Decomposition of δ-Semi Continuity via δsg*-Continuity in Topological Spaces

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Abstract - *The scope of this paper is to introduce the concepts of Dδ*-sets, Dδ**-sets, Dδ*-continuity and Dδ**-continuity. Also we obtain decomposition of δ-semi continuity in topological spaces and some of its properties are derived.*

Keywords - *δ-semi closed, δsg* -closed, Dδ*-sets, Dδ**-sets, Dδ*-continuity, Dδ**-continuity.*

I. Introduction

The study of generalized closed sets was initiated by Levine [8] in order to extend the topological properties of closed sets to a larger family of sets in 1970. The concept of generalized continuous functions was introduced and studied by Balachandran [1] in 1991. In 2005, Ekici [2] introduced δ-semi continuity. In 1963, Levine [7] introduced the concept of semi open sets in topological spaces which is a weaker form of open sets. Veliko [16] introduced a stronger form of closed sets namely δ-open sets in 1968. In 1997, Park [6] introduced the idea of δ-semi open sets. Tong [14] introduced the notions of A-sets and A-continuity in topological spaces and established the decomposition of continuity in 1986. In 1989, Tong [15] introduced the notions of B-sets and B-continuity and used them to obtain another decomposition of continuity. In 2006, Noiri [11] also obtained a decomposition of continuity. Some generalized closed sets [13] δg*-closed and Δ*-closed [9] are introduced by Sudha and Meena respectively. A new class of generalized closed sets called δsg*-closed sets in topological spaces using δ-semi closed sets was introduced by Geethagnanaselvi [3, 4] in 2016. In 2017, she also introduced the notion of δsg*-continuity in topological spaces. The main purpose of this paper is to introduce and study the concept of Dδ*-sets, Dδ**-sets, Dδ*-continuity and Dδ**-continuity. Furthermore the decompositions of δ-semi continuity and δsg*-continuity in topological spaces are obtained.

II. PRELIMINARIES

Throughout the present paper (X, τ) and (Y, σ) represents non empty topological space on which no separation axiom is defined, unless otherwise mentioned. The interior and the closure of a subset A of (X, τ) are denoted by *int(A)* and *cl(A)* respectively. Here we recall the following known definitions and properties.

Definition 2.1 A subset A of a topological space (X, τ) is called a

1) ***Regular open*** set [12] if A = int(cl(A))
2) ***Semi open*** *set* [7] if A ⊆ cl(int(A))
3) ***δ-open*** set [16] if it is the union of regular open sets.
4) ***δ-semi open*** set [6] if A ⊆ cl(δ-int(A))

The complement of the above mentioned open sets are their respective closed sets. The intersection of all regular closed (resp. semi closed, δ-closed and δ-semi closed) subsets of (X, τ) containing A is called the regular closure (resp. semi closure, δ-closure and δ-semi closure) of A and is denoted by rcl(A) (resp. scl(A), δ-cl(A) and δ-scl(A)).

Definition 2.2 A subset A of a space (X, τ) is called

1) ***a t-set*** [11] if int(cl(A)) = int(A)
2) ***an A-set*** [14] if A = U∩F where U is open and F is regular closed, i.e., F = cl(int(F))
3) ***a B-set*** [15] if A = U∩F where U is open and F is a t-set.

Definition 2.3 A subset A of a topological space (X, τ) is called

1) ***g-closed*** [8] if cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ). The complement of a g-closed set is called ***g-open***.
2) ***δg*-closed*** [13] if $cl_\delta(A)$ ⊆ U whenever A ⊆ U and U is g-open in (X, τ). The complement of a δg*-closed set is called ***δg*-open***.
3) ***Δ*-closed*** [9] if δcl(A) ⊆ U whenever A ⊆ U and U is δg-open in (X, τ). The complement of a Δ*-closed set is called ***Δ*-open***.
4) ***δsg*-closed*** [3] if δ-scl(A) ⊆ U whenever A ⊆ U and U is g-open in (X, τ). The complement of a δsg*-closed set is called ***δsg*-open***.

