Effect of Pulse Integration on SNR and Probability of Detection in Radar Receiver

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***Abstract*-This paper present a impact of pulse integration in radar receiver on SNR and probability of detection. In practical radar system noncoherent integration gain is always smaller than the number of noncoherent integrated pulse. Here detection of Swerling Targets I, II, III, IV at probability of Detection 0.8, for pulses integration of 1, 5, 10, 25, 75 has been proposed. The performance of the conventional radar for moving target detection have been evaluated analytically mainly focusing on the Swerling model and their integration loss during the signal processing related with the threshold voltage level of matched filter. Finally, possible sub optimum solutions are found, through which considerable streamlining of processing structures can be achieved.**

***Keywords* - Swerling, SNR, probability of detection and pulse integration.**

I.INTRODUCTION

RADAR theory has been a vibrant scientific field for the last few decade or so [1]–[3]. Radar theory deals with many different and diverse problems. However, the two most important problems are the detection and range estimation problems. The importance of these two problems is not limited to radars, and other engineering disciplines like sonar and communication deal with very similar problems [4]. Over the years, radar systems have developed considerably. These developments can be attributed to the increase in computation power and advances in hardware design. The detection probability is influenced by many random factors, such as the radar system's parameters, the natural environment and most of all, as well as the RCS fluctuation characteristic of the target.

II. PULSE INTEGRATION

When a target is located within the radar beam during a single scan, it may reflect several pulses. By adding the returns from all pulses returned by a given target during a single scan, the radar sensitivity (SNR) can be increased. The number of returned pulses depends on the antenna scan rate and the radar PRF. More precisely, the number of pulses returned from a given target is given by

$$\text{Pulse integration}= n_p = \frac{\theta_a T_{sc} f_r}{2\pi} \quad (1)$$

Where θ_a is the azimuth antenna beamwidth, T_{sc} is the scan time, and f_r is the radar PRF. The number of reflected pulses can be expressed as

$$n_p = \frac{\theta_a f_r}{\dot{\theta}_{scan}} \quad (2)$$

Where $\dot{\theta}_{scan}$ is the antenna scan rate in degrees per second. Note that when using eq 1, θ_a is expressed in radians, while when using eq 2, it is expressed in degrees. As an example, consider a radar with an azimuth antenna beamwidth $\theta_a=3^o$, antenna scan rate $\dot{\theta}_{scan}$ = 45^o/sec (antenna scan time, T_{sc}= 8sec), and a PRF f_r= 300Hz. Using either (1) or (2). Yields n_p=20 pulses.

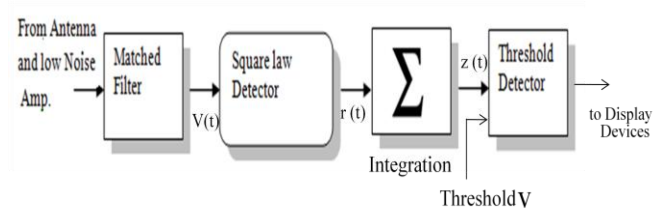

Fig.1. Simplified block diagram of a radar detector when noncoherent integration is used.

The process of adding radar returns from many pulses is called radar pulse integration. Pulse integration can be performed on the quadrature components prior to the envelope detector. This is called coherent integration or predetection integration. Coherent integration preserves the phase relationship between the received pulses. Thus a buildup in the signal amplitude is achieved. Alternatively, pulse integration performed after the envelope detector (where the phase relation is destroyed) is called noncoherent or postdetection integration.

A. Noncoherent Integration.

When the phase of the integrated pulses is not known so that coherent integration is no longer possible, another form of pulse integration is done. In this case pulse integration is performed by adding (integrating) the individual pulses envelope or the square of their envelope. Thus, the term noncoherent integration is adopted. The performance difference (measured in SNR) between the linear envelope detector and quadratic (square law) detector is practically negligible. Robertson [4] showed that this difference is typically less than 0.2 db, he showed that the performance difference is higher than 0.2dB only for cases when $n_p > 100$ and $P_D < 0.01$. Both of these conditions are of no practical significance in radar applications.

