Application of Lagrange's Equation to Undamped vibration absorbers attached to multi-degree freedom system.

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When system required more coordinates to describe its motion then system is multi-degree system. The power is formulated as the inner product between the velocity and force terms integrated over a cycle, conservative terms (mass& stiffness) consider. This method is used to minimize the motion of physical mass or modes of vibration, This method is used for the calculation of absorber parameter and frequencies of Multi Degree Freedom System. Vibration absorber (i,e spring & mass system) are among the mostly commonly used devices for vibration suppression. The identification equation can obtained with model by Lagrange's Equation. As per the vibration analysis procedure basic mathematical model is developed, derived equation of motion.

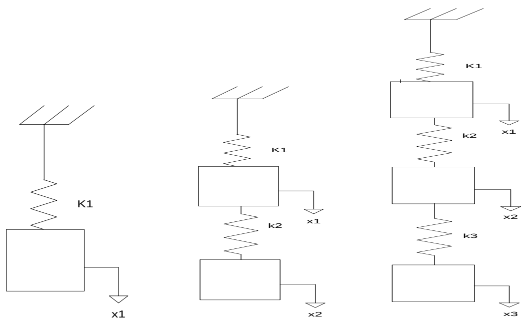
Keywords:

Lagrange's Equation, Multi Degree Freedom System.

INTRODUCTION:

The absorbers (spring and mass) are the oldest vibration absorber devices. A lot of work has been done in vibration by many others. About thirty years back, the vibration analysis of complex multidegree of freedom system was very difficult. But now with the help of finite element method and other advanced techniques the engineers are able to use computers to conduct numerically detailed vibration analysis of complex mechanical system even having a multidegree of freedom. Its theory of operation was presented Frahm in1909 [3].

He introduce the dynamic vibration absorber with the spring mass system, which is attached to the main system. This system only work when it was in properly tunned, and theoretically capable of setting main vibration to zero at particular frequency.In this study , Lagrange's Equation method which provides closed forms of equations for the frequency domain solution of multidegree freedom system is introduced. The method is applicable for multidegree of freedom system with number of elements. The minimum number of independent coordinates required to specify the motion of a system at any instant is known as degree of freedom of the system. In general, it is equal to the number of displacement that are possible. This numbers varies from zero to infinity. The one two and three degree of freedom system are shown in fig1.



(a) (b) (c)

Fig 1 Finite degree of freedom

In single degree of freedom there is only one independent coordinate (x_1) to specify the configuration as shown in 1(a). Similarly, there are two (x_1, x_2) fig (b) and three coordinates (x_1,x_2, and x_3) in fig (c).

2. Application of Lagrange's Equation in mechanical vibration.

The equation of motion of a multi-body harmonic vibrating system are written in terms of generalised coordinates by using the Lagrange's Equation. Lagrange formulated a scalar procedure starting from the scalar quantities of kinetic energy, potential energy and the work expressed in terms of generalized coordinates, and the generalised coordinates are independent parameters which specify the system completely. At a stage of derivation the equation we have ,

$$\frac{d}{dt}\left(\frac{dT}{\partial \dot{x}}\right) - \frac{dT}{\partial xi} + \frac{\partial U}{\partial xi} = Q_i \quad (1)$$

Where,

T and U are the kinetic and potential energies, **x is a vector** of generalized displacements. and Q**i** work term done by non-potential forces. d(T+U)=0 principle of conservation of energy T is the function of generalized coordinates qi & generalized velocity $\dot{x}$.

2.1 Determine the Equations of motion for three degree of freedom.

if the kinetic and potential energies of the system are given as

$$T=\frac{1}{2}m_1\dot{x}_1^2+\frac{1}{2}m_2\dot{x}_2^2+\frac{1}{2}m_3\dot{x}_3^2$$

And

$$U=\frac{1}{2}k_1x_1^2+\frac{1}{2}k_2\,(x_2-x_1)^2+\frac{1}{2}k_3(x_3-x_2)^2$$

Lagrange's Equation can be generate to obtain the equation of motion from the above two equation.

$$\frac{d}{dt}\left(\frac{dT}{\partial x_1}\right) = m_1\dot{x}_1$$

$$\frac{\partial T}{\partial x_1} = 0$$

$$\frac{\partial U}{\partial x_1}=k_1x_1-k_2(x_2-x_1)$$

Since by considering the Lagrange's equation become

$$m_1\ddot{x_1}+x_1(k_1+k_2)-k_2x_2=0 \qquad (a)$$

Similarly for second equation

$$m_1\ddot{x_2}+k_2x_2-k_2x_1-k_3x_3+k_3x_2=0 \qquad (b)$$

Third equation

$$m_3\ddot{x_3}-x_2k_3+x_3k_3=0 \qquad (c)$$

To solve all the above a, b, c , equations these equations are convertated into matrix form as

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{bmatrix} \ddot{x_1} \\ \ddot{x_2} \\ \ddot{x_3} \end{bmatrix} + \begin{bmatrix} k_1+k^2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

2.2 Case Study with analytics

Examples case study are provided to demonstrate the effectiveness of model. A sample three degree of freedom system will be used as shown in fig 2 ,m_1= m_2= m_3= 1kg, and stiffness = 1000 N/m .

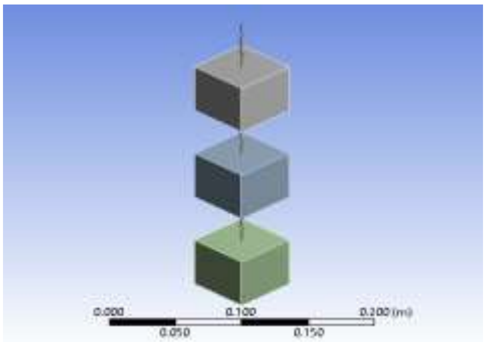


Table 1

TABLE 1
Model (A1) > Modal (A2) > Solution (A3)

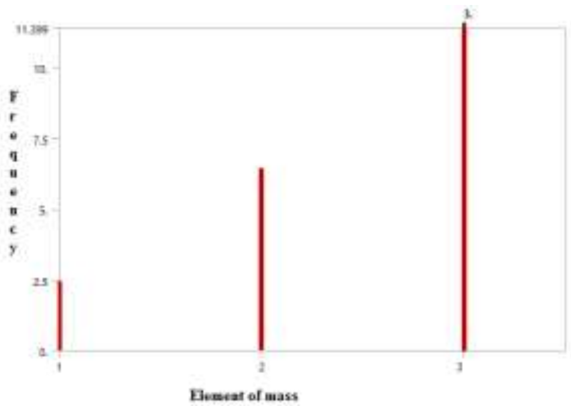
Mode	Frequency [Hz]
1.	2.4588
2.	6.4419
3.	11.399

Table 2

The data table for case study

	1	2	3
Mass (kg)	1	1	1
Stiffness (N/m)	1000	1000	1000
Natural frequency (Hz)	2.4588	6.4419	11.399
Natural frequency (rad/s)	15.44	40.47	71.622
Force (N)	0	0	0

The following bar chart indicates the frequency at each calculated mode



CONCLUSION:

A new approach has been introduce for writing the response of expression of system where several degree of spring and mass system attached to main system .For overall study we applied an energy balance method i,e Lagrange's Equation for estimating damping parameters in multi-degree freedom system . This method involves the inner product between force and velocity in the differential equation. The results of both are similar to each other. We successfully applied the method to numerical simulation of several degree of freedom systems which is compared with the software. Good results are obtained when the numbers of nodes are sufficient.

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