$\hat{g}^{**}$s-closed sets in topological spaces

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Abstract

*In this paper we introduce a new class of sets namely, $\hat{g}^{**}$s-closed sets, which is settled in between the class of g*s-closed sets and the class of gs-closed sets. Also, we compared it with other generalized closed sets and we find that the union of two $\hat{g}^{**}$s-closed sets is not an $\hat{g}^{**}$s-closed set and the intersection of two $\hat{g}^{**}$s-closed sets is again an $\hat{g}^{**}$s-closed set.*

Keyword: *$\hat{g}^{**}$s-closed sets, g*s-closed sets, gs-closed sets.*

I. INTRODUCTION

Levine introduced the class of semi open sets in 1963**[11]**and g-closed sets**[12]** in 1970.M.K.R.S.Veera Kumar defined $\hat{g}$-closed sets**[7]** in 2001 and $\hat{g}^{*}$-closed sets**[9]** in 1996. The authors introduce a new class of sets called $\hat{g}^{**}s$-closed sets, which properly placed in between the class of g*s-closed sets and the class of gs-closed sets.

II.PRELIMINARIES

Throughout this paper (X,τ) represent the non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), $cl(A)$ *and* $int(A)$ denote the closure and interior of A respectively.

A. Definition:

A subset A of a topological space (X,τ) is called

1) *A semi-open set* **[11]** if $A \subseteq cl(int(A))$ and *a semi-closed set* if $int(cl(A)) \subseteq A$.
2) *A semi pre-open set***[1]** if $A \subseteq cl(int(cl(A)))$ and *a semi pre-closed set* if $int(cl(int(A))) \subseteq A$.
3) *An α-open set* **[3]** if $A \subseteq int(cl(int(A)))$ and *an α-closed set* if $cl(int(cl(A))) \subseteq A$.
4) *A regular open set***[16]** if $A = int(cl(A))$ and *a regular closed set* if $A = cl(int(A))$.

B. Definition:

A subset A of a topological space (X,τ) is called

1) *A generalized closed set* (briefly *g- closed*) **[12]** if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X,τ).
2) *A generalized semi-closed set* (briefly *gs-closed*) **[15]** if $scl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X,τ).
3) *A $\hat{g}$-closed* or (*w-closed*) **[7]** set if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is semi open in (X,τ).
4) *A generalized pre-closed set* (briefly *gp-closed*) **[3]** if $pcl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X,τ).
5) *A g^{*}closed set***[6]** if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is g-open in (X,τ).
6) *A $\hat{g}^{*}$-closed* or *(*g-closed) set***[9]** if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is $\hat{g}$-open in (X,τ).
7) *A g**-closed set***[8]** if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is g*-open in (X,τ).
8) *A g*s-closed set***[4]** if $scl(A) \subseteq U$, whenever $A \subseteq U$ and U is gs-open in (X,τ).
9) *A $\hat{g}^{*}$s-closed set***[14]** if $scl(A) \subseteq U$, whenever $A \subseteq U$ and U is $\hat{g}$-open in (X,τ).
10) *A g*s*-closed set* [or (*sg*-closed set*) or ((*sg*)*-*closed set*)] **[10]** if $scl(A) \subseteq U$, whenever $A \subseteq U$ and U is g*-open in (X,τ).
11) *A gs**-closed set***[2]** if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is gs*- open in (X,τ).
12) *A gs*-closed set* or [*(gs)*-closed sets*] **[5]** if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is gs- open in (X,τ).

C. Results:

1)Every $\hat{g}^{*}$-closed sets are gs-closed set. **[9]**
2)Every $\hat{g}^{*}$-closed sets are g-closed set. **[9]**
3)Every closed sets are $\hat{g}^{*}$-closed set. **[9]**
4)Every g^{*}closed sets are $\hat{g}^{*}$-closed set. **[9]**

D. Notations used:

1) **scl(A)** – semi closure of A.
2) **pcl(A)**– pre-closure of A.

III. BASIC PROPERTIES OF $\hat{g}^{**}s$-CLOSED SETS

We now introduce the following definitions.

