to the exact solution. Khoa Viet Nguyen [20] has been investigated the influence of the coupling mechanism between horizontal bending and vertical bending vibrationsdue to the crack on the mode shapes. Due to the coupling mechanism the mode shapes of a beam change from plane curves to space curves. Suggested that distortions in the case using the 3D beam element can be amplified and inspected clearly by using the projections of the mode shapes on appropriate planes than while using 2D beam element, distortions in the mode shapes. F. Bakhtiari-Nejad [21] proposed a analytical estimation method to find the natural frequencies and mode shapes of the beam, to overcome weakness of solving eigen value problem to obtain exact natural frequencies and mode shapes. Jeslin Thalapil [22] examines the methods to detecting longitudinal cracks using changes in natural frequencies in the case of monolithic long and short beams. The objective of this paper is to apply spatial wavelet transform to highlight the sensitivity for detection and quantification and localization of damage in a beam structure, for that wavelet-based damage detection technique was investigated numerically on an example of the cantilever beam with damage in the form of the notch of depth 30%,20%,and 10% of the beam height .The analysis was performed on the first eight mode shapes.

II. Theoretical Analysis of Transverse Vibration of Fixed Fixed-Beam

A. Governing Equations for Vibration Mode of the Beam

A beam which is fixed at both ends is known as fixed beam. From elementary theory of bending of beams also known as Euler-Bernoulli, the relationship between the bending moment and deflection can be expressed as;

$$M = EI\frac{d^2y}{dx^2}$$

Where, E is Young's Modulus and I is the moment of inertia of the beam c/s. For uniform beam we can obtain equation of motion as;

$$\frac{EI}{\rho A}\frac{d^4y}{dx^4}+\frac{d^2y}{dt^2}=0 \qquad (1)$$

Where, ρ is the mass density and A is the cross sectional area of the beam.

$$c^2\frac{d^4y}{dx^4}+\frac{d^2y}{dt^2}=0 \qquad (2)$$

Where $c=\sqrt{\frac{EI}{\rho A}}$

The solution of equation (2) depends on position and another on time

So, the solution is y=w(x)T(t) (3)

By substituting Eq. (3) to Eq. (2) and simplifying it we get;

$$c^2\frac{d^4w(x)}{dx^4}=\frac{1}{T(t)}\frac{d^2T(x)}{dt^2} \qquad (4)$$

$$\frac{d^4w}{dx^4}-\beta^4w(x)=0 \qquad (5.1)$$

$$\frac{d^2T}{dt^2}-\omega^2T(t)=0 \qquad (5.2)$$

$$where, \beta^4=\frac{\omega_i^2}{c^2}=\frac{\rho A\omega_i^2}{EI}$$

To find out the solution of (5) considering the equation;

$$w(x,t) = C_1cosh\beta x + C_2sinh\beta x + C_3cos\beta x + C_4sin\ \beta x \qquad (6)$$

In order to solve Eq.(6) the following boundary conditions for the fixed beam are needed;

At, $x=0; w(0,t)=0$ (Zero deflection at fixed end)

At, $x=L; w(0,t)=0$ (Zero deflection at fixed end)

Now using the expression for mode shape (Eq.6) and applying the boundary conditions following relations can be obtained:

$$w(x,t) = C_1cosh\beta x + C_2sinh\beta x + C_3cos\beta x + C_4sin\ \beta x$$

At W (L)=0

$$C_1cosh\beta L + C_2sinh\beta L + C_3cos\beta L + C_4sin\ \beta L=0 \qquad (8)$$

At, $c_2+c_4=0$ (9)

At $w^{'}(L)=0$

$$C_1\sin h\beta L + C_2\cos h\beta L - C_3\sin\beta L + C_4\cos\ \beta L=0 \qquad (10)$$

From Eq.(7) $c_3=-c_1$

From Eq.(9) $c_4=-c_2$

From Eq.(8) we get

$$C_1cosh\beta L + C_2sinh\beta L - C_1cos\beta L - C_2sin\ \beta L=0 \qquad (11)$$

