A Systematic Survey on Compressed Sensing: Signal Acquisition And Reconstruction Schemes And Applications

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Abstract* — *Compressed sensing is an emerging technique where the signal is compressed at the time of acquisition. The compressed signals can be represented in time domain or transform domain. This technique basically implements efficient acquisition and reconstruction of a signal from fewer no. of measurements .CS exploits sparsity where many coefficients of the signal of interest expressed in some domain are equated to zero. With the help of basis, frames and dictionaries a signal can be expressed in sparse form. CS enables sampling at a rate which is much lesser than Nyquist sampling rate and signal can be recovered from compressed measurements.

This paper deals with the detailed review of existing literatures in effective acquisition, reconstruction of the signal, techniques to solve inverse scattering problem and various applications of CS in several fields such as biomedical applications, communication systems, video processing and so on.

Index Terms — *Compressed Sensing, Sparsity ,CS acquisition techniques, Recovery algorithms ,Inverse Scattering problem,CS applications*

INTRODUCTION

According to Shanon's theorem , any signal can be reconstructed from its samples provided it is sampled at a rate greater than or equal to two times the maximum frequency of the signal. In the conventional method of sampling more no. of samples are required and most of them are discarded at the time of recovery. CS provides a simplified methodology by taking fewer no. of measurements, implementing compression of signal by finding out all the samples' coefficients, taking only the higher coefficients into consideration rejecting the lower coefficients for storage and transmission. Compressed sensing implements reconstruction of a compressed version of signal by taking only few amount of linear and non-adaptive measurements.

The most important aspect of CS technique is use of efficient algorithms for reconstruction of under sampled signal. CS deploys an underdetermined system of measurements having infinitely many solutions which focused on Non-deterministic Polynomial-time hard(NP-hard) problem. Proper measurement matrices are to be taken into consideration.

I. SIGNAL ACQUISITION AND RECONSTRUCTION MODEL

Few non-adaptive random measurements are taken in compressed sensing schemes. Acquisition model of Compressed sensing comprises of the input signal $x \in R^n$ of length n ,$\varphi \in R^{mxn}$ is an mxn measurement matrix and $y \in R^m$ is a measurement vector having length m. The compressive measurements are found out by multiplying the input signal with the random measurement matrix. The no. of measurements taken here is less than the length of the signal .i.e m<<n.

$$y=\varphi x \quad (1)$$

The measurement vector y and reconstruction matrix i..e $A=.\varphi\psi \in R^{mxn}$ where ψ is the sparse basis function of the signal x are taken as inputs to the reconstruction model. The signal x can be expressed as

$$x=\psi s \quad (2)$$

where $s \in R^n$ is a sparse vector of length n,having lesser no. of non-zero entries. The signal of interest can be reconstructed by solving equation (1) which is an undetermined system of linear equations that leads to infinite no. of possible solutions. We can attain an exclusive solution by taking ℓ_0 optimization problem wherein all possible combinations can be tried for getting solution which is very tedious. Various types signal recovery algorithms implementing $\ell 1$ norms and other relevant norms are discussed in this paper so as to get an estimate of sparse representation of x [1]. For perfect reconstruction of signal, restricted isometric propery(RIP) and

incoherence property should be satisfied.

II. MEASUREMENT MATRICES IN COMPRESSED SENSING

An appropriate measurement matrix φ should be selected for successful implementation of CS. The most commonly used random matrices used in CS are Gaussian or Bernoulli, partial Fourier matrics and so on. Though the probability of reconstruction is high so far as usage of random matrices is concerned, they too have demerits. Lot of storage will be needed in case such matrices are used.

There is no such effective algorithm where RIP condition can be verified for these matrices. Deterministic matrices satisfy RIP as well as coherence properties. The advantages of deterministic matrices include less storage requirement, simple sampling and recovery processes. For an accurate and efficient signal recovery ,deterministic matrices can be used provided some a priori information about location of non-zero elements are known. The no. of measurements required for several measurement matrices for perfect recovery is given in the table1

where k is the sparsity of vector s, μ is the relation of coherence between any two elements in a given pair of matrices φ and ψ

,m is the no. of measurements ,n is the length of the input signal

and c is a positive constant.[2]

TABLE 1 No. of Measurements required for different types of Matrices

Type of Matrix	No. of measurements required
Gaussian and Bernouli	m>=ck log n/k
Paartial Fourier	m>=c μk (log n)^4
Any other matrix	m= O(k log n)
Deterministic	m= O(k^{2}log n)

III. CS ACQUISITION TECHNIQUES

The main requirement of CS is proper recovery of signal. The measurements must be taken randomly. To meet this requirement, different techniques have been proposed. This section discusses the operating principles of these acquisition techniques.