III. SWERLING MODELS

B. Detection of Swerling I Targets

The echo pulses received from a target on any one scan are of constant amplitude throughout the entire scan but are independent(uncorrelated) from scan to scan. This assumption ignores the effect of the antenna beam shape on the echo amplitude. The chi-square probability density function with 2N degree of freedom is given as

$$f_X(x) = \frac{N}{(N-1)!\sqrt{\sigma_x^2}}\left(\frac{N_x}{\sigma_x}\right)^{N-1} \exp\left(-\frac{N_x}{\sigma_x}\right) \quad (3)$$

Where σ_x is the standard deviation for the RCS value. Using this equation, the pdf associated with Swerling I and II targets can be obtained by letting N=1, which yields a Rayleigh pdf, Letting N=2 yields the *pdf* for Swerling III and IV type targets.The exact formula for the probability of detection for Swerling I type targets was derived by Swerling. It is

$$P_D = e^{-\frac{v_T}{1+SNR}} \quad ; n_P = 1 \quad (4)$$

$$P_D = 1 - \Gamma_I(v_T, n_P - 1) + \left(1 + \frac{1}{n_P SNR}\right)^{n_P - 1} \Gamma_I\left(\frac{v_T}{1 + \frac{1}{n_P SNR}}, n_P - 1\right) \times e^{-v_T/(1 + n_P SNR)} \; ; \; n_P > 1 \quad (5)$$

Figure 2 shows a plot of the probability of detection as a function of SNR for n_p=1, 5, 10, 25, 75. and $P_{fa} = 10^{-9}$ for both Swerling I and V (Swerling 0) type fluctuations. Note that it requires more SNR, with fluctuation, to achieve the same P_D as in the case with no fluctuation.

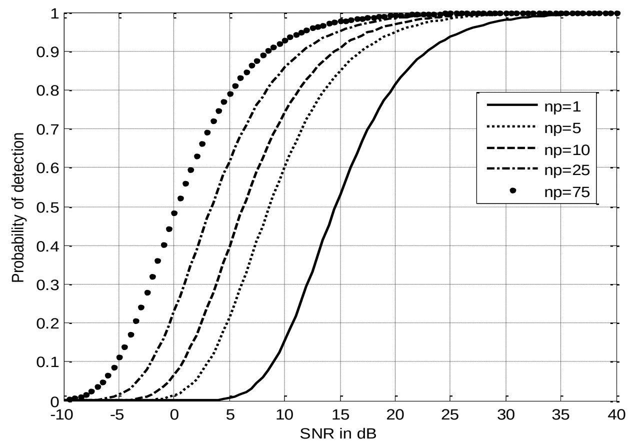

Fig. 2. Probability of detection versus SNR, Swerling I , $P_{fa}=10^{-9}$.

Table I: Effect of pulse integration on SNR for Swerling I.

Number of integrated pulses (np)	Signal to noise ration SNR (dB)
1	19.447
5	13.467
10	11.121
25	8.230
75	5.034

C. Detection of Swerling II Targets

Swerling case 2 – with the same density function as Swerling I but the fluctuations are more rapid than in Swerling I and are taken to be independent from pulse to pulse rather than from scan to scan.In the case of Swerling II targets, the probability of detection is given by

$$P_D = 1 - \Gamma_I\left(\frac{v_T}{(1+SNR)}, n_p\right) \quad ; n_p \le 50 \quad (6)$$

for the case when n_p >50 the probability of detection is computed using the Gram-Charlier series.

$$P_D \cong \frac{erfc(V/\sqrt{2})}{2} - \frac{e^{-V^2/2}}{\sqrt{2\pi}}[C_3(V^2-1) + C_4V(3-V^2) - C_6V(V^4 - 10V^2 + 15)] \quad (7)$$