From Eq.(10) we get

$$C_1\sin h\beta L + C_2\cos h\beta L + C_1\sin\beta L - C_2\cos\ \beta L=0 \qquad (12)$$

From Eq.(11) we get $\cos\beta L\cos h\beta L=0$ (13)

From Eq.(12) we get

$$\frac{C_1}{C_2}=\frac{\cos\beta L-\cos h\beta L}{\sin h\beta L+\sin\beta L} \qquad (14)$$

Hence $\frac{\sin\beta L-\sin h\beta L}{\cos h\beta L-\cos\ \beta L}=\frac{\cos\beta L-\cos h\beta L}{\sin h\beta L+\sin\beta L}$

and finally we get $\cos\beta L\cos h\beta L=0$ (15)

This transcendental equation has an infinite number of solution =1,2,3…n

Corresponding giving an infinite number of frequencies,

$$\omega_i=(\beta_iL)^2\sqrt{\left(\frac{EI}{\rho AL^4}\right)} \qquad (16)$$

The first five roots of the Eq. (15) are shown in **TABLE-1**

Roots(i)	β_iL
1	4.730
2	7.853
3	10.996
4	14.137
5	17.278

B.Specification

The dimensions and the material constant for the uniform fixed beam investigated in this paper are:

- Material of Beam= Aluminum

- Total Length = 600mm
- Width of Beam =25mm
- Height of Beam =10mm
- Moment of Inertia(I)=2.0833x10^{-09} m^4
- Density Of Material= 2770 kg/m^3
- Cross-Section Of Beam= 10*25 mm^2
- Young's Modulus Of Elasticity= 70 GPa
- Poission's ratio=0.33

Natural frequencies of fixed-fixed beam obtained by using Equation (16)

Table-2 : Mode Shape Frequency (Hz)

Mode No	Frequency in HZ
1	143.53
2	341.56
3	399.87
4	782.74
5	953.36

III FEM SIMULATION

A. Modelling of Crack on Cantilever Beam

The finite element analysis is brought out for the cracked beam (Fixed-Fixed) to locate the mode shape of transverse vibration at different locations and crack depth. The crack modelling has been very important aspect, FEM software package ANSYS has been used to modelled the cracks are deemed on Fixed-Fixed beam with relative crack location at 20% ,30% ,50% and 70% of length of the beam from one fixed end and the analysis has been done using general purpose finite element software ANSYS on un-cracked and cracked fixed beam beam to obtain natural frequencies and mode shape of transverse vibration at different locations with different crack depths interval of 10%, 20%,30% and 40% crack depth. Properties the cracked beams of the current research have and dimensions mentioned above:

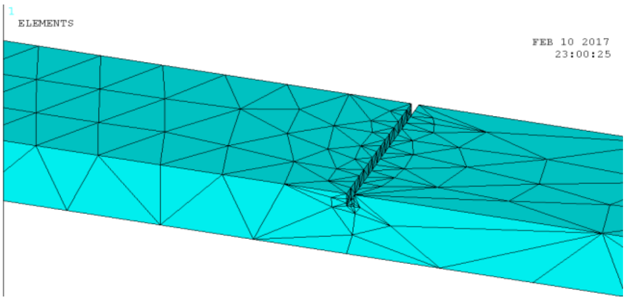

Fig.02.*Rectangular shaped edge crack with a 1 mm width on the top surface of the beam*.