A. RANDOM DEMODULATOR

Random demodulator otherwise known as analog to information converter (AIC), is an efficient wideband signal sampler. Here the input signal is first multiplied with a chipping sequence(pseudorandom code). Then the signal is convolved in frequency domain and the signal frequency is spread to low frequency regions. Then an integrator acting as low pass filter is implemented to attain a unique frequency signature of signal in low frequency region .The original signal information is carried with the help of frequency signature which in turn helps in recovering the original signal from compressed measurements. (1)

B. RANDOM FILTERING

This technique convolves the input signal with the finite impulse response filter h. The filtered signal can be taken into consideration for getting compressive measurements.

C. RANDOM CONVOLUTION

In this approach, the measurement matrix's first row is filled with random values. Then the next row is obtained by performing circular shift operation of the previous row. This process is repeated for the rest of the rows untill the measurement matrix is formed. The measurement vector Y is generated by convolving the measurement matrix and the i/p signal. The matrix formed his a structured matrix. The benefits of using such matrix is faster procurement, easy to store and communication.[3]

IV. CS RECONSTRUCTION STRATEGIES

CS reconstruction algorithms basically deal with sparse representation of original input signal from compressive measurements, represented in some appropriate basis function or dictionary[4]. In this section some of the reconstruction approaches are discussed.

A. CONVEX OPTIMIZATION TECHNIQUE

This method treats the compressed sensing based recovery strategy as convex optimization problem that is going to be solved by implementing solver with the help of linear programming. The convex designs are applied to attain sparsity of the signal of interest.

1) BASIC PURSUIT

Basic pursuit is one of the convex optimization techniques, which is possibly solved using minimum l_1 norm ,

$$\hat{S} = \arg\min \|s\|_1 \quad \text{subject to } \Theta s = y \qquad (3)$$

Basic pursuit algorithm is used in compressed sensing to obtain the sparse estimate of input signal x in dictionary or matrix Θ from minimum no.of measurements of y. BP is implemented for recovery of the signal if the compressed measurements are noise-free.

2) BASIS PURSUIT DENOISING (BPDN)

BPDN accounts for the noise in dimension from the solution having minimum l_1 norm provided relaxed condition on constraint is satisfied.

$$\hat{S} = \arg\min_{s} \|s\|_1 \text{ subject to } 1/2(\|y-\Theta s|_2^2 \leq \epsilon \quad (4)$$

where l_2 is known as Euclidian norm ,which represents the length of a vector [6]

3) SOLVERS FOR CONVEX OPTIMIZATION PROBLEM

Optimization problems can be resolved with the help of solvers.

The solution to Basic Pursuit problem can be obtained by using BP-simplex, interior point algorithm .In simplex algorithm, all probable solutions can be attained by constructing a polyhedron. Fixed point continuation(FPC) ,gradient projection for sparse representation (GPSR),Bregman iteration and so on [7] can also be used for convex optimization approach.

In BP simplex algorithm, a set of n columns which are linearly independent is chosen from dictionary. Then a column in basis is interchanged with a column not in basis that provides considerable improvement in objective function. The steps are repeated until no further improvement is possible. Finally the best possible solution is obtained .

In BP interior algorithm, an initial non-sparse solution solution is found out. Then sparsity is transformed and the solution is moved into the simplex zone.This procedure is repeated until a solution of sufficient no. of non-zero entries is reached. This sort of result is known as vertex simplex.[8]

Both FPC and GPSR deal with the solution to the unconstrained formulation of l_1 minimization problem.

B. GREEDY APPROACH

Greedy approach is an iteration method.

In every step, the solution is upgraded by choosing those columns of the reconstruction matrix which are highly corelated with the compressed measurements. These columns are called atoms. Atoms selected once are not included in further iterative steps. This method minimizes the computational complication of the algorithm and increases the execution speed[9],[10]. The following algorithms are the two types of the Greedy approach algorithms.