Definition: 3.1

A subset A of a topological space (X,τ) is called a $\hat{g}^{**}s$-closed set if $scl(A) \subseteq U$, whenever $A \subseteq U$ and U is $\hat{g}^{*}$-open in (X,τ).

Theorem:3.2

Every closed set is $\hat{g}^{**}s$-closed.

Proof:

Let A be a closed set in X.

Let $A \subseteq U$ and U is $\hat{g}^{*}$-open.

Since A is closed $cl(A) \subseteq U$.

But $scl(A) \subseteq cl(A) \subseteq U$

$\Rightarrow scl(A) \subseteq U$.

Hence A is $\hat{g}^{**}s$-closed.

Remark: 3.3

Example:3.4 shows that the converse of the above theorem is not true

Example:3.4

Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$, $A = \{b\}$ is $\hat{g}^{**}s$-closed but not closed.

Theorem: 3.5

Every semi closed set is $\hat{g}^{**}s$-closed.

Proof:

Let A be a semi closed set in X.

Let $A \subseteq U$ and U is $\hat{g}^{*}$-open.

Since A is semi-closed, $scl(A) \subseteq U$.

Hence A is $\hat{g}^{**}s$-closed.

Remark: 3.6:

Example:3.7 shows that the converse of the above theorem is not true

Example: 3.7

Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{c\}, \{b, c\}\}$,

$A = \{a, c\}$ is $\hat{g}^{**}s$-closed but not semi-closed.

Theorem: 3.8

Every α-closed set is $\hat{g}^{**}s$-closed.

Proof:

Let A be a α-closed set in X.

Let $A \subseteq U$ and U is $\hat{g}^{*}$-open.

Since A is α-closed, $\alpha cl(A) = A \subseteq U$.

$\Rightarrow \alpha cl(A) \subseteq U$.

But every α-closed set is semi-closed.

$\Rightarrow scl(A) \subseteq U$.

Hence A is $\hat{g}^{**}s$-closed.

Remark: 3.9

Example:3.10 shows that the converse of the above theorem is not true

Example: 3.10

Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a, b\}\}$,

$A = \{b\}$ is $\hat{g}^{**}s$-closed but not α -closed.

Theorem: 3.11

Every regular closed set is $\hat{g}^{**}s$-closed.

Proof:

Let A be a regular closed set in X.

Let $A \subseteq U$ and U is $\hat{g}^{*}$-open.

Since A is regular-closed, $rcl(A) \subseteq U$.

But $scl(A) \subseteq rcl(A) \subseteq U$.

$\Rightarrow scl(A) \subseteq U$.

Hence A is $\hat{g}^{**}s$-closed.

Remark: 3.12:

Example:3.13 shows that the converse of the above theorem is not true

Example: 3.13

Let $X = \{a, b, c\}, \tau = \{\phi, X, \{a\}\}, A = \{c\}$ is $\hat{g}^{**}s$-closed but not r-closed.

Theorem: 3.14

Every $g^{*}s$-closed set is $\hat{g}^{**}s$-closed.

Proof:

Let A be a $g^{*}s$ -closed set in X.

Let $A \subseteq U$ and U is $\hat{g}^{*}$-open.

But all $\hat{g}^{*}$-open sets are gs-open.

Thus $A \subseteq U$ and U is gs-open.

$\Rightarrow scl(A) \subseteq U$. [Since A is $g^{*}s$ -closed set]

Hence A is $\hat{g}^{**}s$-closed.

Remark: 3.15:

Example:3.16 shows that the converse of the above theorem is not true

Example:3.16

Let$X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b, c\}\}$,

$A = \{b\}$is$\hat{g}^{**}s$-closed but not $g^{*}s$-closed.

Theorem: 3.17

Every g^{*}-closed set is $\hat{g}^{**}s$-closed.

Proof:

Let A be a g^{*}-closed set in X.

Let $A \subseteq U$ and U is $\hat{g}^{*}$-open.

But all $\hat{g}^{*}$-open sets are g-open sets.

Thus $A \subseteq U$ and U is g-open.