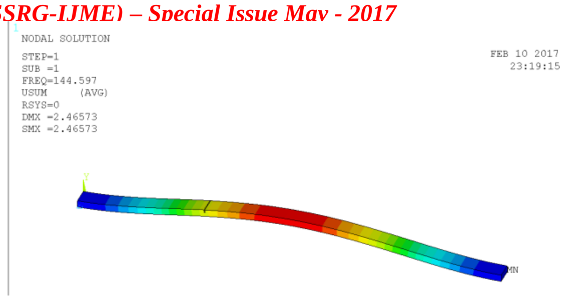

Fig.01.*First Mode shape at crake depth 1mm*

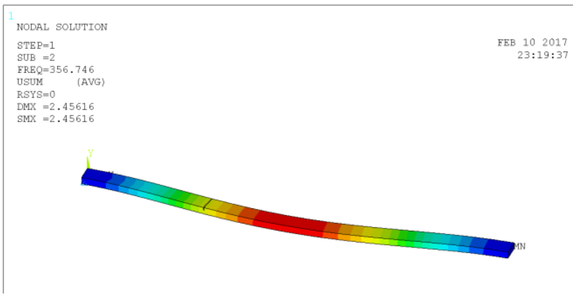

Fig.02.*Second Mode shape at crake depth 1mm*

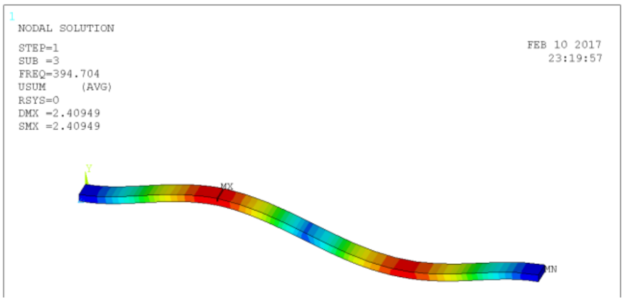

Fig.03.*Third Mode shape at crake depth 1mm*

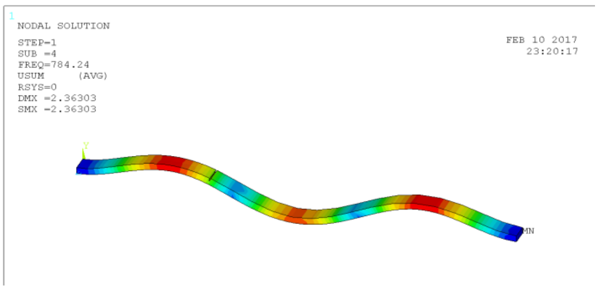

Fig.04.*Fourth Mode shape at crake depth 1mm*

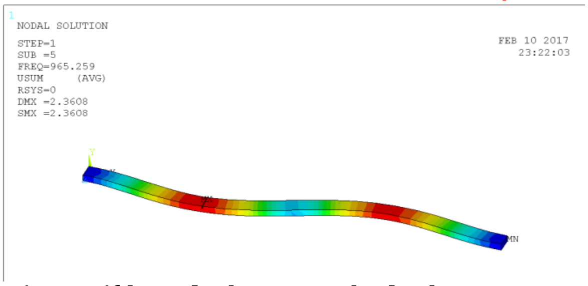

Fig.06.*Fifth Mode shape at crake depth 1.0mm*

Table-3: ***Mode Shape Frequency using ANSYS (Hz)***

Mode No	Frequency in HZ
1	145.13
2	357.21
3	401.57
4	792.94
5	972.32

Table-4:*The percentage error between theoretical and numerical result are shown in*

Mode No	Theoretical frequency in Hz	Numerical frequency from ANSYS program in Hz	Percentage Error (%)
1	143.53	145.13	1.11
2	341.56	357.21	4.58
3	399.87	401.57	4.25
4	782.74	792.94	1.30
5	953.36	972.32	1.98

IV Parametric Studies Of The Fixed Beam :

The effects of the crack on natural frequency of a fixed steel beam were investigated for various crack depths and crack locations

The dimensions and the material *Properties* for the uniform fixed beam investigated in this paper are:

- Material Of Beam= Aluminum
- Total Length = 600mm
- Width of Beam =25mm
- Height of Beam =10mm
- Moment of inertia(I)=$2.0833\text{x}10^{-09}$
- Density Of Material= 2770 kg/m³
- Cross-Section Of Beam= 10*25 mm²
- Young's Modulus Of Elasticity= 70 GPa
- Poission's ratio=0.33