1) SERIAL GREEDY ALGORITHMS

Matching pursuit(MP),orthogonal matching pursuit(OMP) and gradient Pursuit(GP) are the examples of serial greedy algorithms. The fundamental steps of these algorithms are shown below.

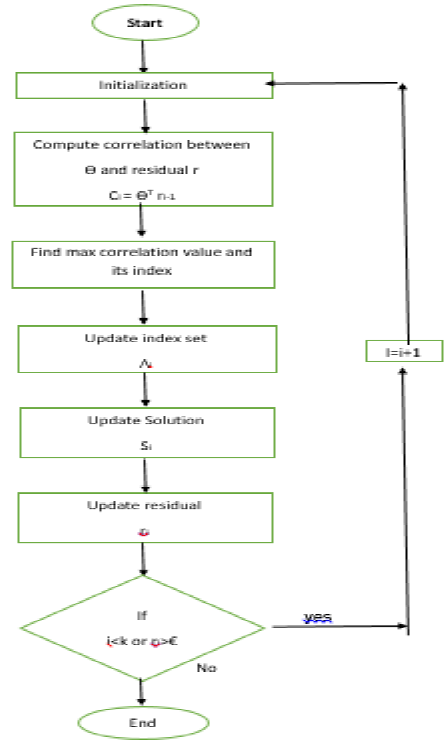

Fig 1. Flow chart for serial greedy algorithms

2) PARALLEL GREEDY ALGORITHMS

Compressive sampling matching pursuit (CoSaMP) and subspace pursuit(SP) can be categorized as parallel greedy algorithms. k atoms or multiple of k atoms are chosen at the same time from the reconstruction/recovery matrix .Hence they are named as parallel greedy algorithms. The residual steps are similar to serial greedy algorithms. These algorithms are more accurate than serial algorithms. The wrong atoms can be dropped if at all selected during iterations.

C. THRESHOLDING APPROACH

In this approach, k atoms of reconstruction matrix are chosen at same time. Here thresholding technique is used to upgrade the solution set Si .The remaining steps are similar to greedy algorithms. Approximate Message Passing(AMP) deploys this approach. The steps followed in this technique are shown below.

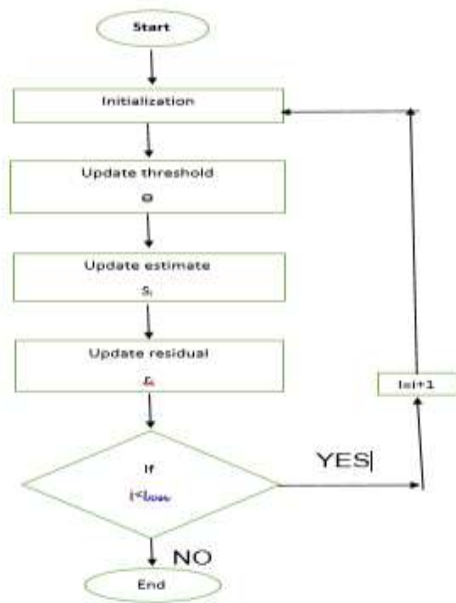

Figure 2.Flowchart for AMP algorithm

D. COMBINATORIAL APPROACH

Random Fourier Sampling, chaining pursuits and sparse sequential pursuit algorithms use this technique. A specific measurement pattern is generated in such type of approach.. The measurement matrix ϕ is constructed with the help of certain discrete valued functions.Each measurement y_i is generated by the combination of equal no. of samples of the given i/p signal [11]. The steps of this algorithm are described below.

