α Generalized Closed Sets in Neutrosophic Topological Spaces

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Abstract: In this paper a new concept of neutrosophic closed sets called neutrosophic α generalized closed sets is introduced and their properties are thoroughly studied and analyzed. Some new interesting theorems based on the newly introduced set are presented.

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1. Introduction

The concept of neutrosophic sets was first introduced by Florentin Smarandache [3] in 1999 which is a generalization of intuitionistic fuzzy sets by Atanassov [1]. A. A. Salama and S. A. Alblowi [6] introduced the concept of neutrosophic topological spaces after Coker [2] introduced intuitionistic fuzzy topological spaces. Further the basic sets like semi open sets, pre open sets, α open sets and semi-α open sets are introduced in neutrosophic topological spaces and their properties are studied by various authors [4,5]. The purpose of this paper is to introduce and analyze a new concept of neutrosophic closed sets called neutrosophic α generalized closed sets.

2. Preliminaries:

Here in this paper the neutrosophic topological space is denoted by (X, τ). Also the neutrosophic interior, neutrosophic closure of a neutrosophic set A are denoted by NInt(A) and NCl(A). The complement of a neutrosophic set A is denoted by C(A) and the empty and whole sets are denoted by θ_X and 1_X respectively.

Definition 2.1: Let X be a non-empty fixed set. A neutrosophic set (NS) A is an object having the form A = {x, μ_A(x), σ_A(x), ν_A(x)) : x ∈ X} where μ_A(x), σ_A(x) and ν_A(x) represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element x ∈ X to the set A.

A Neutrosophic set A = {(x, μ_A(x), σ_A(x), ν_A(x)) : x ∈ X} can be identified as an ordered triple (μ_A, σ_A, ν_A) in ]0, 1] on X.

Definition 2.2: Let A = (μ_A, σ_A, ν_A) be a NS on X, then the complement C(A) may be defined as

1. C(A) = {x, 1-μ_A(x), 1-ν_A(x)) : x ∈ X}
2. C(A) = {x, μ_A(x), σ_A(x)) : x ∈ X}
3. C(A) = {x, 1-ν_A(x), 1-μ_A(x)) : x ∈ X}

Note that for any two neutrosophic sets A and B,
4. C(A ∪ B) = C(A) ∩ C(B)
5. C(A ∩ B) = C(A) ∪ C(B)

Definition 2.3: For any two neutrosophic sets A = {x, μ_A(x), σ_A(x), ν_A(x)) : x ∈ X} and B = {x, μ_B(x), σ_B(x), ν_B(x)) : x ∈ X} we may have

1. A ⊆ B ⇔ μ_A(x) ≤ μ_B(x), σ_A(x) ≤ σ_B(x) and ν_A(x) ≥ ν_B(x) ∀ x ∈ X
2. A ⊆ B ⇔ μ_A(x) ≤ μ_B(x), σ_A(x) ≥ σ_B(x) and ν_A(x) ≥ ν_B(x) ∀ x ∈ X
3. A ∩ B = {x, μ_A(x) ∧ μ_B(x), σ_A(x) ∨ σ_B(x), ν_A(x) ∨ ν_B(x))
4. A ∩ B = {x, μ_A(x) ∨ μ_B(x), σ_A(x) ∧ σ_B(x), ν_A(x) ∧ ν_B(x))
5. A ∪ B = {x, μ_A(x) ∨ μ_B(x), σ_A(x) ∨ σ_B(x), ν_A(x) ∧ ν_B(x))
6. A ∪ B = {x, μ_A(x) ∧ μ_B(x), σ_A(x) ∧ σ_B(x), ν_A(x) ∨ ν_B(x))

Definition 2.4: A neutrosophic topology (NT) on a non-empty set X is a family of neutrosophic subsets in X satisfies the following axioms:

\[(NT_1)\quad \theta_X, \, 1_X \in \tau\]
\[(NT_2)\quad G_1 \cap G_2 \in \tau \text{ for any } G_1, G_2 \in \tau\]
\[(NT_3)\quad \bigcup_{i=1}^{n} G_i \in \tau \quad \forall \{ G_i : i \in I \} \subseteq \tau\]