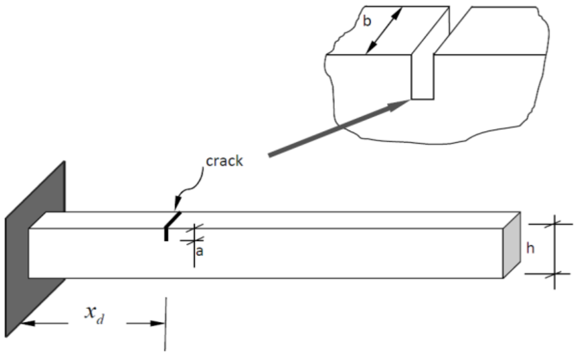

Fig.01. Geometry of the cracked beam

Table-5:*Variation of Natural frequency of the fixed beam with different crack depth for Crack position* $\left(\frac{X_d}{L}\right)$ ***=0.2***

Natural frequency	Crack depth ratio H=(a/h)	Normalized Natural frequency
Un-cracked beam		1.0000
Mode-01	*0.1*	1
	0.2	1
	0.3	0.99979329
	0.4	0.99965548
Mode-02	*0.1*	1
	0.2	1
	0.3	0.99972
	0.4	0.999228
Mode-03	*0.1*	1
	0.2	0.998755
	0.3	0.996514
	0.4	0.993326
Mode-04	*0.1*	1
	0.2	0.996683
	0.3	0.990579
	0.4	0.982659
Mode-05	*0.1*	1
	0.2	0.994689
	0.3	0.982307
	0.4	0.967501

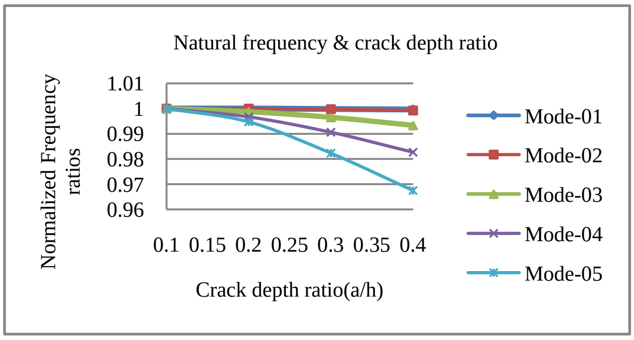

Fig.07.*Mode Nature Frequency versus Crack depth*

Table-6*:Natural frequency of the fixed beam with a crack for the first mode =145.13Hz*

Crack position = $\left(\frac{X_d}{L}\right)$	Crack depth ratio H=(a/h)	Normalized Natural frequency
Un-cracked beam		1.0000
0.2	*0.1*	1
	0.2	1
	0.3	0.999793289
	0.4	0.999655481
0.3	*0.1*	1
	0.2	0.998689
	0.3	0.998689
	0.4	0.997792
0.5	*0.1*	1
	0.2	0.997374421
	0.3	0.993436053
	0.4	0.993436053
0.7	*0.1*	1
	0.2	0.999655
	0.3	0.995859
	0.4	0.997929

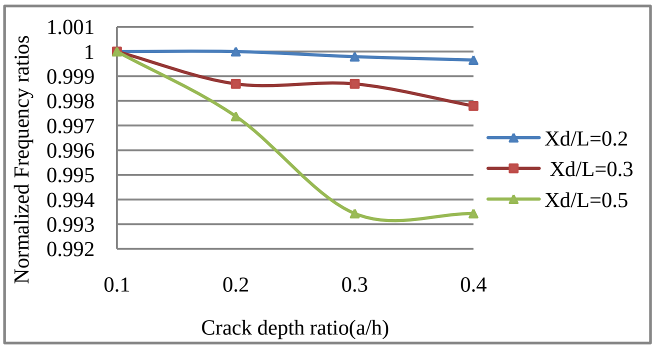

Fig.08.*Natural frequency of the fixed beam with a crack for the first mode*

Table-7:*Natural frequency of the fixed beam with a crack for the second mode =357.21Hz*