$\Rightarrow cl(A) \subseteq U$. [Since A is g^{*}-closed set]

But $scl(A) \subseteq cl(A) \subseteq U$

$\Rightarrow scl(A) \subseteq U$.

Hence A is $\hat{g}^{**}s$-closed.

Remark: 3.18:

Example:3.19 shows that the converse of the above theorem is not true

Example: 3.19

Let$X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b, c\}\}$,

$A = \{b\}$ is a$\hat{g}^{**}s$-closed but not g^{*}-closed.

Theorem: 3.20

Every $(gs)^{*}$-closed set is $\hat{g}^{**}s$-closed.

Proof:

Let A be a $(gs)^{*}$-closed set in X.

Let $A \subseteq U$ and U is $\hat{g}^{*}$-open.

But all $\hat{g}^{*}$-open sets are gs-open sets.

Thus $A \subseteq U$ and U is gs-open.

$\Rightarrow cl(A) \subseteq U$. [Since A is $(gs)^{*}$-closed set]

But $scl(A) \subseteq cl(A) \subseteq U$

$\Rightarrow scl(A) \subseteq U$.

Hence A is $\hat{g}^{**}s$-closed.

Remark: 3.21:

Example:3.22 shows that the converse of the above theorem is not true

Example: 3.22

Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$, $A = \{b\}$is a$\hat{g}^{**}s$-closed but not $(gs)^{*}$-closed.

Theorem: 3.23

Every $\hat{g}^{**}s$-closed set is gs-closed.

Proof:

Let A be a $\hat{g}^{**}s$-closed set in X.

Let $A \subseteq U$ and U is open.

But all open sets are $\hat{g}^{*}$-open sets.

Thus $A \subseteq U$ and U is $\hat{g}^{*}$-open.

$\Rightarrow scl(A) \subseteq U$. [Since A is $\hat{g}^{**}s$-closed set]

Hence A is gs-closed.

Remark: 3.24:

Example:3.25shows that the converse of the above theorem is not true

Example:3.25

Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$,

$A = \{a, b\}$is ags-closed but not $\hat{g}^{**}s$-closed.

Theorem: 3.26

Every $\hat{g}^{**}s$-closed set is $(sg)^{*}$-closed.

Proof:

Let A be a $\hat{g}^{**}s$-closed set in X.

Let $A \subseteq U$ and U is open.

But all g^{*}- open sets are $\hat{g}^{*}$- open sets.

Thus $A \subseteq U$ and U is $\hat{g}^{*}$- open.

$\Rightarrow scl(A) \subseteq U$. [Since A is $\hat{g}^{**}s$-closed set]

Hence A is $(sg)^{*}$-closed.

Remark: 3.27:

Example:3.28 shows that the converse of the above theorem is not true

Example: 3.28

Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$,

$A = \{a, b\}$is a$(sg)^{*}$-closed but not $\hat{g}^{**}s$-closed.

Remark: 3.29

$\hat{g}^*s^*$-closed and $\hat{g}^{**}s$-closed sets are independent.

Proof:

Let $X = \{a, b, c\}$.Consider the topological space$\tau_1 = \{\phi, X, \{a\}\}$.

Let $A = \{\phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ is a $\hat{g}^*s^*$-closed set and $B = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$ isa $\hat{g}^{**}s$-closed set in the topological space τ_1.

Here, the sets$\{a, b\}$and $\{a, c\}$ belongs to A but does not belongs to B.

Therefore, the sets$\{a, b\}$and $\{a, c\}$ are a $\hat{g}^*s^*$-closed set but not a $\hat{g}^{**}s$-closed set.

Hence, a $\hat{g}^*s^*$-closed set need not be a $\hat{g}^{**}s$-closed set. ⇢ (1)

Similarly, Consider the topological space$\tau_2 = \{\phi, X, \{a\}, \{b, c\}\}$.

Let$A = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ is a $\hat{g}^{**}s$-closed set and $B = \{\phi, X, \{a\}, \{b, c\}\}$ is a $\hat{g}^*s^*$-closed set in the topological space τ_2.