In this case the pair (X, τ) is a neutrosophic topological space (NTS) and any neutrosophic set in τ is known as a neutrosophic open set (NOS) in X. A neutrosophic set A is a neutrosophic closed set (NCS)
Definition 2.5: Let $(X, \tau)$ be a NTS and $A = \{(x, \mu_A(x), \sigma_A(x), \nu_A(x)) : x \in X\}$ be a NS in X. Then the neutrosophic interior and the neutrosophic closure of $A$ are defined by

$$NCl(A) = \{K : K is a NCS in X and A \subseteq K\}$$

$$NInt(A) = \{x : x \in X\}$$

Note that for any NS $A$, $NCl(A) = C(NInt(A))$ and $NInt(C(A)) = C(NInt(A))$.

Definition 2.6: A NS $A$ of a NTS $X$ is said to be

(i) a neutrosophic pre-open set (NP-OS) if $A \subseteq NInt(NCl(A))$

(ii) a neutrosophic semi-open set (NS-OS) if $A \subseteq NCl(NInt(A))$

(iii) a neutrosophic $\alpha$-open set (N$\alpha$-OS) if $A \subseteq NInt(NCl(NInt(A)))$

(iv) a neutrosophic semi-$\alpha$-open set (NS$\alpha$-OS) if $A \subseteq NCl(\alpha NInt(A))$

Definition 2.7: A NS $A$ of a NTS $X$ is said to be

(i) a neutrosophic pre-closed set (NP-CS) if $NCl(NInt(A)) \subseteq A$

(ii) a neutrosophic semi-closed set (NS-CS) if $NInt(NCl(A)) \subseteq A$

(iii) a neutrosophic $\alpha$-closed set (N$\alpha$-CS) if $NCl(NInt(A)) \subseteq A$

(iv) a neutrosophic semi-$\alpha$-closed set (NS$\alpha$-CS) if $NInt(\alpha NCl(A)) \subseteq A$

3. $\alpha$ generalized closed sets in neutrosophic topological spaces

In this section we introduce neutrosophic $\alpha$ closure, neutrosophic $\alpha$ interior and $\alpha$ generalized closed set and its respective open set in neutrosophic topological spaces and discuss some of their properties.

Definition 3.1: A NS $A$ in a NTS $X$ is said to be a neutrosophic regular closed set (NRCS) if $NCl(NInt(A)) = A$ and neutrosophic regular open set if $NInt(NCl(A)) = A$.

Definition 3.2: A NS $A$ in a NTS $X$ is said to be a neutrosophic $\beta$ closed set (N$\beta$CS) if $NCl(NInt(NCl(A))) \subseteq A$, and neutrosophic $\beta$ open set if $A \subseteq NCl(NInt(NCl(A)))$.

Definition 3.3: Let $A$ be a NS of a NTS $(X, \tau)$. Then the neutrosophic $\alpha$ interior and the neutrosophic $\alpha$ closure are defined as

$$N\alpha Int(A) = \cup \{G : G is a N\alpha OS in X and G \subseteq A\}$$

$$N\alpha Cl(A) = \cap \{K : K is a N\alpha CS in X and A \subseteq K\}$$

Result 3.4: Let $A$ be a NS in $X$. Then $N\alpha Cl(A) = A \cup NCl(NInt(NCl(A)))$.

Proof: Since $N\alpha Cl(A) = A \cup NCl(NInt(NCl(A))) \subseteq N\alpha Cl(A)$ and $A \subseteq N\alpha Cl(NInt(NCl(A))) \subseteq A \cup N\alpha Cl(A) = N\alpha Cl(A)$.

Definition 3.5: A NS $A$ in a NTS $X$ is said to be a neutrosophic $\alpha$ generalized closed set (N$\alpha$CS) if $N\alpha Cl(A) \subseteq A$ whenever $A \subseteq U$ and $U$ is a NOS in $X$. The complement $C(A)$ of a N$\alpha$CS in $X$.