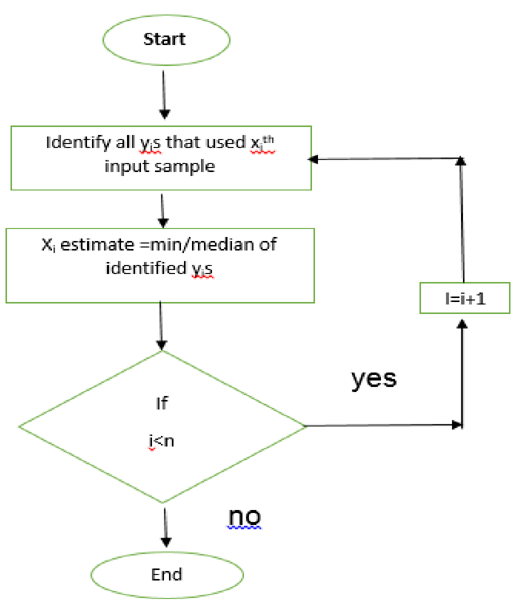

Figure 3.Flowchart for steps involved in combinatorial approach

E. NON-CONVEX APPROACH

This approach uses lesser no. of measurements than convex optimization technique. Here instead of l_1 norm ,l_p norm is used where $0<p<1$. [12].

F. BAYESIAN APPROACH

This method is meant for deterministic input signals. Maximum likelihood estimate or maximum a posteriori estimate is used to find out input signal coefficients. Reconstruction error is not taken into consideration here.

Table 2 A brief Comparison of CS recovery techniques

Approach	**Characteristics**	**Benefits**	**Drawbacks**
Convex	**Minimization of l_1 norm to obtain a solution**	**Noise Robustness**	**Slow,complex**
	Correlation based iteration method	**Faster,less complex,prone to noise**	**Prior information of signal sparsity is required Convergence issues**
Thresholding	**Exploits thresholding criteria to select atoms**	**Able to add/remove multiple entries per iteration**	**Adaptive step size is taken for performance improvement.**
Combinatorial	**Computes min or median of measurements**	**Faster and simpler**	**Needs a specific pattern in measurement**
Non-Convex	**Minimization of l_p norm to obtain a solution**	**Recovers from fewer measurements than l_1 counterpart**	**Slower, complex**
Bayesian	**Used for recovery of signals with known probability distribution**	**Faster and provides more sparser solution**	**High computational cost**

V.POTENTIAL AREAS OF APLLICATIONS OF COMPRESSED SENSING

CS field is growing in leaps and bounds and is implemented in various areas. Some of the major areas where hhhhCS can be deployed are discussed below.

A. IMAGE PROCESSING USING COMPRESSED SENSING

CS techniques can be used in image acquisition. It can be used in single pixel cameras and radar imaging systems. CS can also be applied in parallel imaging, microwave imaging and under water imaging too [14] [15].

B. BIOMEDICAL APPLICATIONS

CS can be applied in the field of biomedical imaging. CS theory
can also be implemented in processing biological signals like ECG, EEG ,ENG and so on by exploiting sparsity. This technique can also be applied in DNA micro arrays ,study of proteins and so on [16],[17] .

C. COMMUNICATION SYSTEMS

CS theory has wide range of applications in communication systems. Data acquisition for wireless sensor network is done by implementing data compressibility. In wireless body area networks ,tele health monitoring system, compressed sensing is used . With respect to IoT, CS can also be implemented in IoT [18].

CS can be applied in antenna array so that no. of array elements can be minimized and background noise/interference can be reduced.

D. PATTERN RECOGNITION

CS can be utilized in face recognition and speech recognition techniques from missing data by exploiting sparsity [19].

E. VIDEO PROCESSING

Compressive sensing techniques can be implemented for 3D video acquisition and processing using distributed video sensing , adaptive video sensing and so on. [20].

F. SPEECH PROCESSING

CS has wide range of applications in speech processing. This technique can be deployed in differentiating voiced and unvoiced speeches. CS can be implemented in speech enhancement, ocean sound monitoring and so on[21].

G. VLSI APPLICATIONS

In VLSI domain also CS techniques can be deployed. Nano scale ICs can be modelled and designed exploiting sparsity. CS can also be implemented in low cost silicon nano scale integrated circuits and so on[22].

VI. CONCLUSION

Exploitation of CS has transformed many zones in signal processing. Some of the major applications include improved MRI , superior quality image and video procurement with the usage of single pixel camera ,acquiring ultra wideband signals etc . In CS based sparsity technique, a signal can be simultaneously sampled and compressed. In this paper, a organized review of compressed sensing techniques and its applications are discussed. Various acquisition and recovery schemes based on compressed sensing are also conferred in this paper. Many CS techniques deploy the most suitable sensing matrix or sparse dictionary. However,CS is relatively a new technique which can further be improved by optimizing the reconstruction quality.

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