Where the constants C_3, C_4, and C_6 are the Gram-Charlier series coeffiecients, and the variable V is

$$V = \frac{v_T - n_{p(1+SNR)}}{\varpi} \quad (8)$$

Values for C_3, C_4, and C_6 and ϖ is

$$C_3 = -\frac{1}{3\sqrt{n_p}} \quad , \; C_4 = \frac{1}{4n_p} \; , \; C_6 = \frac{C_3^2}{2} \; , \; \varpi = \sqrt{n_p}(1 + SNR) \tag{9}$$

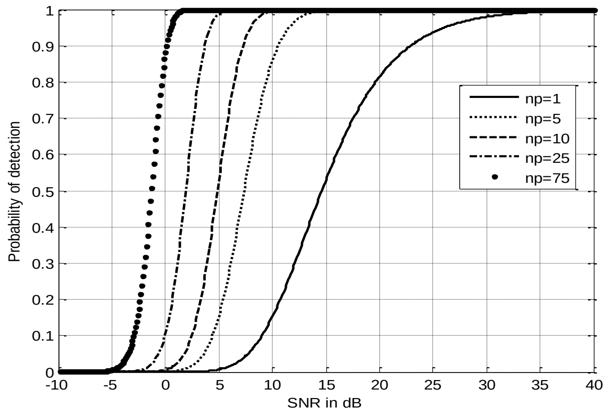

Fig. 3. Probability of detection versus SNR, Swerling II, P_{fa}=10^{-9}.

Table II: Effect of pulse integration on SNR forSwerling II model.

Number of integrated pulses (np)	Signal to noise ration SNR (dB)
1	19.447
5	9.242
10	6.268
25	2.929
75	0.100

D. Detection of Swerling III Targets

The fluctuation is assumed to be independent form scan to scan, as in case of swerling I, but the probability density function is given by the chi-square distribution with four degree of freedom.

$$f_X(x) = \frac{4x}{\sigma_x^2} exp\left(-\frac{2x}{\sigma_x}\right) \qquad x \geq 0 \tag{10}$$

The exact formulas, developed my Marcum, for the probability of detection for Swerling III type targets when $n_p = 1,2$.

$$P_D = exp\left(\frac{-v_T}{1+n_p SNR/2}\right)\left(1 + \frac{2}{n_p SNR}\right)^{n_p - 2} \times K_0 \tag{11}$$

$$K_0 = 1 + \frac{v_T}{1+n_p SNR/2} - \frac{2}{n_p SNR}(n_p - 2) \tag{12}$$

For $n_p > 2$ the expression is

$$P_D = \frac{v_T^{n_p-1} e^{-V_T}}{(1+n_p SNR/2)(n_p-2)!} + 1 - \Gamma_I(v_T, n_p - 1) + K_0 \times \Gamma_I\left(\frac{v_T}{1+2/n_p SNR}, n_p - 1\right) \tag{13}$$

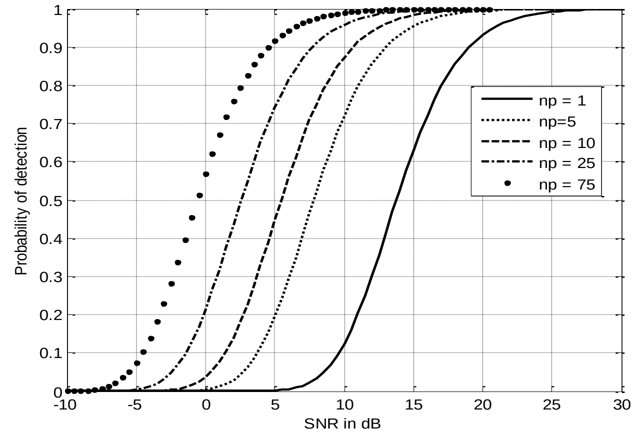

Fig. 4. Probability of detection versus SNR,Swerling III, P_{fa}=10^{-9}.

Table III: Effect of pulse integration on SNR for Swerling III model.