Example 3.6: Let $X = \{a, b\}$ and $\tau = \{\emptyset, A, B, X\}$ where $A = (x, (0.5, 0.6), (0.3, 0.2), (0.4, 0.1))$ and $B = (x, (0.4, 0.4), (0.4, 0.3), (0.5, 0.4))$. Then $\tau$ is a NT. Here $\mu_A(a) = 0.5$, $\mu_B(b) = 0.6$, $\sigma_A(a) = 0.3$, $\sigma_B(b) = 0.2$, $\nu_A(a) = 0.4$ and $\nu_B(b) = 0.1$. Also $\mu_B(a) = 0.4$, $\mu_B(b) = 0.3$, $\sigma_B(b) = 0.3$, $\nu_B(b) = 0.4$. Let $M = (x, (0.5, 0.4), (0.5, 0.4), (0.4, 0.5))$ be any NS in $X$. Then $M \subseteq A$ where $A$ is a NOS in $X$. Now $N\alpha Cl(M) = M \cup C(B) \subseteq (a, \emptyset, C(B) \subseteq A$. Therefore $M$ is a N$\alpha$CS in $X$.

Proposition 3.7: Every NCS $A$ is a N$\alpha$CS in $X$ but not conversely in general.

Proof: Let $A \subseteq U$ where $U$ is a NOS in $X$. Now $N\alpha Cl(A) = A \cup NCl(NInt(NCl(A))) \subseteq A \cup NCl(A) = A \subseteq U$, by hypothesis. Therefore $A$ is a N$\alpha$CS in $X$.

Example 3.8: In Example 3.6, $M$ is a N$\alpha$CS in $X$ but not a NCS in $X$ as $NCl(M) = C(B) \neq M$. 

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Remark 3.9: Every NS-CS and every N_{ag}CS in a NTS X are independent to each other in general.

Example 3.10: In Example 3.6, M is a N_{ag}CS but not a NS-CS as NInt(NCl(M)) = B \nsubseteq M.

Example 3.11: Let X = \{a, b\} and \tau = \{0_{N}, A, B, C, 1_{N}\}, where A = \langle x, (0.5, 0.4), (0.3, 2.0), (0.5, 0.6) \rangle, B = \langle x, (0.8, 0.7), (0.4, 0.3), (0.2, 0.3) \rangle and C = \langle x, (0.2, 0.1), (0.3, 0.2), (0.8, 0.9) \rangle. Then \tau is a NT. Let M = \langle x, (0.5, 0.3), (0.3, 0.2), (0.5, 0.7) \rangle. Then M is a NS-CS but not a N_{ag}CS as M \nsubseteq A, B and N_{Cl}(M) = M \cup C(A) = C(A) \nsubseteq A.

Remark 3.12: Every NP-CS and every N_{ag}CS in a NTS X are independent to each other in general.

Example 3.13: In Example 3.11, M is a NP-CS but not a N_{ag}CS as seen in the respective example.

Example 3.14: Let X = \{a, b\} and \tau = \{0_{N}, A, B, 1_{N}\}, where A = \langle x, (0.5, 0.4), (0.3, 0.2), (0.5, 0.6) \rangle and B = \langle x, (0.4, 0.3), (0.3, 0.1), (0.6, 0.7) \rangle. Then \tau is a NT. Let M = \langle x, (0.5, 0.5), (0.2, 0.1), (0.4, 0.4) \rangle. Then M is a N_{ag}CS but not a NP-CS as NCl(NInt(M)) = C(A) \nsubseteq M.

Proposition 3.15: Every N_{\alpha}-CS A is a N_{ag}CS in X but not conversely in general.
Proof: Let A \nsubseteq U, where U is a NOS in X. Then N_{\alpha}Cl(A) = A \nsubseteq NCl(NInt(NCl(A))) \nsubseteq A \nsubseteq U, by hypothesis. Hence A is a N_{ag}CS in X.

Example 3.16: In Example 3.6, M is a N_{ag}CS in X but not a N_{\alpha}-CS as NCl(NInt(NCl(M))) = C(B) \nsubseteq M.

Proposition 3.17: Every NOS, N_{\alpha}-OS are N_{ag}OS but not conversely in general.
Proof: Obvious.

Example 3.18: In Example 3.6, C(M) is a N_{ag}OS but not a NOS, N_{\alpha}-OS in X.