Lemma 2.4 [6] **(i)** The intersection (resp. union) of arbitrary collection of δ-semi closed (resp. δ-semi open) sets in (X, τ) is δ-semi closed (resp. δ-semi open). **(ii)** $A \subseteq X$ is δ-semi closed if and only if A= δ-scl(A).

Definition 2.5 A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called

a) ***A-continuous*** [14] if the inverse image of every open set in (Y, σ) is A-set in (X, τ).
b) ***B-continuous*** [15] if the inverse image of every open set in (Y, σ) is B-set in (X, τ).
c) ***g-continuous*** [1] if the inverse image of every open set in (Y, σ) is g-open in (X, τ).
d) ***δ-semi continuous*** [10] if the inverse image of every open set in (Y, σ) is δ-semi open in (X, τ).
e) ***δsg^*-continuous*** [5] if the inverse image of every open set in (Y, σ) is δsg^*-open in (X, τ).

Remark 2.6

1) Every δ-semi closed set is a δsg^*-closed set but not conversely [3].
2) Every δ-semi continuous function is a δsg^*-continuous function but not conversely [5].

III. $D\delta^*$-SETS AND $D\delta^{**}$-SETS

In this section we introduce and study the notions of $D\delta^*$-sets and $D\delta^{**}$-sets in topological spaces.

Definition 3.1 A subset A of a topological space (X, τ) is said to be

(1) ***an $D\delta^*$-set*** if $A = U \cap F$ where U is g-open and F is δ-semi closed in (X, τ).
(2) ***an $D\delta^{**}$-sets*** if $A = U \cap F$ where U is δsg^*-open and F is a A-set in (X, τ).

Theorem 3.2 For a subset A of a topological space (X, τ), the following are equivalent.

(a) A is $D\delta^*$-set
(b) $A = U \cap \delta\text{-scl}(A)$ for some g-open set U.

Proof: (a) ⇒ (b) Since A is an $D\delta^*$-set then $A = U \cap F$, where U is g-open and F is δ-semi closed. So, $A \subseteq U$ and $A \subseteq F$. Hence $\delta\text{-scl}(A) \subseteq \delta\text{-scl}(F) = F$. Therefore $A \subseteq U \cap \delta\text{-scl}(A) \subseteq U \cap \delta\text{-scl}(F) = U \cap F = A$. Hence $A = U \cap \delta\text{-scl}(A)$.

(b) ⇒ (a) it is obvious because δ-scl(A) is a δ-semi closed set.

Remark 3.3 In a topological space (X, τ),

1) the intersection of two $D\delta^*$-sets is an $D\delta^*$-set.
2) the union of two $D\delta^{**}$-sets is an $D\delta^{**}$-set.

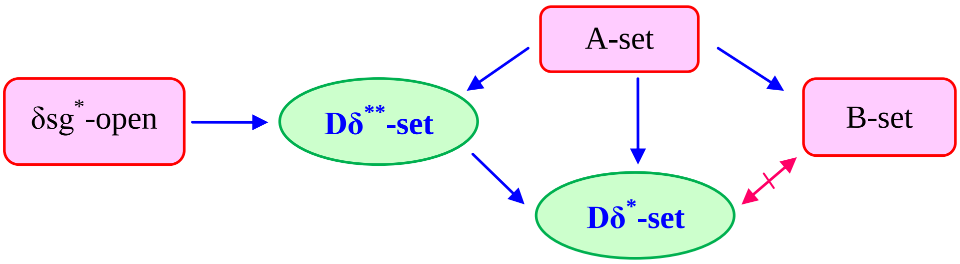
Remark 3.4 The union of two $D\delta^*$-sets need not be an $D\delta^*$-set as seen from the following example.

Example 3.5 Let X = {u,v,w}, τ = {X, ϕ, {u}}. The sets {v} and {w} are $D\delta^*$-sets in (X, τ) but their union {v,w} is not an $D\delta^*$-set in (X, τ).