Number of integrated pulses(np)	Signal to noise ration SNR (dB)
1	16.867
5	10.876
10	8.526
25	5.627
75	2.423

E. Detection of Swerling IV Targets

The expression for the probability of detection for Swerling IV targets for $n_P < 50$ is

$$P_D = 1 - \left[\gamma_0 + \left(\frac{SNR}{2}\right) n_p \gamma_1 + \left(\frac{SNR}{2}\right)^2 \frac{n_p(n_p-1)}{2!}\gamma_2 + \ldots + \left(\frac{SNR}{2}\right)^{n_p}\gamma_{n_p}\right]\left(1+\frac{SNR}{2}\right)^{-n_p} \tag{14}$$

$$\gamma_i = \Gamma_I\left(\frac{v_T}{1 + \frac{SNR}{2}}, n_p + i\right) \tag{15}$$

For the case when $n_p \geq 50$, the Gram-Charlier series can be used to calculate the probability of detection.

$$P_D \cong \frac{erfc(V/\sqrt{2})}{2} - \frac{e^{-V^2/2}}{\sqrt{2\pi}}[C_3(V^2 - 1) + C_4 V(3 - V^2) - C_6 V(V^4 - 10V^2 + 15)]$$

Where the constants C_3, C_4, and C_6 are the Gram-Charlier series coefficients,

$$C_3 = \frac{1}{3\sqrt{n_p}}\frac{2\beta^3 - 1}{(2\beta^2 - 1)^{1.5}} \; , \; C_6 = \frac{C_3^2}{2}, \; C_4 = \frac{1}{4n_p}\frac{2\beta^4 - 1}{(2\beta^2 - 1)^2}$$

$$\varpi = \sqrt{n_{p(2\beta^2-1)}} \; , \; \beta = 1 + (SNR)/2 \tag{16}$$

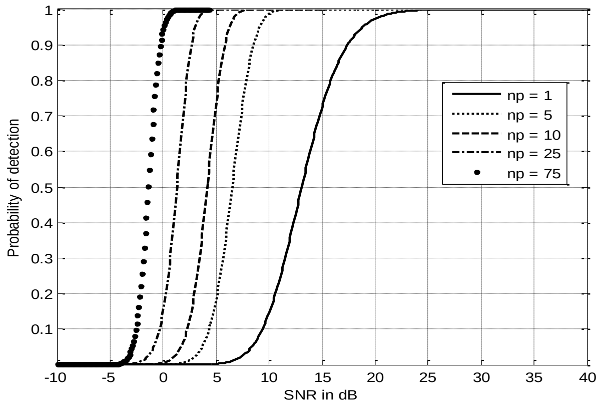

Fig. 5. Probability of detection versus SNR, Swerling IV, P_{fa}=10^{-9}.

Table IV: Effect of pulse integration on SNR for Swerling IV model.

Number of integrated pulses (np)	Signal to noise ration SNR (dB)
1	15.646
5	7.684
10	5.111
25	2.067
75	0.100

F. Detection of Swerling V Targets

For Swerling 0 (Swerling V) target fluctuations, the probability of detection is calculated using Gram-Charlier Series

$$P_D \cong \frac{erfc\,(V/\sqrt{2})}{2} - \frac{e^{-V^2/2}}{\sqrt{2\pi}}[C_3(V^2-1)+ C_4 V\,(3-V^2) - C_6 V(V^4-10V^2+15)]$$

$$C_3 = -\frac{SNR+1/3}{\sqrt{n_p}(2SNR+1)^{1.5}}\ ,\ C_4 = \frac{SNR+1/4}{n_p(2SNR+1)^2}\ ,$$