Crack position = $\left(\frac{X_d}{L}\right)$	Crack depth ratio H=(a/h)	Normalized Natural frequency
Un-Cracked beam		1.0000
0.2	*0.1*	1
	0.2	1
	0.3	0.99972
	0.4	0.999228
0.3	*0.1*	1
	0.2	0.999496
	0.3	0.999496
	0.4	0.99902
0.5	*0.1*	1
	0.2	0.998992
	0.3	0.9972
	0.4	0.9972
0.7	*0.1*	1
	0.2	0.999888
	0.3	0.997592
	0.4	0.999104

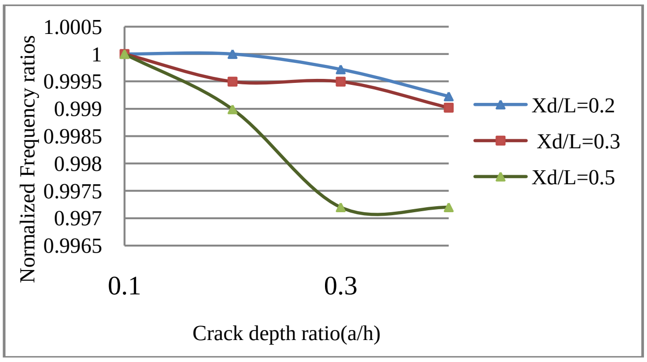

Fig.09.*Natural frequency of the fixed beam with a crack for the second mode*

Table-8:*Natural frequency of the fixed beam with a crack for the third mode =401.57Hz*

Crack position = $\left(\frac{X_d}{L}\right)$	Crack depth ratio H=(a/h)	Normalized Natural frequency
Un-Cracked beam		1.0000
0.2	*0.1*	1
	0.2	0.998755
	0.3	0.996514
	0.4	0.993326
0.3	*0.1*	1
	0.2	0.991656
	0.3	0.991656
	0.4	0.98606
0.5	*0.1*	1
	0.2	0.999592
	0.3	0.999025
	0.4	0.999225
0.7	*0.1*	1
	0.2	0.996827
	0.3	0.981711
	0.4	0.986108

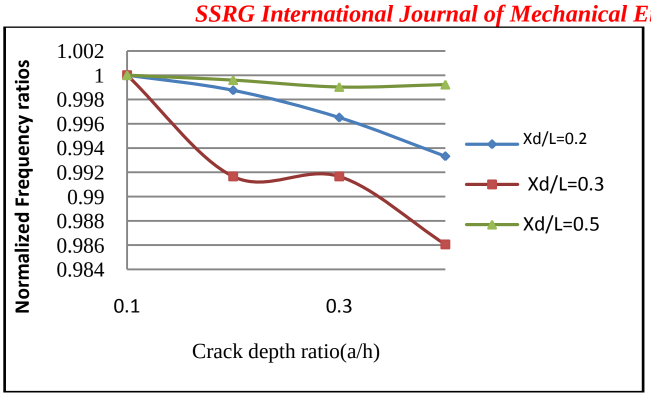

Fig.10.*Natural frequency of the fixed beam with a crack for the third mode*

Table-9:Natural frequency of the fixed beam with a crack for the fourth mode =792.94Hz

Crack position = $\left(\frac{X_d}{L}\right)$	Crack depth ratio H=(a/h)	Normalized Natural frequency
Un-Cracked beam		1.0000
0.2	*0.1*	1
	0.2	0.996683
	0.3	0.990579
	0.4	0.982659
0.3	*0.1*	1
	0.2	0.997283
	0.3	0.997283
	0.4	0.995557
0.5	*0.1*	1
	0.2	0.996434
	0.3	0.990944
	0.4	0.990944
0.7	*0.1*	1
	0.2	0.999416
	0.3	0.9875
	0.4	0.987766