Here, the sets $\{b\}$, $\{c\}, \{a, b\}$ and $\{a, c\}$

belongs to A but does not belongs to B.

Therefore, the sets$\{b\}$, $\{c\}, \{a, b\}$ and $\{a, c\}$

are a $\hat{g}^{**}s$-closed set but not a $\hat{g}^*s^*$-closed set.

Hence, a $\hat{g}^{**}s$-closed set need not be a $\hat{g}^*s^*$-closed set. ⇢ (2)

Therefore, from (1)and (2), we can say that $\hat{g}^*s^*$-closed and $\hat{g}^{**}s$-closed sets are independent.

Remark: 3.30

gs^{**}-closed and $\hat{g}^{**}s$-closed sets are independent.

Proof:

Let $X = \{a, b, c\}$.Consider the topological space$\tau_1 = \{\phi, X, \{a\}\}$.

Let $A = \{\phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ is a gs^{**} -closed set and $B = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$ is a $\hat{g}^{**}s$-closed set in the topological space τ_1.

Here, the sets$\{a, b\}$and $\{a, c\}$ belongs to A but does not belongs to B.

Therefore, the sets$\{a, b\}$and $\{a, c\}$ are a gs^{**}-closed set but not a $\hat{g}^{**}s$-closed set.

Hence, a gs^{**}-closed set need not be a $\hat{g}^{**}s$-closed set. ⇢ (1)

Similarly, Consider the topological space

$\tau_2 = \{\phi, X, \{c\}, \{b, c\}\}$.

Let $A = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$ is $\hat{g}^{**}s$-closed set and $B = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ is a gs^{**}-closed set in the topological space τ_2.

Here, the set $\{b\}$ belongs to A but does not belongs to B.

Therefore, the set $\{b\}$ is a $\hat{g}^{**}s$-closed set but not a gs^{**}-closed set.

Hence, a $\hat{g}^{**}s$-closed set need not be a gs^{**}-closed set. ⇢ (2)

Therefore, from (1)and (2), we can say that gs^{**}-closed and $\hat{g}^{**}s$-closed sets are independent.

Remark: 3.31

$\hat{g}^*$-closed and $\hat{g}^{**}s$-closed sets are independent.

Proof:

Let $X = \{a, b, c\}$.Consider the topological space$\tau_1 = \{\phi, X, \{a\}\}$.

Let $A = \{\phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ is a $\hat{g}^*$-closed set and $B = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$ is a $\hat{g}^{**}s$-closed set in the topological space τ_1.

Here, the sets$\{a, b\}$and $\{a, c\}$ belongs to A but does not belongs to B.

Therefore, the sets$\{a, b\}$and $\{a, c\}$ are a $\hat{g}^*$-closed set but not a $\hat{g}^{**}s$-closed set.

Hence, a $\hat{g}^*$-closed set need not be a $\hat{g}^{**}s$-closed set. ⇢ (1)

Similarly, Consider the topological space

$\tau_2 = \{\phi, X, \{a\}, \{b, c\}\}$.

Let$A = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ is $\hat{g}^{**}s$-closed set and $B = \{\phi, X, \{a\}, \{b, c\}\}$ is a $\hat{g}^*$-closed set in the topological space τ_2.

Here, the sets $\{b\}, \{c\}, \{a, b\}$and$\{a, c\}$ belongs to A but does not belongs to$B$.

Therefore, the sets$\{b\}, \{c\}, \{a, b\}$ and$\{a, c\}$are a $\hat{g}^{**}s$-closed set but not a $\hat{g}^*$-closed set.

Hence, a $\hat{g}^{**}s$-closed set need not be a $\hat{g}^*$-closed set. ⇢ (2)

Therefore, from (1) and (2), we can say that $\hat{g}^*$-closed and $\hat{g}^{**}s$-closed sets are independent.

Remark: 3.32

g^{**}-closed and $\hat{g}^{**}s$-closed sets are independent.

Proof:

Let $X = \{a, b, c\}$.Consider the topological space$\tau_1 = \{\phi, X, \{a\}\}$.