$$C_6 = \frac{C_3^2}{2},\quad \varpi = \sqrt{n_{p(2SNR+1)}} \tag{17}$$

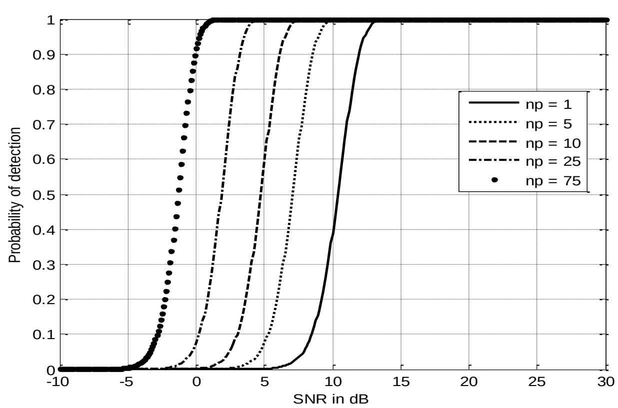

Fig. 6. Probability of detection versus SNR, Swerling 0 (Swerling V), P_{fa}=10^{-9}

Table V: Effect of pulse integration on SNR for Swerling V model.

Number of integrated pulses (np)	Signal to noise ration SNR (dB)
1	12.155
5	7.931
10	5.549
25	2.609
75	0.100

IV. IMPROVEMENT FACTOR AND INTEGRATION LOSS

The non coherent integration gain is always smaller than the number of non coherently integrated pulses. This loss in integration is referred to as post detection or square-law detector loss.

Define $(\mathrm{SNR})_{\mathrm{NCI}}$ as the SNR required to achieve a specific P_D given a particular P_{fa} when n_p pulses are integrated noncoherently. Also denote the single pulse SNR as $(SNR)_1$. It follows that

$$(\mathrm{SNR})_{\mathrm{NCI}} = (\mathrm{SNR})_1 \times \mathrm{I}(\mathrm{n_P}) \tag{18}$$

Where $\mathrm{I}(\mathrm{n_P})$ is called the integration improvement factor. An empirically derived expression for the improvement factor that is accurate within 0.8dB reported in Peebles [5] as

$$[\mathrm{I}(\mathrm{n_P})]_{dB} = 6.79(1+0.253P_D)\left(1+\frac{\log\left(\frac{1}{P_{fa}}\right)}{46.6}\right)\log(n_p)$$

$$\left(1-0.140\log(n_p)+0.018310(\log\,n_p)^2\right) \tag{19}$$

The integration loss in dB is defined as

$$[L_{NCI}]_{dB} = 10\log n_p - [I(n_p)]_{dB} \tag{20}$$

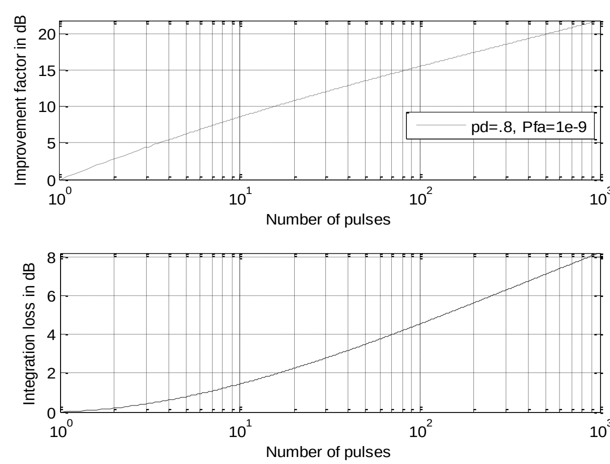

Fig.7. Plot for the improvement factor and integration loss versus number of noncoherently integrated pulses.

Table VI: Improvement factor of pulse integration.

Number of integrated pulses (np)	Improvement factor (dB)
1	0
5	6.203
10	8.555
25	11.439
75	14.645

It can be observed from the above table that with increase in number of integration pulses leads to improvement in detection of target.

V CONCLUSION

Traditional RADAR system have more ambiguity in recognition analysis of target without pulse integration of echo signal from the target. With pulse integration swerling I model has optimum value of SNR for integrated pulses from scan to scan according to a chi-square probability density function with two degree of freedom then swerling III model of four degree of freedom for 75 pulses. Similarly from the plot of 10 pulse integration value for swerling II target fluctuation independently from pulse to pulse according to a chi-square probability density function with two degree of freedom have higher SNR value then swerling IV model with four degree of freedom.

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