Remark 3.6 The intersection of two $D\delta^{**}$-sets need not be an $D\delta^{**}$-set as seen from the following example.

Example 3.7 Let X = {u,v,w}, τ = {X, ϕ, {u}, {v}, {u,v}}. The sets {u,w} and {v,w} are $D\delta^{**}$-sets in (X, τ) but their intersection {w} is not an $D\delta^{**}$-set in (X, τ).

Remark 3.8 We have the following implications



None of the implications is reversible as seen from the following examples.

Example 3.9 Let X = {u,v,w}, τ = {X, ϕ, {u}, {u,v}, {u,w}}. Then the set {u,v} is an $D\delta^{**}$-set but not a δsg^*-open set in (X, τ).

Example 3.10 Let X = {u,v,w}, τ = {X, ϕ, {u}, {v}, {u,v}}. Then the set {w} is an $D\delta^*$-set but not an $D\delta^{**}$-set in (X, τ).

Example 3.11 Let X = {u,v,w}, τ = {X, ϕ, {u,v}}. Then the set {u} is an $D\delta^{**}$-set but not an A-set in (X, τ).

Example 3.12 Let X = {u,v,w}, τ = {X, ϕ, {u}}. Then the set {u,w} is an $D\delta^*$-set but not an A-set in (X, τ).

Example 3.13 Let X = {u,v,w}, τ = {X, ϕ, {u}, {u,v}}. Then the set {v,w} is a B-set but not a A-set in (X, τ).

Remark 3.14 The notions of $D\delta^*$-sets and B-sets are independent as seen from the following examples.

Example 3.15 Let X = {u,v,w}, τ = {X, ϕ, {u}}. Then the set {u,v} is an $D\delta^*$-set but not a B-set in (X, τ) and also that {v,w} is a B-set but not an $D\delta^*$-set in (X, τ).

Theorem 3.16 For a subset A of a topological space (X, τ), the following are equivalent.

(a) A is δ-semi closed
(b) A is an $D\delta^*$-set and δsg^*-closed.

Proof: (a) ⇒ (b) Obvious.

(b) ⇒ (a) Since A is an $D\delta^*$-set, then by Theorem 3.2, A = U∩δ-scl(A) where U is g-open in (X, τ). So A ⊆ U and since A is δsg^*-closed, then δ-scl(A) ⊆ U. Therefore δ-scl(A) ⊆ U∩δ-scl(A) = A. Hence A is δsg^*-closed.

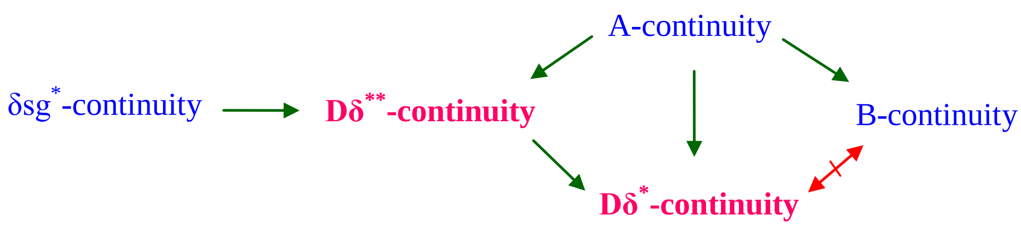
IV. $D\delta^*$- CONTINUITY AND $D\delta^{**}$- CONTINUITY

Definition 4.1 A function f : (X, τ) → (Y, σ) is said to be **$D\delta^*$-continuous** (resp. **$D\delta^{**}$-continuous**) if $f^{-1}(V)$ is an $D\delta^*$-set (resp. $D\delta^{**}$-set) in (X, τ) for every open subset V of (Y, σ).

Definition 4.2 A function f : (X, τ) → (Y, σ) is said to be **$D^*\delta^*$-continuous** if $f^{-1}(V)$ is an $D\delta^*$-set in (X, τ) for every closed subset V of (Y, σ).