Let $A = \{\phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ is a g^{**}-closed set and $B = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$ is a $\hat{g}^{**}s$-closed set in the topological space τ_1.

Here, the sets$\{a, b\}$and $\{a, c\}$ belongs to A but does not belongs to B.

Therefore, the sets$\{a, b\}$and $\{a, c\}$ are a g^{**}-closed set but not a $\hat{g}^{**}s$-closed set.

Hence, a g^{**}-closed set need not be a $\hat{g}^{**}s$-closed set. ⇢ (1)

Similarly, Consider the topological space

$\tau_2 = \{\phi, X, \{c\}, \{b, c\}\}$.

Let $A = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$is a $\hat{g}^{**}s$-closed set and $B = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$ is a g^{**}-closed set in the topological space τ_2.

Here, the sets $\{b\}$ belongs to A but does not belongs to B.

Therefore, the set$\{b\}$is a $\hat{g}^{**}s$-closed set but not a g^{**}-closed set.

Hence, a $\hat{g}^{**}s$-closed set need not be a g^{**}-closed set. ⇢ (2)

Therefore, from (1) and (2), we can say that g^{**}-closed and $\hat{g}^{**}s$-closed sets are independent.

Remark: 3.33

sp-closed and $\hat{g}^{**}s$-closed sets are independent.

Proof:

Let $X = \{a, b, c\}$.Consider the topological space$\tau_1 = \{\phi, X, \{a, b\}\}$.

Let $A = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ is a sp-closed set and $B = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ is a $\hat{g}^{**}s$-closed set in the topological space τ_1.

Here, the sets$\{a\}$and $\{b\}$ belongs to A but does not belongs to B.

Therefore, the sets$\{a\}$and $\{b\}$ are a sp-closed set but not a $\hat{g}^{**}s$-closed set.

Hence, a sp-closed set need not be a $\hat{g}^{**}s$-closed set. ⇢ (1)

Similarly,Consider the topological space

$\tau_2 = \{\phi, X, \{c\}, \{b, c\}\}$.

Let $A = \{\phi, X, \{c\}, \{b, c\}, \{a, c\}\}$ is $\hat{g}^{**}s$-closed set and $B = \{\phi, X, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$ is a sp-closed set in the topological space τ_2.

Here, the set $\{a, c\}$ belongs to A but does not belongs to B.

Therefore, the set $\{a, c\}$is a $\hat{g}^{**}s$-closed set but not a sp-closed set.

Hence, a $\hat{g}^{**}s$-closed set need not be a sp-closed set. ⇢ (2)

Therefore, from (1) and (2), we can say that sp-closed and $\hat{g}^{**}s$-closed sets are independent.

Remark: 3.34

gp-closed and $\hat{g}^{**}s$-closed sets are independent.

Proof:

Let $X = \{a, b, c\}$.Consider the topological space$\tau_1 = \{\phi, X, \{a\}\}$.

Let $A = \{\phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ is a gp-closed set and $B = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$ is a $\hat{g}^{**}s$-closed set in the topological space τ_1.

Here, the sets$\{a, b\}$and $\{a, c\}$ belongs to A but does not belongs to B.

Therefore, the sets$\{a, b\}$and $\{a, c\}$ are a gp-closed set but not a $\hat{g}^{**}s$-closed set.

Hence, a gp-closed set need not be a $\hat{g}^{**}s$-closed set. ⇢ (1)

Similarly,Consider the topological space

$\tau_2 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$.

Let $A = \{\phi, X, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$ is a $\hat{g}^{**}s$-closed set and $B = \{\phi, X, \{c\}, \{b, c\}, \{a, c\}\}$ is a gp-closed set in the topological space τ_2.

Here, the sets $\{a\}$ and $\{b\}$ belongs to A but does not belongs to B.

Therefore, the sets$\{a\}$and $\{b\}$ are a $\hat{g}^{**}s$-closed set but not a gp-closed set.

Hence, a$\hat{g}^{**}s$-closed set need not be a gp-closed set. ⇢ (2)

Therefore, from (1) and (2), we can say that, gp-closed and $\hat{g}^{**}s$-closed sets are independent.