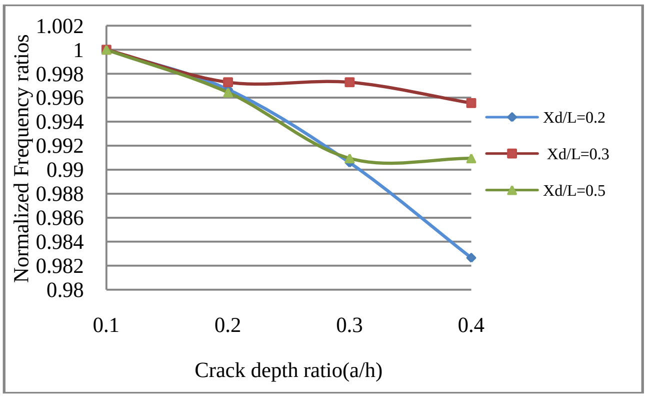

Fig.11.*Natural frequency of a fixed beam with a crack for the fourth mode*

Table-10:*Natural frequency of the fixed beam with a crack for the Fifth mode =972.32Hz*

Crack position = $\left(\frac{X_d}{L}\right)$	Crack depth ratio H=(a/h)	Normalized Natural frequency
Un-Cracked beam		1.0000
0.2	*0.1*	1
	0.2	0.994689
	0.3	0.982307
	0.4	0.967501
0.3	*0.1*	1
	0.2	0.996521
	0.3	0.996521
	0.4	0.993475
0.5	*0.1*	1
	0.2	1.000021
	0.3	0.999938
	0.4	0.999938
0.7	*0.1*	1
	0.2	0.998847
	0.3	0.991756
	0.4	0.993536

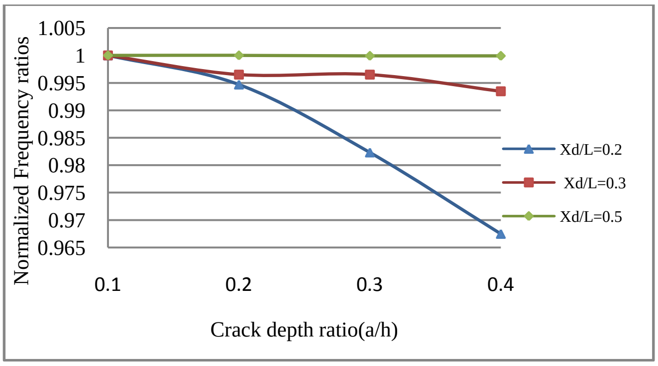

Fig.12.*Natural frequency of a fixed beam with a crack for the fifth mode*

V. Observations

Fig.0 7, show that natural frequencies of the fixed beam with a single edge crack at various crack depths for first, second, third, fourth and fifth modes of vibration respectively. Results show that there is an appreciable variation between natural frequency of cracked and un-cracked fixed beam. For the particular the position of the crack, variation in natural frequency is observed by increasing crack depth it is observed that natural frequency of the cracked beam decreases not only with increase in crack depth due to reduction in stiffness but also change in mode shape for higher modes reduction stiffness is more, which indicates that higher modes are more sensitive to the presence of the damage. Fig.08, Fig.09, Fig.10., Fig.11.,shows that When the position of the crack is at that point where amplitude of vibration is zero there is no change in natural frequency in spite of change in crack depth. Natural frequency changes drastically when crack is on that point where amplitude of vibration is maximum. It is observed that natural frequency of the cracked beam decreases both with increase in crack distance and crack depth due to reduction in stiffness. It appears therefore that the change in frequencies is not only a function of crack depth and crack location but also of the mode number.

IV. Conclusion

• It has been observed that the natural frequency changes substantially due to the presence of cracks depending upon location and size of cracks but also of the mode number.

• When the crack positions are constant i.e. at particular crack location, the natural frequencies of a cracked beam are inversely proportional to the crack depth and mode number which indicates that higher modes are more sensitive to the presence of the damage compared to lower mdes.

• It has been observed that the change in frequencies is not only a function of crack depth, and crack location, but also of the mode number

• As largest effects are observed at the centre for fixed beam we can say, decrease in frequencies is more for a crack located where the bending moment is higher.

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