Remark 4.3 It is clear that, a function f : (X, τ) → (Y, σ) is δ-semi continuous if and only if $f^{-1}(V)$ is a δ-semi closed set in (X, τ) for every closed subset V of (Y, σ).

Remarks 4.4 We have the following diagram



Example 4.5 Let X = Y = {u,v,w} with τ = {X, ϕ, {u}, {u,v}, {u,w}} and σ = {Y, ϕ, {u}, {u,v}}. Let f : (X, τ) → (Y, σ) be the identity map. Then f is $D\delta^{**}$-continuous but not δsg^*-continuous, since for the open set {u,v} in (Y, σ), f^{-1}({u,v}) = {u,v} is an $D\delta^{**}$-set but not δsg^*-open in (X, τ).

Example 4.6 Let X = Y = {u,v,w} with τ = {X, ϕ, {u}, {u,v}} and σ = {Y, ϕ, {u}, {v}, {u,v}}. Let f : (X, τ) → (Y, σ) be the identity map. Then f is $D\delta^{**}$-continuous but not A-continuous, since for the open set {v} in (Y, σ), f^{-1}({v}) = {v} is an $D\delta^{**}$-set but not A-set in (X, τ).

Example 4.7 Let X = Y = {u,v,w} with τ = {X, ϕ, {u,v}} and σ = {Y, ϕ, {u}, {v}, {u,v}}. Let f : (X, τ) → (Y, σ) be the identity map. Then f is $D\delta^*$-continuous but not A-continuous, since for the open set {u} in (Y, σ), f^{-1}({u}) = {u} is an $D\delta^*$-set but not an A-set in (X, τ).

Example 4.8 Let X = Y = {u,v,w} with τ = {X, ϕ, {u}} and σ = {Y, ϕ, {u}, {v}, {u,v}, {u,w}}. Let f : (X, τ) → (Y, σ) be the identity map. Then f is $D\delta^*$-continuous but not $D\delta^{**}$-continuous, since for the open set {u,w} in (Y, σ), f^{-1}({u,w}) = {u,w} is an $D\delta^*$-set but not $D\delta^{**}$-set in (X, τ).

Example 4.9 Let X = Y = {u,v,w} with τ = {X, ϕ, {u}, {u,v}} and σ = {Y, ϕ, {u}, {v,w}}. Let f : (X, τ) → (Y, σ) be the identity map. Then f is B-continuous but not an A-continuous, since for the open set {v,w} in (Y, σ), f^{-1}({v,w}) = {v,w} is a B-set but not an A-set in (X, τ).

Remark 4.10 The following examples shows the concepts of $D\delta^*$-continuity and B-continuity are independent as seen from the following examples.

Example 4.11 Let X = Y = {u,v,w} with τ = {X, ϕ, {u}} and σ = {Y, ϕ, {u,v}}. Let f : (X, τ) → (Y, σ) be the identity map. Then f is $D\delta^*$ -continuous but not a B-continuous, since for the open set {u,v} in (Y, σ), f^{-1}({u,v}) = {u,v} is an $D\delta^*$-set vut not a B-set in (X, τ).

Example 4.12 Let X = Y = {u,v,w} with τ = {X, ϕ, {u}, {u,v}} and σ = {Y, ϕ, {u}, {v,w}}. Let f : (X, τ) → (Y, σ) be the identity map. Then f is B-continuous but not a $D\delta^*$-continuous, since for the open set {v,w} in (Y, σ), f^{-1}({v,w}) = {v,w} is a B-set but not an $D\delta^*$-set in (X, τ).

The following theorem exhibits a new decomposition for δ-semi continuous functions.

Theorem 4.13 For a function f : (X, τ) → (Y, σ), the following are equivalent.

(a) f is δ-semi continuous
(b) f is $D^*\delta^*$-continuous and δsg^*-continuous.

Proof: The proof follows from Definitions 4.2 and 2.5 (e), Remark 4.3 and Theorem 3.13.

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