Remark: 3.35

g-closed and $\hat{g}^{**}s$-closed sets are independent.

Proof:

Let $X = \{a, b, c\}$.Consider the topological space$\tau_1 = \{\phi, X, \{a\}\}$.

Let $A = \{\phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ is a g-closed set and $B = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$ is a $\hat{g}^{**}s$-closed set in the topological space τ_1.

Here, the sets$\{a, b\}$and $\{a, c\}$ belongs to A but does not belongs to B.

Therefore, the sets$\{a, b\}$and $\{a, c\}$ are a g-closed set but not a $\hat{g}^{**}s$-closed set.

Hence, a g-closed set need not be a $\hat{g}^{**}s$-closed set. ⇢ (1)

Similarly,Consider the topological space

$\tau_2 = \{\phi, X, \{c\}, \{b, c\}\}$.

Let $A = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ is a $\hat{g}^{**}s$-closed set and $B = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$ is a g-closed set in the topological space τ_2.

Here, the set $\{b\}$ belongs to A but does not belongs to B.

Therefore, the sets$\{a, b\}$and $\{a, c\}$ are a $\hat{g}^{**}s$-closed set but not a g-closed set.

Hence, a $\hat{g}^{**}s$-closed set need not be a g-closed set. ⇢ (2)

Therefore, from (1) and (2), we can say that g-closed and $\hat{g}^{**}s$-closed sets are independent.

Remark: 3.36

Union of two $\hat{g}^{**}s$-closed sets need not be $\hat{g}^{**}s$-closed.

Proof:

Let $X = \{a, b, c\}$.Consider the topology

$\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$.

Here,$\{\phi, X, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$ is a $\hat{g}^{**}s$-closed set.

Consider, two $\hat{g}^{**}s$-closed elements $A = \{a\}$and

$B = \{b\}$.

Here A and B are $\hat{g}^{**}s$-closed sets but

$A \cup B = \{a\} \cup \{b\} = \{a, b\}$ is not $\hat{g}^{**}s$-closed set.

Theorem: 3.37

Let A and B be any two$\hat{g}^{**}s$-closed sets in a topological space X. Then $A \cap B$ is also a $\hat{g}^{**}s$-closed set in X.

Proof:

Let A and B be any two $\hat{g}^{**}s$-closed sets.

Then $A \subseteq U$, U is a $\hat{g}^{*}$-open set and $B \subseteq U$, U is a $\hat{g}^{*}$-open set.

Hence, $scl(A) \subseteq U$ and $scl(B) \subseteq U$.

Therefore, $scl(A \cap B) \subseteq scl(A) \cap scl(B) \subseteq U$

$\Rightarrow scl(A \cap B) \subseteq U$, U is a $\hat{g}^{*}$-open set in X.

Hence, $A \cap B$ is also a $\hat{g}^{**}s$-closed set in X.

Remark:3.38

From the above discussion, we obtain the following implications.

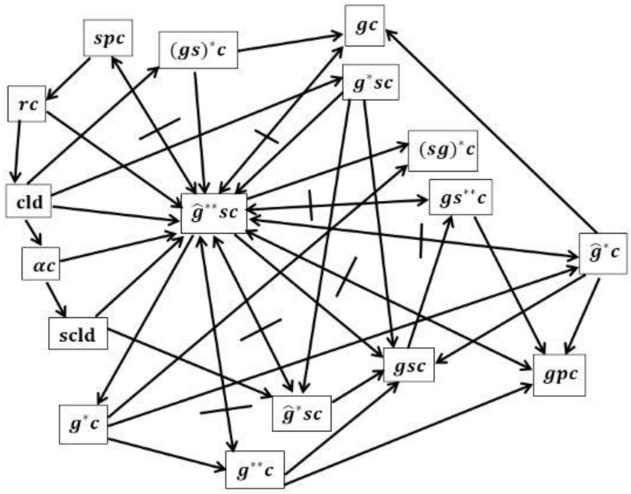

Figure:1

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