Remark 3.19: Both NS-OS and NP-OS are independent to N_{ag}OS in X in general.

Example 3.20: The above Remark can be proved easily from the Examples 3.10, 3.11 and 3.13, 3.14 respectively.

Proposition 3.21: The union of any two N_{ag}CSs is a N_{ag}CS in a NTS X.
U ∈ N\square-O(X) and N. Cl(A) ⊆ N. Cl(U) = U, since U ∈ N\square-C(X), by hypothesis. Therefore A is an N\square-CS in X.

**Proposition 3.27:** If A is a NOS and a N\square-CS in (X, τ), then A is a NROS in (X, τ).

**Proof:** Let A be a NOS and a N\square-CS in (X, τ). Then A is a N\square-CS in X. Now NInt(NCl(A)) \cap NCl(NInt(NCl(A))) ⋄ A. Since A is a NOS, A = NInt(A) \cap NInt(NCl(A)). Hence NInt(NCl(A)) = A and A is a NROS in X.

**Definition 3.28:** A NS A in (X, τ) is a neutrosophic Q-set (NQ-S) in X if NInt(NCl(A)) = NCl(NInt(A)).

**Proposition 3.29:** For a NOS A in (X, τ), the following conditions are equivalent:

(i) A is a NCS in (X, τ),
(ii) A is a N\square-CS and a NQ-S in (X, τ).

**Proof:** (i) ⇒ (ii) Since A is a NCS, it is a N\square-CS in (X, τ). Now NInt(NCl(A)) = NInt(A) = A = NCl(A) = NCl(NInt(A)), by hypothesis. Hence A is a NQ-S in (X, τ).

(ii) ⇒ (i) Since A is a NOS and a N\square-CS in (X, τ), by Theorem 3.27, A is a NROS in (X, τ). Therefore A = NInt(NCl(A)) = NCl(NInt(A)) = NCl(A), by hypothesis. Hence A is a NCS in (X, τ).

**Proposition 3.30:** Let (X, τ) be a NTS. Then for every A ∈ N\square-O(X) and for every B ∈ NS(X), N. Int (A) ⊆ B ⊆ A implies B ∈ N\square-O(X).

**Proof:** Let A be any N\square-O(X) of X and B be any NS of X. By hypothesis N. Int(A) ⊆ B ⊆ A. Then C(A) is a N\square-CS in X and C(A) ⊆ C(B) ⊆ N. Cl(C(A)). By Theorem 3.24, C(B) is a N\square-CS in (X, τ). Therefore B is a N\square-CS in (X, τ). Hence B ∈ N\square-O(X).

**Proposition 3.31:** Let (X, τ) be a NTS. Then for every A ∈ NS(X) and for every B ∈ NS-O(X), B ⊆ A ⊆ NInt(NCl(B)) implies A ∈ N\square-O(X).

**Proof:** Let B be a NS-O in (X, τ). Then B ⊆ NCl(NInt(B)). By hypothesis, A ⊆ NInt(NCl(B)) ⊆ NInt(NCl(NCl(NInt(B)))) ⊆ NInt(NCl(NInt(NInt(B)))) ⊆ NInt(NInt(NCl(NInt(A)))). Therefore A is a N\square-OS and by Proposition 3.17, A is a N\square-OS in (X, τ). Hence A ∈ N\square-O(X).

**Proposition 3.32:** A NS A of a NTS (X, τ) is a N\square-OS in (X, τ) if and only if F ⊆ N. Int(A) such that F is a NCS in (X, τ) and F ⊆ A.

**Proof:** Necessity: Suppose A is a N\square-OS in (X, τ). Let F be a NCS in (X, τ) such that F ⊆ A. Then C(F) is a NOS and C(A) ⊆ C(F). By hypothesis C(A) is a N\square-CS in (X, τ), we have N. Cl(C(A)) ⊆ C(F). Therefore F ⊆ N. Int(A).

Sufficiency: Let U be a NOS in (X, τ) such that C(A) ⊆ U. By hypothesis, C(U) ⊆ N. Int(A). Therefore N. Cl(C(A)) ⊆ U and C(A) is an N\square-CS in (X, τ). Hence A is a N\square-OS in (X, τ).

References: